8.3 Compound Interest: Future Value

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

GOAL

Determine the future value of an investment that earns compound interest.

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COMPOUND Interest

Interest is added to the principal periodically throughout the year. New interest may be paid on the principal plus the interest. The interest rate is stated per annum and is divided by the number of **compounding periods**.

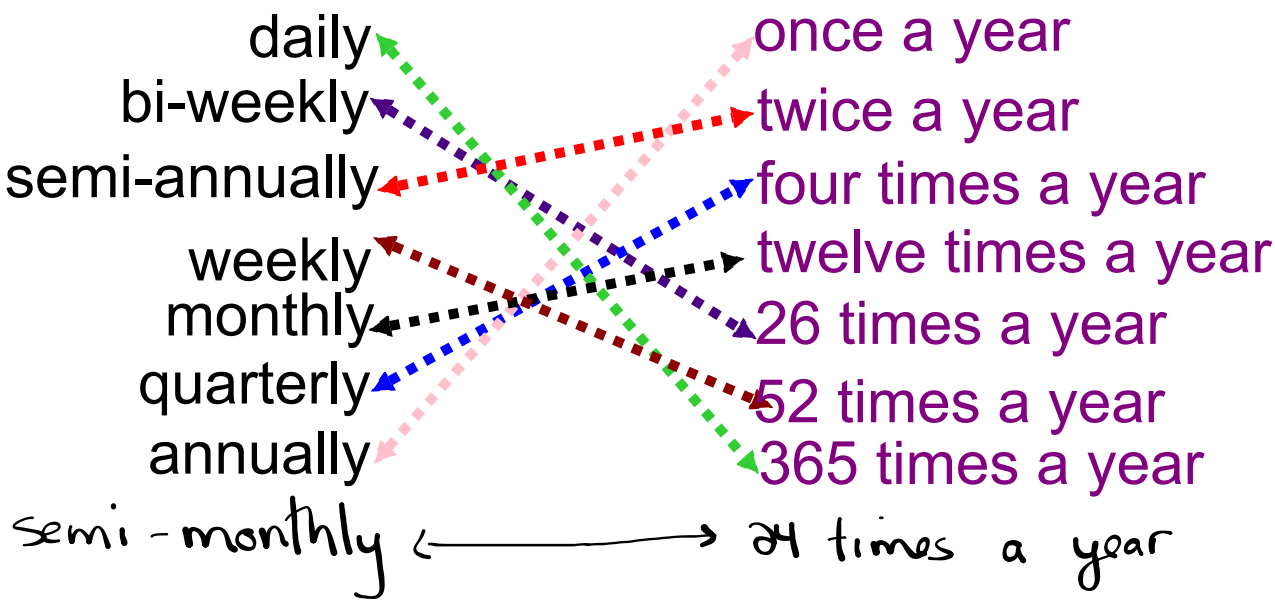
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$I = A - P$$

A = final value of the investment ...(principal + interest)
P = principal
r = annual interest rate
n = number of compounding periods in a year
t = term of the investment or loan in number of years

Terminology Tango

Click on the picture to verify the match.



EXAMPLE #1: If $1000 is invested at 8 %/a compounded semi-annually for 2 years, how much will the investment be worth?

Using the simple interest formula...

I = 1000(0.08)(6/12)
= $40 (after 1st interest period)

New principal = 1000 + 40
= $1040

I = 1040(0.08)(6/12)
= $41.60 (after 2nd interest period)

New Principal = 1040 + 41.60
= $1081.60

I = 1081.60(0.08)(6/12)
= $43.26 (after 3rd interest period)

New Principal = 1081.60 + 43.26
= $1124.86

I = 1124.86(0.08)(6/12)
= $44.99 (after 4th interest period)

New Principal = 1124.86 + 44.99
= $1169.85

Compound Interest Formula...

Given:
P = 1000
r = 8%
r = 0.08
n = 2
t = 2
A = ?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 1000\left(1 + \frac{0.08}{2}\right)^{(2)(2)}$$

$$A = 1000\left(1 + 0.04\right)^{4}$$

$$A = 1000\left(1.04\right)^{4}$$ y^x

$$A = 1000(1.1699)$$ xy $\wedge$

$$A = \$1169.86$$

EXAMPLE #2:

Calculate the final value of an initial investment of $6000.00. Interest is paid at 4% per annum, compounded semi-annually, for three years.

