6.2 Properties of Graphs of Quadratic Functions

GOAL

Identify the characteristics of graphs of quadratic functions, and use the graphs to solve problems.

LEARN ABOUT the Math

Nicolina plays on her school's volleyball team. At a recent match, her Nonno, Marko, took some time-lapse photographs while she warmed up. He set his camera to take pictures every 0.25 s. He started his camera at the moment the ball left her arms during a bump and stopped the camera at the moment that the ball hit the floor. Marko wanted to capture a photo of the ball at its greatest height. However, after looking at the photographs, he could not be sure that he had done so. He decided to place the information from his photographs in a table of values.

From his photographs, Marko observed that Nicolina struck the ball at a height of 2 ft above the ground. He also observed that it took about 1.25 s for the ball to reach the same height on the way down.

x y

Time (s)	Height (ft)
0.00	2
0.25	6
0.50	8
0.75	8
1.00	6
1.25	2

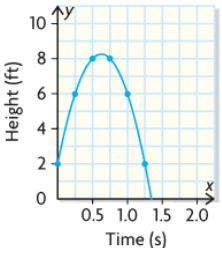
} max height

? When did the volleyball reach its greatest height?

EXAMPLE 1 Using symmetry to estimate the coordinates of the vertex

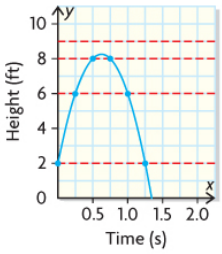
Marko's Solution

D: $\{x \mid x \geq 0, x \in R\}$



I plotted the points from my table, and then I sketched a graph that passed through all the points.

The graph looked like a parabola, so I concluded that the relation is probably quadratic.



I knew that I could draw horizontal lines that would intersect the parabola at two points, except at the **vertex**, where a horizontal line would intersect the parabola at only one point.

Using a ruler, I drew horizontal lines and estimated that the coordinates of the vertex are around (0.6, 8.2).

This means that the ball reached maximum height at just over 8 ft, about 0.6 s after it was launched.

vertex
The point at which the quadratic function reaches its maximum or minimum value.

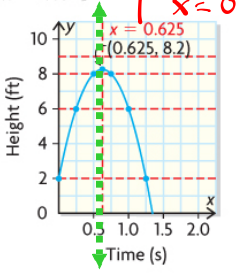
Equation of the axis of symmetry:

$$x = \frac{0 + 1.25}{2}$$

$$x = 0.625$$

$x = \frac{0.5 + 0.75}{2}$

$x = 0.625$



I used points that have the same *y*-value, (0, 2) and (1.25, 2), to determine the equation of the **axis of symmetry**. I knew that the axis of symmetry must be the same distance from each of these points.

axis of symmetry
A line that separates a 2-D figure into two identical parts.
For example, a parabola has a vertical axis of symmetry passing through its vertex.

From the equation, the *x*-coordinate of the vertex is 0.625. From the graph, the *y*-coordinate of the vertex is close to 8.2.

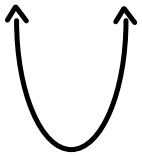
Therefore, 0.625 s after the volleyball was struck, it reached its maximum height of approximately 8 ft 2 in.

I revised my estimate of the coordinates of the vertex.

vertex: (0.625, 8.2) approx

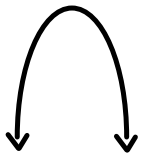
axis of symmetry: $x = 0.625$

↑ x-value of vertex



Domain: {x| x ε R }

Range: {y| y ≥ y-value of vertex, y ε R }



Domain: {x| x ε R }

Range: {y| y ≤ y-value of vertex, y ε R }

EXAMPLE Graphing a quadratic function using a table of values

Sketch the graph of the function: $y = x^2 + x - 2$

Determine the y-intercept, any x-intercepts, the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function.

a = 1 opens up b = 1 c = -2 y int = -2 (0, -2)

Anthony's Solution

$y = x^2 + x - 2$

The function is a quadratic function in the form
$y = ax^2 + bx + c$
$a = 1$
$b = 1$
$c = -2$

The degree of the given equation is 2, so the graph will be a parabola.

