Function Operations

To combine two functions, $f(x)$ and $g(x)$, add or subtract as follows:

Sum of Functions	Difference of Functions
$h(x) = f(x) + g(x)$	$h(x) = f(x) - g(x)$
$h(x) = (f + g)(x)$	$h(x) = (f - g)(x)$
↑ adding	↑ subtracting

polynomial functions:

- $y = 3x + 5$ D: $\{x \mid x \in R\}$
- $y = x^2 - x - 6$ D: $\{x \mid x \in R\}$
- $y = x^3 + 7x^2 - 5x + 6$ D: $\{x \mid x \in R\}$
- $y = x^4 - 2x^2 - 1$ D: $\{x \mid x \in R\}$

Radical Function:

- $f(x) = \sqrt{x+3}$ $x + 3 \geq 0$, $x \geq -3$ D: $\{x \mid x \geq -3, x \in R\}$
- $y = \sqrt{2x-5}$ $2x - 5 \geq 0$, $2x \geq 5$, $x \geq \frac{5}{2}$ D: $\{x \mid x \geq \frac{5}{2}, x \in R\}$
- $g(x) = \sqrt{4-x}$ $4 - x \geq 0$, $-x \geq -4$, $x \leq 4$ D: $\{x \mid x \leq 4, x \in R\}$

  $4 - x \geq 0$

  $4 \geq x$

exponential function:

- $y = 3^x$ D: $\{x \mid x \in R\}$
- $y = 3(2)^{x+5} - 1$ D: $\{x \mid x \in R\}$

logarithmic function:

$y = \log_3 x$ D: $\{x \mid x > 0, x \in R\}$

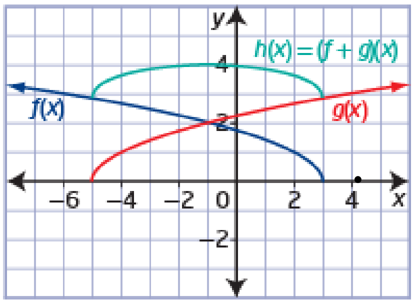
Rational function

$y = \frac{x+1}{x-3}$ $x - 3 \neq 0$, $x \neq 3$ D: $\{x \mid x \neq 3, x \in R\}$

Key Ideas

- You can add two functions, $f(x)$ and $g(x)$, to form the combined function $h(x) = (f + g)(x)$.
- You can subtract two functions, $f(x)$ and $g(x)$, to form the combined function $h(x) = (f - g)(x)$.
- The domain of the combined function formed by the sum or difference of two functions is the domain common to the individual functions. For example,

  Domain of $f(x)$: $\{x \mid x \leq 3, x \in \mathrm{R}\}$
  Domain of $g(x)$: $\{x \mid x \geq -5, x \in \mathrm{R}\}$
  Domain of $h(x)$: $\{x \mid -5 \leq x \leq 3, x \in \mathrm{R}\}$



- The range of a combined function can be determined using its graph.
- To sketch the graph of a sum or difference of two functions given their graphs, add or subtract the y-coordinates at each point.

Example 1

Determine the Sum of Two Functions

Consider the functions $f(x) = 2x + 1$ and $g(x) = x^2$.

a) Determine the equation of the function $h(x) = (f + g)(x)$. (Add)

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

c) State the domain and range of $h(x)$.

d) Determine the values of $f(x)$, $g(x)$, and $h(x)$ when $x = 4$.

a) $h(x) = (f+g)(x)$

$h(x) = f(x) + g(x)$

$h(x) = 2x+1 + x^2$

$h(x) = x^2+2x+1$

Domain:

