6.7 Solving Quadratic Equations Using the Quadratic Formula

GOAL

Use the quadratic formula to determine the roots of a quadratic equation.

Question from Homework:

- Solve by Factoring

$7x^2-38x-24=0$ $\_+\_=-38$

$(x+\frac{4}{7})(x-\frac{42}{7})=0$ $\_\times\_=-168$

$(7x+4)(x-6)=0$

-168
1 x -168
2 x -84
3x -56
4x -42

$7x+4=0$ | $x-6=0$

$7x=-4$ | $x=6$

$x=-\frac{4}{7}$ | $(6,0)$

$(-\frac{4}{7},0)$ 2 x-intercepts

- Solve using Quadratic Formula

$7x^2-38x-24=0$

$a=7$ $b=-38$ $c=-24$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-38)\pm\sqrt{(-38)^2-4(7)(-24)}}{2(7)}$

$x=\frac{38\pm\sqrt{1444+672}}{14}$

$x=\frac{38\pm\sqrt{2116}}{14}$

$x=\frac{38\pm 46}{14}$

$x=\frac{38+46}{14}$ | $x=\frac{38-46}{14}$

$x=\frac{84}{14}$ | $x=\frac{-8}{14}$

$x=6$ | $x=-\frac{4}{7}$

$(6,0)$ | $(-\frac{4}{7},0)$

x-intercepts

THE QUADRATIC FORMULA

The Quadratic Formula can be used to solve **ALL** quadratic equations, even the ones that you cannot factor!

The solution to any quadratic equation: $ax^2 + bx + c = 0$; where $a \neq 0$, is given by:

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since the solutions to quadratic equations are linked to the x-intercepts of quadratic functions, it makes sense that quadratic equations may also have 0, 1, or 2 solutions.

In the next few examples, we will use the quadratic formula to find the solution to various quadratic equations. These examples will illustrate the three possible results that can be obtained when solving quadratics.

Example 1: Two REAL Solutions

Solve $x^2 + 7x + 12 = 0$

Solution:

a = 1; b = 7; c = 12

Therefore, $x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(12)}}{2(1)}$

$x = \frac{-7 \pm \sqrt{49-48}}{2}$

$x = \frac{-7 \pm \sqrt{1}}{2}$

$x = \frac{-7 \pm 1}{2}$

$x = \frac{-6}{2}$ or $x = \frac{-8}{2}$

$x = -3$ or $x = -4$

(-3,0) + (-4,0)

Example 2: One REAL Solution

Solve $2x^2 + 24x + 72 = 0$

Solution:

a = 2; b = 24; c = 72

Therefore, $x = \frac{-24 \pm \sqrt{(24)^2 - 4(2)(72)}}{2(2)}$

$x = \frac{-24 \pm \sqrt{576-576}}{4}$

$x = \frac{-24 \pm \sqrt{0}}{4}$

$x = \frac{-24 \pm 0}{4}$

$x = \frac{-24}{4}$

$x = -6$

(-6,0)

<u>Example 3: No REAL Solutions.</u>

Solve $x^2 - 4x + 8 = 0$

<u>Solution:</u>

$a = 1$; $b = -4$; $c = 8$

Therefore, $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16-32}}{2}$

$x = \frac{4 \pm \sqrt{-16}}{2}$ ← cannot take the square root of a negative

No real solution

APPLY the Math

Connecting the quadratic formula to factoring

Solve the following equation:

$$6x^2 - 3 = 7x$$

Adrianne's Solution

$$6x^2 - 3 = 7x$$

more terms to one side

$$6x^2 - 7x - 3 = 0$$

$a = 6$, $b = -7$, and $c = -3$

First, I rewrote the equation in standard form to determine the values of *a*, *b*, and *c*.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

I wrote the quadratic formula and substituted the values of *a*, *b*, and *c*.

I simplified the right side. I realized that 121 is a perfect square.

$$x = \frac{18}{12} \quad \text{or} \quad x = \frac{-4}{12}$$

$$x = \frac{3}{2} \quad \text{or} \quad x = \frac{-1}{3}$$

I determined the two solutions.

Verify:

$$6x^2 - 7x - 3 = 0$$

$$(3x + 1)(2x - 3) = 0$$

$$3x + 1 = 0 \quad \text{or} \quad 2x - 3 = 0$$

$$3x = -1 \qquad 2x = 3$$

$$x = \frac{-1}{3} \qquad x = \frac{3}{2}$$

The solutions match those I got using the quadratic formula.

If the radicand in the quadratic formula is a perfect square, then the original equation can be factored. I decided to verify my solution by factoring the original equation.

In Summary

Key Idea

- The roots of a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, can be determined by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Need to Know

- The quadratic formula can be used to solve any quadratic equation, even if the equation is not factorable.
- If the radicand in the quadratic formula simplifies to a perfect square, then the equation can be solved by factoring.
- If the radicand in the quadratic formula simplifies to a negative number, then there is no real solution for the quadratic equation.