Since the coefficient of x^2 is positive, the parabola opens up.

Since the y-intercept is less than zero and the parabola opens up, there must be two x-intercepts and a **minimum value**.

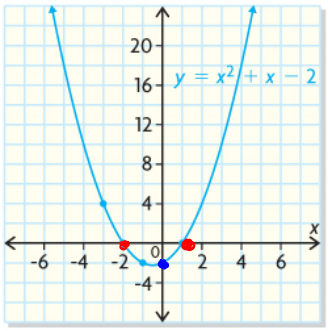
minimum value
The least value of the dependent variable in a relation.

x	−3	−2	−1	0	1
y	4	0	−2	−2	0

$y = x^2 + x - 2$

x	y
-3	4
-2	0
-1	-2
0	-2
1	0
2	4
3	10

x-int y-int x-int



I made a table of values. I included the y-intercept, (0, −2), and determined some other points by substituting values of x into the equation.

I stopped determining points after I had identified both x-intercepts, because I knew that I had enough information to sketch an accurate graph.

I graphed each coordinate pair and then drew a parabola that passed through all the points.

Equation of the axis of symmetry:

(-2, 0) + (1, 0)

$x = \frac{-2 + 1}{2}$

$x = \frac{-1}{2}$

$x = -0.5$

I used the x-intercepts to determine the equation of the axis of symmetry. both have a height of zero

y-coordinate of the vertex:
$y = (-0.5)^2 + (-0.5) - 2$
$y = 0.25 - 0.5 - 2$
$y = -2.25$
The vertex is $(-0.5, -2.25)$.

I knew that the vertex is a point on the axis of symmetry. The x-coordinate of the vertex must be −0.5. To determine the y-coordinate of the vertex, I substituted −0.5 for x in the given equation.

D: {x | x ∈ R}
R: {y | y ≥ -2.25, y ∈ R}

The y-intercept is −2.
The x-intercepts are −2 and 1.
The equation of the axis of symmetry is $x = -0.5$.
The vertex is (−0.5, −2.25).

The vertex, (−0.5, −2.25), defines the minimum value of y.

Domain and range:
$\{(x, y) \mid x \in R, y \geq -2.25, y \in R\}$

No restrictions were given for x, so the domain is all real numbers.

EXAMPLE Graphing a quadratic function using a table of values

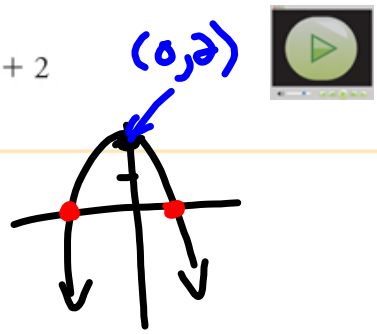
Sketch the graph of the function:

$$y = x^2 + x - 2$$

Determine the y-intercept, any x-intercepts, the equation of the axis of symmetry, the coordinates of the vertex, and the domain and range of the function.

Your Turn

Explain how you could decide if the graph of the function $y = -x^2 + x + 2$ has x-intercepts.



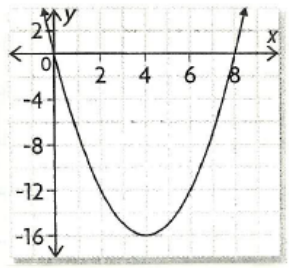
Answer

First, look at the direction of the opening. The given function must open downward, because the coefficient of the x^2 term is negative. Next, look at the y-intercept, which is 2 in this case. Since the parabola opens downward and the y-intercept is positive, the vertex must lie above the x-axis, and the parabola must have two x-intercepts.

