

Questions from Quiz

$$\textcircled{1} \text{ a) } \lim_{x \rightarrow 3} \frac{(\sqrt{x^2+7} - \sqrt{x+13})(\sqrt{x^2+7} + \sqrt{x+13})}{(x-3)(\sqrt{x^2+7} + \sqrt{x+13})}$$

$$\lim_{x \rightarrow 3} \frac{x^2+7-x-13}{(x-3)(\sqrt{x^2+7} + \sqrt{x+13})}$$

$$\lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(\sqrt{x^2+7} + \sqrt{x+13})} = \boxed{\frac{5}{8}}$$

$$\text{g) } \lim_{x \rightarrow 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x^2+5x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{-1}{(x+5)^2}}{\frac{2x+5}{x^2+5x}} \quad \lim_{x \rightarrow 0} \frac{\frac{-1}{25}}{5} = \boxed{\frac{-1}{125}}$$

$$\text{g) } \lim_{x \rightarrow 0} \frac{5(x+5) \frac{1}{x+5} - \frac{1}{5} 5(x+5)}{x^2+5x (5)(x+5)}$$

$$\lim_{x \rightarrow 0} \frac{5-x-5}{5x(x+5)(x+5)}$$

$$\lim_{x \rightarrow 0} \frac{-x}{5x(x+5)^2} = \boxed{\frac{-1}{125}}$$

$$\textcircled{1} \text{ h) } \lim_{x \rightarrow b} \frac{x^2-b^2}{x^8-b^8}$$

$$\lim_{x \rightarrow b} \frac{(x^2-b^2)}{(x^4-b^4)(x^4+b^4)}$$

$$\lim_{x \rightarrow b} \frac{\cancel{(x^2-b^2)}}{\cancel{(x^2-b^2)}(x^2+b^2)(x^4+b^4)} = \frac{1}{(b^2)(b^4)} = \boxed{\frac{1}{4b^6}}$$

$$\textcircled{1} \text{ f) } \lim_{x \rightarrow 0} \frac{\sin 7x}{2x}$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin 7x}{7x} \right) \left(\frac{7}{2} \right)$$

$$\textcircled{4} \lim_{x \rightarrow 1} \frac{\sqrt[6]{x} - 1}{\sqrt[4]{x} - 1}$$

$$\lim_{x \rightarrow 1} \frac{\frac{1}{6} x^{-5/6}}{\frac{1}{4} x^{-3/4}} \quad \lim_{x \rightarrow 1} \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

Bonus:

$$\begin{array}{lll} y = ax^2 + bx + c & 4 = 2a(1) + b & 8 = 2a(-1) + b \\ y' = 2ax + b & 2a + b = 4 & -2a + b = 8 \end{array}$$

$$\begin{array}{l} 2a + b = 4 \\ (+) - 2a + b = 8 \\ \hline 2b = 12 \\ \boxed{b = 6} \end{array} \quad \begin{array}{l} 2a + 6 = 4 \\ 2a = -2 \\ \boxed{a = -1} \end{array}$$

$$\begin{array}{l} y = -x^2 + 6x + c \quad \text{passes through } (2, 15) \\ 15 = -4 + 12 + c \\ \boxed{7 = c} \end{array}$$

$\therefore y = -x^2 + 6x + 7$ is the equation

$$d) \lim_{x \rightarrow -1^-} \frac{|x+1|}{x^2-1}$$

$$\lim_{x \rightarrow -1^-} \frac{|x+1|}{(x+1)(x-1)}$$

$$\lim_{x \rightarrow -1^-} \frac{\overset{-1}{|1-1.0001+1|}}{\cancel{(-1.0001+1)}(-1.0001-1)} = \frac{-1}{-2} = \boxed{\frac{1}{2}}$$

Questions from Homework

$$\textcircled{6} \text{ a) } f(x) = \sqrt{x} - \sqrt{1-x} = x^{1/2} - (1-x)^{1/2}$$

$$F(x) = \frac{2x^{3/2}}{3} - \frac{2(1-x)^{3/2}}{3} (-1) + C$$

$$F(x) = \frac{2x^{3/2}}{3} + \frac{2(1-x)^{3/2}}{3} + C$$

$$\textcircled{6} \text{ b) } f(x) = \frac{1}{x} - \frac{1}{1-x}$$

$$F(x) = \ln|x| - \ln|1-x|(-1) + C$$

$$F(x) = \ln(x) + \ln(1-x) + C$$

$$F(x) = \ln(x-x^2) + C$$

Warm Up

Determine the general antiderivative of the following:

$$f(x) = 2x^2 - x + 7$$

$$F(x) = \frac{2x^3}{3} - \frac{x^2}{2} + 7x + C$$

$$f(x) = \cos x - \sin x$$

$$F(x) = \sin x + \cos x + C$$

$$f(x) = -3e^{-x} + 6e^{2x}$$

$$F(x) = \frac{-3e^{-x}}{-1} + \frac{6e^{2x}}{2} + C$$

$$F(x) = 3e^{-x} + 3e^{2x} + C$$

$$f(x) = \frac{2}{x^2} - \frac{5}{x} + x = 2x^{-2} - \frac{5}{x} + x$$

$$F(x) = \frac{2x^{-1}}{-1} - 5\ln|x| + \frac{x^2}{2} + C$$

$$F(x) = -\frac{2}{x} - 5\ln(x) + \frac{1}{2}x^2 + C$$

$$f(x) = \cos 5x - x^2 \csc^2 x^3 + 5x \sin 2x^2$$

$$F(x) = \frac{1}{5} \sin 5x + \frac{1}{3} \cot x^3 - \frac{5}{4} \cos 2x^2 + C$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ -\csc^2 x^3 (3x^2) & & -\sin 2x^2 (4x) \\ \underline{-3x^2 \csc^2 x^3} & & -4x \sin 2x^2 \\ 3 & & \end{array}$$

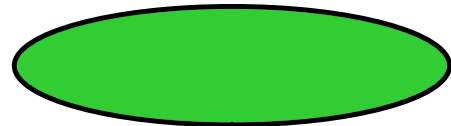
Differential Equations

An equation that involves the derivative of a function is called a differential equation:

As discussed previously, in applications of calculus it is very common to have a situation where it is required to find a function, given knowledge about its derivatives.

Find all functions g such that:

$$g'(x) = 4 \sin x - 3x^5 + 6\sqrt[4]{x^3}$$



Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

Find f if given $f'(x)$: and $f(0) = -2$

$$f'(x) = e^x + \frac{20}{1+x^2}$$



Find f if given $f''(x)$: and $f(0) = 4$, and $f(1) = 1$

$$f''(x) = 12x^2 + 6x - 4$$



Practice Problems...

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Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$$f'(x) = n[g(x)]^{n-1} g'(x)$$

Let's look at the following:

$$f'(x) = (x^2 - 3)^5 (2x)$$

$$f'(x) = x^2 \sqrt{x^3 - 1}$$

$$f'(x) = \frac{3x}{\sqrt{1 - 5x^2}}$$

$$f'(x) = \frac{\cos 8x}{(1 + \sin 8x)^4}$$