A = final value of the investment ...(principal + interest)
P = principal
r = annual interest rate
n = number of compounding periods in a year
t = term of the investment or loan in number of years

$P = 6000$

$r = 4\%$

$r = 0.04$

$n = 2$

$t = 3$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 6000\left(1 + \frac{0.04}{2}\right)^{(2)(3)}$$

$$A = 6000(1.02)^6$$

$$A = 6000(1.1262)$$

$$A = \$6756.97$$

EX #3: Maggie invests $30 000 at 10% /a compounded quarterly for 20 years. Determine...

a) How much will this investment be worth?

b) How much interest did you earn?

a) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 30000\left(1+\frac{0.10}{4}\right)^{(4)(20)}$

$A = 30000(1.025)^{80}$

$A \doteq \$216287.03$

b) $I = A - P$

$I = 216287.03 - 30000$

$I = \$186287.03$

EXAMPLE #4...

A keen JMH student wants to save some money from their summer employment. They decide to take out a Canada Savings Bond which pays 2.5 % interest per year compounded monthly. If the student invests $850 into the bond, how much interest will they earn if they don't touch the money for 3 years?

Given:

$P = 850$

$r = 2.5\%$

$r = 0.025$

$n = 12$

$t = 3$

$I = ?$

① Find A

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 850\left(1 + \frac{0.025}{12}\right)^{(12)(3)}$$

$$A = 850(1.002083333)^{36}$$

$$A = \$916.13$$

② Find I:

$$I = A - P$$

$$I = 916.13 - 850$$

$$I = \$66.13$$

HOMEWORK...

p. 457: #1, 2

p. 468: #2, 6, 7

Simple

$I = Prt$

&

$A = P + I$

$A = P + Prt$

$A = P(1 + rt)$

Compound

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$I = A - P$

Solutions

p.457

1. Determine the difference in the interest earned at maturity on these two investments. Explain any discrepancies.

- Eve earned $3000 in a GIC for a term of 5 years with a simple interest rate of 4%, paid annually.
- Larry invested $3000 in a GIC for a term of 5 years with a compound interest rate of 4%, paid annually.

Eve

$$A = P(1+rt)$$
$$= \$3000[1+(0.04)(5)]$$
$$= \$3000(1+0.2)$$
$$= \$3000(1.2)$$
$$= \$3600$$

Larry

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= \$3000\left(1+\frac{0.04}{1}\right)^{(1)(5)}$$
$$= \$3000(1.04)^5$$
$$= \$3649.96$$

Difference $\Rightarrow$ $\$3649.96 - \$3600$
$= \$49.96$

2. Sydney wants to open a savings account. He has $6500 to deposit. He intends to keep the account for 4 years and then use the money to rebuild the engine of his car. Which account should he choose? Justify your choice.

A. 5.1% simple interest, paid weekly.
B. 4.8% compound interest, paid annually.

Account A

$A = P(1+rt)$
$= \$6500[1+(0.051)(4)]$
$= \$6500(1+0.204)$
$= \$6500(1.204)$
$= \$7826$

Account B

$A = P\left(1+\frac{r}{n}\right)^{nt}$
$= \$6500\left(1+\frac{0.048}{1}\right)^{(1)(4)}$
$= \$6500(1.048)^4$
$= \$7840.77$

Sydney should choose Account B.

p.468

2. Determine the future value and the total interest earned for each investment.

a) $520 invested for 8 years at 4.5% compounded monthly.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= \$520\left(1 + \frac{0.045}{12}\right)^{(12)(8)}$$

$$= \$744.83$$

$$I = A - P$$

$$= \$744.83 - \$520$$

$$= \$224.83$$

b) $1400 invested for 15 years at 8.6% compounded semi-annually.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= \$1400\left(1 + \frac{0.086}{2}\right)^{(2)(15)}$$

$$= \$4950.59$$

$$I = A - P$$

$$= \$4950.59 - \$1400$$

$$= \$3550.59$$

6. Trust funds are investments that are set up for a specific purpose. A local business invested $250 000 in a charitable trust fund so that a school can offer schlorships. The interest rate is 3.8%, compounded semi-annually. Only the interest earned can be used to provide the scholarships. How much is available from the trust fund for scholarships each year?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= \$250\,000\left(1 + \frac{0.038}{2}\right)^{(2)(1)}$$

$$= \$259\,590.25$$

$$I = A - P$$

$$= \$259\,590.25 - \$250\,000$$

$$= \$9590.25$$

There will be $9590.25 available for scholarships each year.