Assignment: pages 287 - 288

Questions 1, 3, 4, 6, 7, 9

SOLUTIONS => 6.2 Properties of Graphs of Quadratic Functions

1. 

a) Determine the equation of the axis of symmetry for the parabola.

↳ $x = 4$

b) Determine the coordinates of the vertex of the parabola.

↳ $(4, -16)$

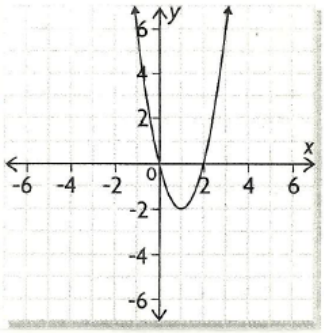
c) State the domain and range of the function.

↳ Domain: $\{x \mid x \in R\}$

↳ Range: $\{y \mid y \geq -16, y \in R\}$

3. For each function, identify the x- and y-intercepts, determine the equation of the axis of symmetry and the coordinates of the vertex, and state the domain and range.

a)



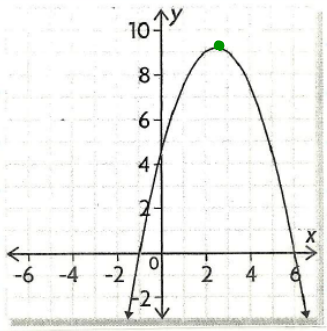
x-intercepts: (0,0) and (2,0)
y-intercept: (0,0)

Equation of axis of symmetry => $x = 1$

Vertex => (1,-2)

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq -2, y \in R\}$

b)



x-intercepts: (-1,0) and (6,0)
y-intercept: (0, 4.5)

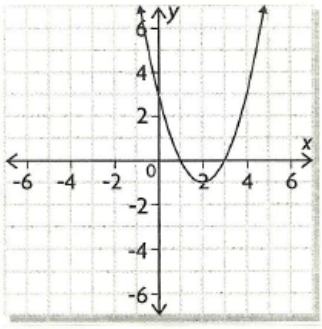
Equation of axis of symmetry => $x = 2.5$

Vertex => (2.5, 9.2)

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \leq 9.2, y \in R\}$

4. For each function, identify the equation of the axis of symmetry, determine the coordinates of the vertex, and state the domain and range.

a)

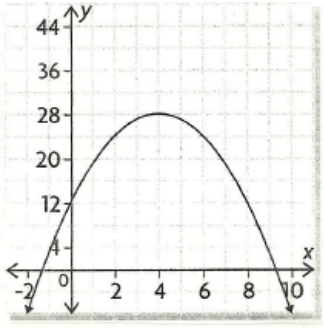


Equation of axis of symmetry => $x = 2$

Vertex => $(2, -1)$

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \geq -1, y \in R\}$

b)

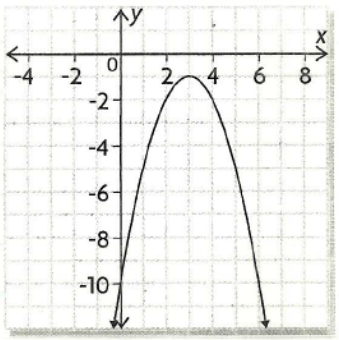


Equation of axis of symmetry => $x = 4$

Vertex => $(4, 28)$

Domain: $\{x \mid x \in R\}$
Range: $\{y \mid y \leq 28, y \in R\}$

c)



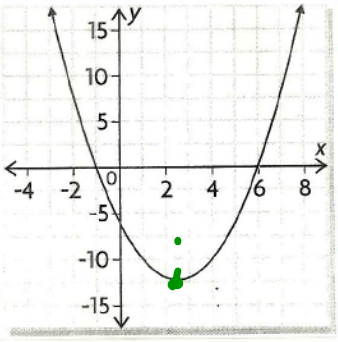
Equation of axis of symmetry => $x=3$

Vertex => $(3,-1)$

Domain : $\{x \mid x \in R\}$

Range : $\{y \mid y \leq -1, y \in R\}$

d)