$f(x) = 2x+1$

$\{x \mid x \in R\}$

$(-\infty, \infty)$

Domain

$g(x) = x^2$

$\{x \mid x \in R\}$

$(-\infty, \infty)$

b) $f(x) = 2x+1$

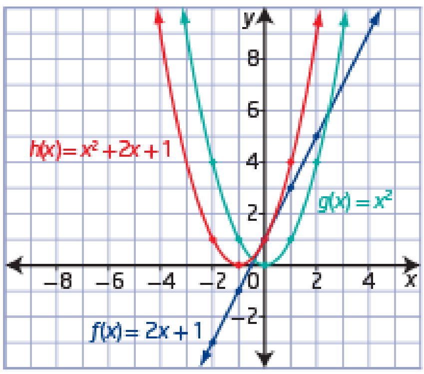
x	y
-2	-3
-1	-1
0	1
1	3
2	5

$g(x) = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$h(x) = x^2+2x+1$

x	y
-2	1
-1	0
0	1
1	4
2	9



How are the y-coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

c) $h(x) = x^2+2x+1$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

d) When $x = 4$

$f(4) = 2(4)+1$

$f(4) = 9$

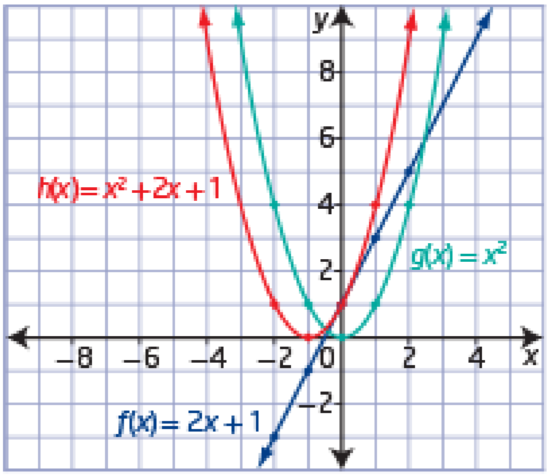
$g(4) = (4)^2$

$g(4) = 16$

$h(4) = (4)^2+2(4)+1$

$h(4) = 25$

or $9+16 = 25$



How are the *y*-coordinates of points on the graph of *h*(*x*) related to those on the graphs of *f*(*x*) and *g*(*x*)?

c) The function $f(x) = 2x + 1$ has domain $\{x \mid x \in R\}$.
The function $g(x) = x^2$ has domain $\{x \mid x \in R\}$.
The function $h(x) = (f + g)(x)$ has domain $\{x \mid x \in R\}$, which consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$.
The range of $h(x)$ is $\{y \mid y \geq 0, y \in R\}$.

d) Substitute $x = 4$ into $f(x)$, $g(x)$, and $h(x)$.

$f(x) = 2x + 1$	$g(x) = x^2$	$h(x) = x^2 + 2x + 1$
$f(4) = 2(4) + 1$	$g(4) = 4^2$	$h(4) = 4^2 + 2(4) +$
$f(4) = 8 + 1$	$g(4) = 16$	$h(4) = 16 + 8 + 1$
$f(4) = 9$		$h(4) = 25$

Example 2

Determine the Difference of Two Functions

Consider the functions $f(x) = \sqrt{x - 1}$ and $g(x) = x - 2$.

a) Determine the equation of the function $h(x) = (f - g)(x)$. (subtract)

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

c) State the domain of $h(x)$.

d) Use the graph to approximate the range of $h(x)$.

a) $h(x) = (f - g)(x)$

$h(x) = f(x) - g(x)$

$h(x) = \sqrt{x-1} - (x-2)$

$h(x) = \sqrt{x-1} - x + 2$

Domain:

$f(x) = \sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

$\{x \mid x \geq 1, x \in R\}$

Domain:

