6.5 Solving Quadratic Equations by Factoring

* ① Try a common factor
② Count terms:
↳ 2 (Check for diff. of squares)
↳ 3 (Simple or Hard Trinomial)

Solving Quadratic Equations ($ax^2 + bx + c = 0$)

Method #1 Factoring

Add ⇩ Multiply ⇩

Example 1: $x^2 + 6x + 8 = 0$

$2 + 4 = 6$

$2 \times 4 = 8$

8
1 x 8
2 x 4

"Simple Trinomial"

Stop here for "factor"

$(x + 2)(x + 4) = 0$

Either $x + 2 = 0$ or $x + 4 = 0$

Stop here for "solve"

$x = -2$ $x = -4$

$(-2, 0)$ x-ints $(-4, 0)$

Let's try one more...

$x^2 - 5x - 24 = 0$

$3 + -8 = -5$

$3 \times -8 = -24$

-24
1 x -24
2 x -12
3 x -8
4 x -6

$(x + 3)(x - 8) = 0$

$x + 3 = 0$ | $x - 8 = 0$

$x = -3$ | $x = 8$

Example 2: $2x^2 + 11x - 15 = 0$

$(x + \frac{6}{2})(x + \frac{-5}{2}) = 0$

$(x + 3)(2x - 5) = 0$

Either $x+3 = 0$ or $2x-5 = 0$

$x = -3$ $\quad\quad$ $\frac{2x}{2} = \frac{5}{2}$

$x = \frac{5}{2}$

"Hard Trinomial"

Multiply ⇨ -30 Add ⇨ 1

★ + 6 and -5

Stop here for "factor"

$-5 + 6 = 1$

$-5 \times 6 = -30$ $(2 \cdot -15)$

Stop here for "solve"

Let's try a few more...

$3x^2 + 5x + 2 = 0$

$(x+\frac{2}{3})(x+\frac{3}{3}) = 0$

$(3x+2)(x+1) = 0$

$3x+2=0$ | $x+1=0$

$x = -\frac{2}{3}$ | $x = -1$

$2 \times 3 = 6$ (3×2)

$2 + 3 = 5$

6
1 x 6
2 x 3

$6x^2 + 14x + 8 = 0$

$2(3x^2 + 7x + 4) = 0$

$2(x+\frac{3}{3})(x+\frac{4}{3}) = 0$

$2(x+1)(3x+4) = 0$ factor

$x+1=0$ | $3x+4=0$

$x = -1$ | $3x = -4$

$x = -\frac{4}{3}$ Solve

EXTRA STEP

$3 \times 4 = 12$ (3×4)

$3 + 4 = 7$

12
1 x 12
2 x 6
3 x 4

Example 3: $7x^2 + 4x = 0$ "Common Factor"

$x(7x + 4) = 0$ Stop here for "factor"

Either $x = 0$ or $7x + 4 = 0$

$$\frac{7x}{7} = \frac{-4}{7}$$

$$x = \frac{-4}{7}$$ Stop here for "solve"

***Sometimes you may remove a common factor first and then end up with a simple trinomial, a hard trinomial, or a difference of squares.

Let's try one more...

Example 4: $4x^2 - 9 = 0$ "Difference of Squares"

Perfect Square ⇧ Perfect Square ⇧

$(2x - 3)(2x + 3) = 0$ Stop here for "factor"

Either $2x - 3 = 0$ or $2x + 3 = 0$

$\frac{2x}{2} = \frac{3}{2}$ $\frac{2x}{2} = \frac{-3}{2}$

$x = \frac{3}{2}$ $x = \frac{-3}{2}$ Stop here for "solve"

Let's try one more...

Using reasoning to write an equation from its roots

Tori says she solved a quadratic equation by graphing. She says the roots were -5 and 7. How can you determine an equation that she might have solved?

Philip's Solution

$x = -5$ or $x = 7$

The x-intercepts of the quadratic function are the roots of the equation.

$x + 5 = 0$ $x - 7 = 0$
One factor is $x + 5$.
The other factor is $x - 7$.

I decided to use the roots to help me write the factors of the equation.

$(x + 5)(x - 7) = 0$
$x^2 + 5x - 7x - 35 = 0$
$x^2 - 2x - 35 = 0$

I wrote the factors as a product. Since each root is equal to 0, their product is also equal to 0.

I simplified to write the equation in standard form.

In Summary

Key Idea

- Some quadratic equations can be solved by factoring.

Need to Know

- To factor an equation, start by writing the equation in standard form.
- You can set each factor equal to zero and solve the resulting linear equations. Each solution is a solution to the original equation.
- If the two roots of a quadratic equation are equal, then the quadratic equation is said to have one solution.