7. Suppose that you are searching online for the best interest rates on a GIC. You find these rates:

- Bank A offers 6.6%, compounded annually.
- Bank B offers 6.55%, compounded semi-annually.
- Bank C offers 6.5%, compounded quarterly.

Rank these rates from greatest to least return on an investment of $20 000 for a term of 2 years.

Bank A

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= \$20000\left(1 + \frac{0.066}{1}\right)^{(1)(2)}$$
$$= \$22\,727.12$$

Bank B

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= \$20\,000\left(1 + \frac{0.0655}{2}\right)^{(2)(2)}$$
$$= \$22\,751.54$$

Bank C

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= \$20\,000\left(1 + \frac{0.065}{4}\right)^{(4)(2)}$$
$$= \$22\,752.78$$

1st Bank C, 2nd Bank B, 3rd Bank A

More Practice With Compound Interest...

Worksheet - Introduction to Compound Interest.doc

SOLUTIONS

Foundations of Math 11
Worksheet - Compound Interest

1. Complete the following chart:

Principal	Rate/a	Time	Compounded	Formula	Amount	Interest
$1200	12%	5 a	Semi-annually	$A = 1200\left(1+\frac{0.12}{2}\right)^{10}$	$2149.02	$949.02
$480	6%	3 a	Quarterly	$A=480\left(1+\frac{0.06}{4}\right)^{(4)(3)}$	$573.90	$93.90
$10000	8%	12 a	Annually	$A=10000\left(1+\frac{0.08}{1}\right)^{(1)(12)}$	$25 181.70	$15 181.70
$5600	$7\frac{1}{4}\%$	10 a	Semi-annually	$A=5600\left(1+\frac{0.0725}{2}\right)^{(2)(10)}$	$11415.08	$5815.08
$80	$10\frac{1}{2}\%$	20 a	Monthly	$A=80\left(1+\frac{0.105}{12}\right)^{(12)(20)}$	$647.35	$567.35
$1 200 000	5%	7 a	Quarterly	$A=1200000\left(1+\frac{0.05}{4}\right)^{(4)(7)}$	$1699190.76	$499190.76

2. Examine how varying interest rates and compounding intervals affects the following investment.

Principal	Rate/a	Time	Compounded	Formula	Amount	Interest
$12 000	8%	15 a	Annually	$A=12000\left(1+\frac{0.08}{1}\right)^{(1)(15)}$	$38066.03	$26066.03
$12 000	8%	15 a	Semi-Annually	$A=12000\left(1+\frac{0.08}{2}\right)^{(2)(15)}$	$38920.77	$26920.77
$12 000	8%	15 a	Quarterly	$A=12000\left(1+\frac{0.08}{4}\right)^{(4)(15)}$	$39372.37	$27372.37
$12 000	8%	15 a	Monthly	$A=12000\left(1+\frac{0.08}{12}\right)^{(12)(15)}$	$39683.06	$27683.06
$12 000	8%	15 a	Daily	$A=12000\left(1+\frac{0.08}{365}\right)^{(365)(15)}$	$39836.16	$27836.16
$12 000	8%	15 a	Simple Interest	$A=12000\left[1+(0.08)(15)\right]$	$26400	$14400

3. Which of the following investments would be worth the most money after 20 years?

$5000 at 8%/a compounded semi-annually	$7000 at 6%/a compounded daily	$17000 at 2%/a compounded monthly
$A = \$5000\left(1 + \frac{0.08}{2}\right)^{(2)(20)}$ $= \$24005.10$	$A = \$7000\left(1 + \frac{0.06}{365}\right)^{(365)(20)}$ $= \$23238.53$	$A = \$17000\left(1 + \frac{0.02}{12}\right)^{(12)(20)}$ $= \$25352.58$

Attachments

Worksheet - Introduction to Compound Interest.doc