$g(x) = x - 2$

$\{x \mid x \in R\}$

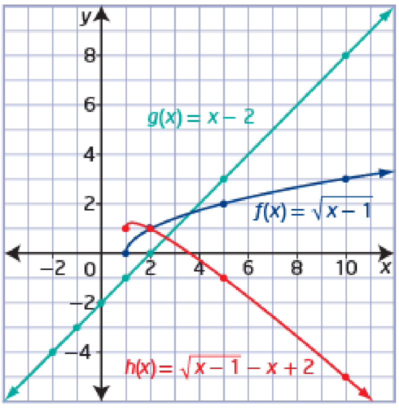
b) **Method 1: Use Paper and Pencil**

For the function $f(x) = \sqrt{x - 1}$, the value of the radicand must be greater than or equal to zero: $x - 1 \geq 0$ or $x \geq 1$.

x	$f(x) = \sqrt{x - 1}$	$g(x) = x - 2$	$h(x) = \sqrt{x - 1} - x + 2$
−2	undefined	−4	undefined
−1	undefined	−3	undefined
0	undefined	−2	undefined
1	0	−1	1
2	1	0	1
5	2	3	−1
10	3	8	−5

Why is the function $h(x)$ undefined when $x < 1$?

How could you use the values in the columns for $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$?



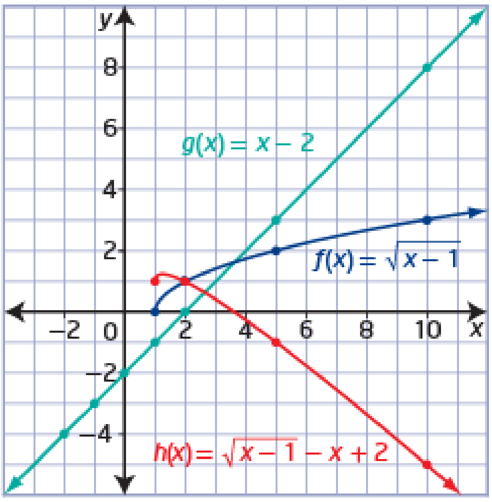
How could you use the y-coordinates of points on the graphs of $f(x)$ and $g(x)$ to create the graph of $h(x)$?

c) $h(x) = \sqrt{x-1} - x + 2$

D: $\{x \mid x \geq 1, x \in R\}$ or $[1, \infty)$

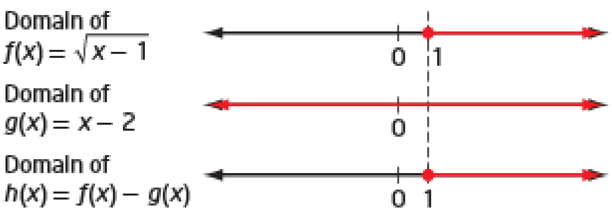
d) $h(x) = \sqrt{x-1} - x + 2$

R: $\{y \mid y \leq 1.2, y \in R\}$ or $(-\infty, 1.2]$



How could you use the *y*-coordinates of points on the graphs of *f*(*x*) and *g*(*x*) to create the graph of *h*(*x*)?

c) The function $f(x) = \sqrt{x-1}$ has domain $\{x \mid x \geq 1, x \in R\}$.
The function $g(x) = x - 2$ has domain $\{x \mid x \in R\}$.
The function $h(x) = (f - g)(x)$ has domain $\{x \mid x \geq 1, x \in R\}$, which consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$.



What values of *x* belong to the domains of both *f*(*x*) and *g*(*x*)?

d) From the graph, the range of $h(x)$ appears to be approximately $\{y \mid y \leq 1.2, y \in R\}$.

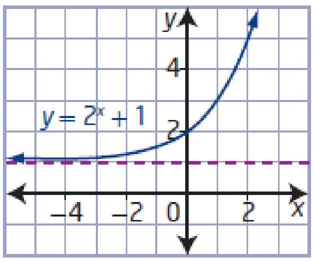
How can you use a graphing calculator to verify the range?