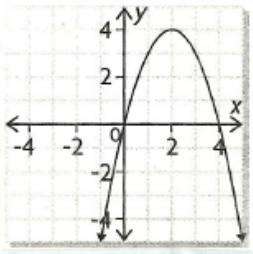
Equation of axis of symmetry => $x=2.5$

Vertex => $(2.5,-12.25)$

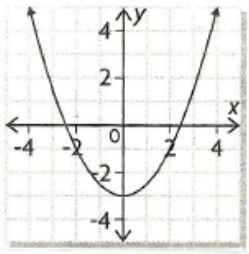
Domain : $\{x \mid x \in R\}$

Range : $\{y \geq -12.25, y \in R\}$

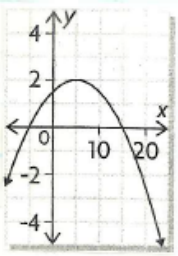
6. State whether each parabola has a minimum or maximum value, and then determine this value.

a) 

Maximum Value => 4

b) 

Minimum Value => -3

c) 

Maximum Value => 2

7.a) Complete the table of values shown for each of the following functions.

x	−4	−2	0	2	4
y					

i) $y = -\frac{1}{2}x^2 + 5$

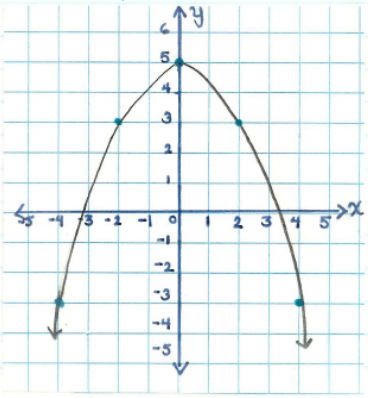
x	-4	-2	0	2	4
y	-3	3	5	3	-3

ii) $y = \frac{3}{2}x^2 - 2$

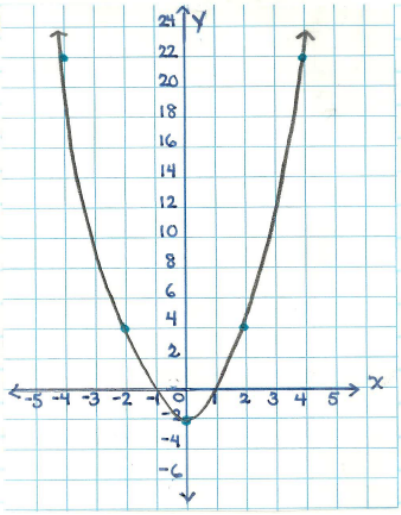
x	-4	-2	0	2	4
y	22	4	-2	4	22

b) Graph the points in your table of values.

i)



ii)



c) State the domain and range of the function.

i) Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \leq 5, y \in R\}$

ii) Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq -2, y \in R\}$

9. For each of the following, both points, (x,y), are located on the same parabola. Determine the equation of the axis of symmetry for each parabola.

a) $(0,2)$ and $(6,2)$.

$\hookrightarrow x = \frac{0+6}{2}$

$x = \frac{6}{2}$

$x = 3$

b) $(1,-3)$ and $(9,-3)$

$\hookrightarrow x = \frac{1+9}{2}$

$x = \frac{10}{2}$

$x = 5$

c) $(-6,0)$ and $(2,0)$

$\hookrightarrow x = \frac{-6+2}{2}$

$x = \frac{-4}{2}$

$x = -2$

d) $(-5,-1)$ and $(3,-1)$

$\hookrightarrow x = \frac{-5+3}{2}$

$x = \frac{-2}{2}$

$x = -1$

Attachments

7s2e2 final.mp4

7s2e3 final.mp4

7s2e4 final.mp4

fm7s2-p8.tns