Assignment: pages 323 - 324

Questions 1, 2(a-d), 6, 7, 11

SOLUTIONS => 6.5 Solving Quadratic Equations by Factoring

1.

a) $x^2 - 11x + 28 = 0$ (A, M)

$-4 \times -7 = 28$

$-4 + -7 = -11$

$(x-4)(x-7)=0$

$x-4=0$ or $x-7=0$

$x=4 \quad x=7$

b) $x^2 - 7x - 30 = 0$ (A, M)

$3 \times -10 = -30$

$3 + -10 = -7$

$(x+3)(x-10)=0$

$x+3=0$ or $x-10=0$

$x=-3 \quad x=10$

c) $2y^2 + 11y + 5 = 0$

$(y + \frac{1}{2})(y + \frac{10}{2})$

$1 \times 10 = 10$
$1 + 10 = 11$

$(2y+1)(y+5) = 0$
$2y+1=0$ or $y+5=0$
$\frac{2y}{2} = \frac{-1}{2}$ $y = -5$
$y = -\frac{1}{2}$

d) $4t^2 + 7t - 15 = 0$

$(t - \frac{5}{4})(t + \frac{12}{4})$

$-5 \times 12 = -60$
$-5 + 12 = 7$

$(4t-5)(t+3) = 0$
$4t-5=0$ or $t+3=0$
$\frac{4t}{4} = \frac{5}{4}$ $t = -3$
$t = \frac{5}{4}$

2.

{a-d} a) $x^2 - 121 = 0$ (Difference of Squares)

$(x-11)(x+11)=0$

$x-11=0$ or $x+11=0$

$x=11$ $\quad$ $x=-11$

b) $9r^2 - 100 = 0$ (Difference of Squares)

$(3r-10)(3r+10)=0$

$3r-10=0$ or $3r+10=0$

$\frac{3r}{3} = \frac{10}{3}$ $\quad$ $\frac{3r}{3} = \frac{-10}{3}$

$r = \frac{10}{3}$ $\quad$ $r = -\frac{10}{3}$

c) $x^2 - 15x = 0$ (Common Factor)

$x(x-15) = 0$

$x = 0$ or $x - 15 = 0$

$x = 15$

d) $3y^2 + 48y = 0$ (Common Factor)

$3y(y+16) = 0$

$\frac{3y}{3} = \frac{0}{3}$ or $y + 16 = 0$

$y = 0$ $y = -16$

6. Determine the roots of each equation.

a) $5u^2 - 10u - 315 = 0$

$5(u^2 - 2u - 63) = 0$

$5(u+7)(u-9) = 0$

$u+7=0$ or $u-9=0$

$u=-7$ $u=9$

$7 \times -9 = -63$

$7 + -9 = -2$

b) $0.25x^2 + 1.5x + 2 = 0$

$0.25(x^2 + 6x + 8) = 0$

$0.25(x+4)(x+2) = 0$

$x+4=0$ or $x+2=0$

$x=-4$ $x=-2$

$4 \times 2 = 8$

$4 + 2 = 6$

c) $1.4y^2 + 5.6y - 16.8 = 0$

$1.4(y^2 + 4y - 12) = 0$

$1.4(y+6)(y-2) = 0$

$y+6=0$ or $y-2=0$

$y=-6$ $y=2.$

$6 \times -2 = -12$

$6 + -2 = 4$

d) $\frac{1}{2}K^2 + 5K + 12.5 = 0$

$\frac{1}{2}(K^2 + 10K + 25) = 0$

$\frac{1}{2}(K+5)(K+5) = 0$

$\frac{1}{2}(K+5)^2 = 0$

$K+5=0$

$K=-5$

$5 \times 5 = 25$

$5 + 5 = 10$

7. The graph of a quadratic function has x-intercepts -5 and -12. Write a quadratic equation that has these roots.

$y = a(x-r)(x-s)$

Assuming $a=1$:

$y = (x--5)(x--12)$

$y = (x+5)(x+12)$

$y = x^2+12x+5x+60$

$y = x^2+17x+60$

Quadratic Equation => $x^2+17x+60=0$

11. Identify and correct the errors in this solution:

$4r^2 - 9r = 0$

$(2r-3)(2r+3) = 0 \rightarrow$ Error

$2r - 3 = 0$ or $2r + 3 = 0$

$2r = 3$ $\quad$ $2r = -3$

$r = 1.5$ or $\quad$ $r = -1.5$

Correction:

$4r^2 - 9r = 0$

$r(4r-9) = 0$

$r = 0$ or $4r - 9 = 0$

$\frac{4r}{4} = \frac{9}{4}$

$r = \frac{9}{4}$

Attachments

7s5e2 finalt.mp4

7s5e3 finalt.mp4

7s5e4 finalt.mp4

7s5e5 finalt.mp4

FM11-7s5.gsp