Homework

finish #1-11 on page 483-484

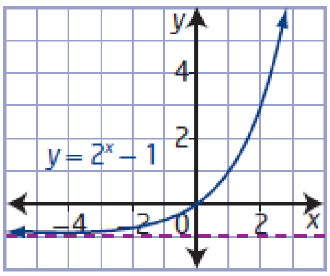
10.1 Sums and Differences of Functions, pages 483 to 487

1. a) $h(x) = |x - 3| + 4$ b) $h(x) = 2x - 3$
   c) $h(x) = 2x^2 + 3x + 2$ d) $h(x) = x^2 + 5x + 4$
2. a) $h(x) = 5x + 2$ b) $h(x) = -3x^2 - 4x + 9$
   c) $h(x) = -x^2 - 3x + 12$ d) $h(x) = \cos x - 4$
3. a) $h(x) = x^2 - 6x + 1$; $h(2) = -7$
   b) $m(x) = -x^2 - 6x + 1$; $m(1) = -6$
   c) $p(x) = x^2 + 6x - 1$; $p(1) = 6$
4. a) $y = 3x^2 + 2 + \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in R\}$
   b) $y = 4x - 2 - \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in R\}$
   c) $y = \sqrt{x + 4} - 4x + 2$; domain $\{x \mid x \geq -4, x \in R\}$
   d) $y = 3x^2 + 4x$; domain $\{x \mid x \in R\}$
5. a)



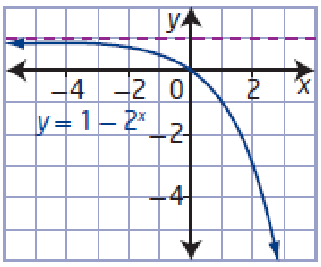
domain $\{x \mid x \in R\}$, range $\{y \mid y > 1, y \in R\}$

b)



domain $\{x \mid x \in R\}$, range $\{y \mid y > -1, y \in R\}$

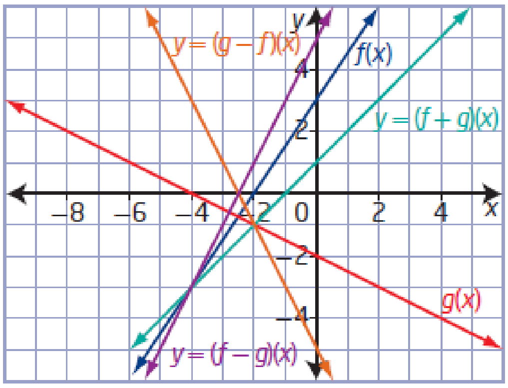
c)



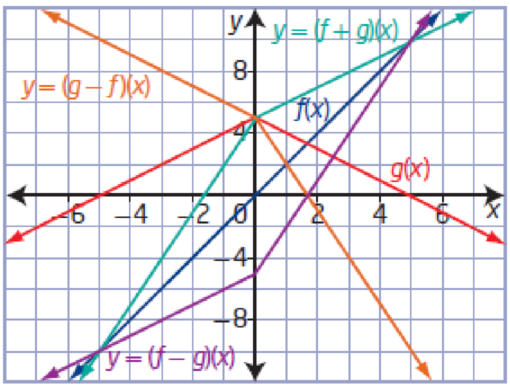
domain $\{x \mid x \in R\}$, range $\{y \mid y < 1, y \in R\}$

6. a) 8 b) 6 c) 7
   d) not in the domain
7. a) B b) C c) A

8. a)



b)



9. a) $y = 3x^2 + 11x + 1$ **b)** $y = 3x^2 - 3x + 3$

c) $y = 3x^2 + 3x + 1$ **d)** $y = 3x^2 - 11x + 3$

10. a) $g(x) = x^2$ **b)** $g(x) = \sqrt{x + 7}$

c) $g(x) = -3x + 1$ **d)** $g(x) = 3x^2 - x - 4$

11. a) $g(x) = x^2 - 1$ **b)** $g(x) = -\sqrt{x - 4}$

c) $g(x) = 8x - 9$ **d)** $g(x) = 2x^2 - 11x - 6$