Questions from Homework

⑥a) $f(x) = \sqrt{x} - \sqrt{1-x} = x^{1/2} - (1-x)^{1/2}$

$F(x) = \dfrac{x^{3/2}}{3/2} - \dfrac{(1-x)^{3/2}}{3/2}(-1) + C$

$F(x) = \dfrac{2}{3}x^{3/2} + \dfrac{2}{3}(1-x)^{3/2} + C$

⑥ b) $f(x) = \dfrac{1}{x} - \dfrac{1}{1-x}$

$F(x) = \ln|x| - \ln|1-x|(-1) + C$

$F(x) = \ln|x| + \ln|1-x| + C$

$F(x) = \ln|x-x^2| + C$

⑥ c) $f(x) = \dfrac{1}{\sqrt{1-x}} + \dfrac{1}{\sqrt{x}}$

$f(x) = (1-x)^{-1/2} + (x)^{-1/2}$

$F(x) = \dfrac{(1-x)^{1/2}}{1/2}(-1) + \dfrac{x^{1/2}}{1/2} + C$

$F(x) = -2\sqrt{1-x} + 2\sqrt{x} + C$ ✓

$F(x) = 2(\sqrt{x} - \sqrt{1-x}) + C$

Warm Up

Determine the general antiderivative of the following:

$$f(x) = 2x^2 - x + 7$$

$$F(x) = \frac{2x^3}{3} - \frac{1x^2}{2} + 7x + C$$

$$f(x) = \cos x - \sin x$$

$$F(x) = \frac{1}{1}\sin 1x - \frac{-1}{1}\cos 1x + C$$

$$F(x) = \sin x + \cos x + C$$

$$f(x) = -3e^{-x} + 6e^{2x}$$

$$F(x) = \frac{-3e^{-1x}}{-1} + \frac{6e^{2x}}{2} + C$$

$$F(x) = 3e^{-x} + 3e^{2x} + C$$

$$f(x) = \frac{2}{x^2} - \frac{5}{x} + x \quad = 2x^{-2} - \frac{5}{x} + x$$

$$F(x) = \frac{2x^{-1}}{-1} - 5\ln|x| + \frac{1x^2}{2} + C$$

$$F(x) = \frac{-2}{x} - 5\ln x + \frac{1x^2}{2} + C$$

Differential Equations

An equation that involves the derivative of a function is called a differential equation:

As discussed previously, in applications of calculus it is very common to have a situation where it is required to find a function, given knowledge about its derivatives.

Find all functions g such that: (general antiderivative) indefinite integral

$$g'(x) = 4\sin x - 3x^5 + 6\sqrt[4]{x^3}$$

$$g'(x) = 4\sin x - 3x^5 + 6x^{3/4}$$

$$g(x) = -\frac{4}{1}\cos 1x - \frac{3x^6}{6} + \frac{6x^{7/4}}{7/4} + C$$

$6 \div \frac{7}{4}$

$6 \times \frac{4}{7} = \frac{24}{7}$

$$g(x) = -4\cos x - \frac{1}{2}x^6 + \frac{24}{7}x^{7/4} + C$$

Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

Find f if given $f'(x)$: and $f(0) = -2$ → $(0, -2)$

$$f'(x) = e^x + \frac{20}{1+x^2}$$

$f(x) = \frac{1}{1}e^{1x} + \frac{20}{1}\tan^{-1} 1x + C$

$f(x) = e^x + 20\tan^{-1}(x) + C$

$-2 = e^0 + 20\tan^{-1}(0) + C$

$-2 = 1 + 20(0) + C$

$-2 = 1 + 0 + C$

$-3 = C$

$\boxed{f(x) = e^x + 20\tan^{-1}(x) - 3}$

$$f(x) = e^x + 20\tan^{-1} x - 3$$

Find f if given $f''(x)$: and $f(0) = 4$, and $f(1) = 1$

$x=0$ $y=4$ $x=1$ $y=1$

$$f''(x) = 12x^2 + 6x - 4$$

$f'(x) = \frac{12x^3}{3} + \frac{6x^2}{2} - 4x + C$

$f'(x) = 4x^3 + 3x^2 - 4x + C$

$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + Cx + d$

$f(x) = x^4 + x^3 - 2x^2 + Cx + d$

$f(x) = x^4 + x^3 - 2x^2 + cx + d$

$4 = (0)^4 + (0)^3 - 2(0)^2 + c(0) + d$

$4 = d$

$f(x) = x^4 + x^3 - 2x^2 + cx + 4$

$1 = (1)^4 + (1)^3 - 2(1)^2 + c(1) + 4$

$1 = 1 + 1 - 2 + c + 4$

$1 = 4 + c$

$-3 = c$

$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$

$$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

Practice Problems...

Page 408 — #1 - 6

Page 411 — #3, 4

$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$

③ b) $F'(x) = 3\sqrt{2x} \quad \rightarrow (2,3)$

$F'(x) = 3\sqrt{2}\cdot\sqrt{x}$

$F'(x) = 3\sqrt{2}\, x^{1/2}$

$F(x) = \dfrac{3\sqrt{2}\, x^{3/2}}{3/2} + C$

$3\sqrt{2} \div \frac{3}{2}$

$\cancel{3}\sqrt{2} \times \frac{2}{\cancel{3}}$

$2\sqrt{2}$

$F(x) = 2\sqrt{2}\, x^{3/2} + C$

To find c:

$3 = 2\sqrt{2}\,(2)^{3/2} + C$

$3 = 2\sqrt{2}\,(2\sqrt{2}) + C$

$3 = 4(2) + C$

$3 = 8 + C$

$-5 = C$

$\boxed{F(x) = 2\sqrt{2}\, x^{3/2} - 5}$

Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$$f'(x) = n[g(x)]^{n-1} g'(x)$$

Let's look at the following:

$$f'(x) = (x^2 - 3)^5 (2x)$$

$$f'(x) = x^2 \sqrt{x^3 - 1}$$

$$f'(x) = \frac{3x}{\sqrt{1 - 5x^2}}$$

$$f'(x) = \frac{\cos 8x}{(1 + \sin 8x)^4}$$