Assignment: **pages 345 - 346**

Questions 2, 3, 5, 6

SOLUTIONS ⇒ 6.7 Solving Quadratic Equations Using the Quadratic Formula

a) $x^2 + 5x - 6 = 0$

$a = 1,\ b = 5,\ c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{2}$$

$$x = \frac{-5 \pm \sqrt{49}}{2}$$

$$x = \frac{-5 \pm 7}{2}$$

$x = \frac{-5+7}{2}$ or $x = \frac{-5-7}{2}$

$x = \frac{2}{2}$ $\quad$ $x = \frac{-12}{2}$

$x = 1$ $\quad$ $x = -6$

b) $4x + 9x^2 = 0$

$$9x^2 + 4x = 0$$

$$a = 9, b = 4, c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(9)(0)}}{2(9)}$$

$$x = \frac{-4 \pm \sqrt{16 - 0}}{18}$$

$$x = \frac{-4 \pm \sqrt{16}}{18}$$

$$x = \frac{-4 \pm 4}{18}$$

$$x = \frac{-4+4}{18} \quad \text{or} \quad x = \frac{-4-4}{18}$$

$$x = \frac{0}{18} \qquad x = \frac{-8}{18}$$

$$x = 0 \qquad x = -\frac{4}{9}$$

c) $25x^2 - 121 = 0$

$$a = 25, b = 0, c = -121$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{0 \pm \sqrt{(0)^2 - 4(25)(-121)}}{2(25)}$$

$$x = \frac{0 \pm \sqrt{0 + 12100}}{50}$$

$$x = \frac{\pm \sqrt{12100}}{50}$$

$$x = \pm \frac{110}{50}$$

$$x = \pm \frac{11}{5}$$

$$x = \frac{11}{5} \quad \text{or} \quad x = \frac{-11}{5}$$

d) $12x^2-17x-40=0$

$a=12, b=-17, c=-40$

$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x=\frac{17 \pm \sqrt{(-17)^2-4(12)(-40)}}{2(12)}$

$x=\frac{17 \pm \sqrt{289+1920}}{24}$

$x=\frac{17 \pm \sqrt{2209}}{24}$

$x=\frac{17 \pm 47}{24}$

$x=\frac{17+47}{24}$ or $x=\frac{17-47}{24}$

$x=\frac{64}{24}$ $\qquad x=\frac{-30}{24}$

$x=\frac{8}{3}$ $\qquad x=-\frac{5}{4}$

3. Solve each equation in question 2 by factoring. Which method did you prefer for each equation? Explain.

a) $x^2+5x-6=0$ $\quad \underline{-1} \times \underline{6} = -6$

$(x-1)(x+6)=0$ $\quad \underline{-1} + \underline{6} = 5$

$x-1=0$ or $x+6=0$

$x=1$ $\quad x=-6$

b) $4x+9x^2=0$

$9x^2+4x=0$

$x(9x+4)=0$

$x=0$ or $9x+4=0$

$\frac{\cancel{9}x}{\cancel{9}}=\frac{-4}{9}$

$x=-\frac{4}{9}$

c) $25x^2-121=0$

$(5x-11)(5x+11)=0$

$5x-11=0$ or $5x+11=0$

$\frac{\cancel{5}x}{\cancel{5}}=\frac{11}{5}$ $\quad \frac{\cancel{5}x}{\cancel{5}}=\frac{-11}{5}$

$x=\frac{11}{5}$ $\quad x=-\frac{11}{5}$

d) $12x^2 - 17x - 40 = 0$

$(x + \frac{15}{12})(x - \frac{32}{12}) = 0$

$(x + \frac{5}{4})(x - \frac{8}{3}) = 0$

*Must reduce fractions!

$15 \times -32 = -480$

$15 + -32 = -17$

$(4x+5)(3x-8) = 0$

$4x + 5 = 0$ or $3x - 8 = 0$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{-5}{4}$ $\qquad$ $\frac{\cancel{3}x}{\cancel{3}} = \frac{8}{3}$

$x = -\frac{5}{4}$ $\qquad$ $x = \frac{8}{3}$

5. The roots for the quadratic equation

$$1.44a^2 + 2.88a - 21.6 = 0$$

are $a = 3$ and $a = -5$. Verify these roots.

[a=3] L.S

$1.44a^2 + 2.88a - 21.6$

$1.44(3)^2 + 2.88(3) - 21.6$

$1.44(9) + 8.64 - 21.6$

$12.96 - 12.96$

0

R.S

0

Since L.S = R.S, $a = 3$ is a valid solution.

[a=-5] L.S.

$1.44a^2 + 2.88a - 21.6$

$1.44(-5)^2 + 2.88(-5) - 21.6$

$1.44(25) - 14.4 - 21.6$

$36 - 36$

0

R.S.

0

Since L.S = R.S, $a = -5$ is a valid solution.

6. Solve each equation. State the solutions as exact values.

a) $3x^2-6x-1=0$

$a=3,\ b=-6,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{6\pm\sqrt{(-6)^2-4(3)(-1)}}{2(3)}$$

$$x=\frac{6\pm\sqrt{36+12}}{6}$$

$$x=\frac{6\pm\sqrt{48}}{6}$$

We can stop here for now!

b) $x^2+8x+3=0$

$a=1,\ b=8,\ c=3$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4(1)(3)}}{2(1)}$$

$$x=\frac{-8\pm\sqrt{64-12}}{2}$$

$$x=\frac{-8\pm\sqrt{52}}{2}$$

We can stop here for now!

c) $8x^2+8x-1=0$

$a=8,\ b=8,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4(8)(-1)}}{2(8)}$$

$$x=\frac{-8\pm\sqrt{64+32}}{16}$$

$$x=\frac{-8\pm\sqrt{96}}{16}$$

We can stop here for now!

d) $9x^2-12x-1=0$

$a=9,\ b=-12,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{12\pm\sqrt{(-12)^2-4(9)(-1)}}{2(9)}$$

$$x=\frac{12\pm\sqrt{144+36}}{18}$$

$$x=\frac{12\pm\sqrt{180}}{18}$$

We can stop here for now!

7s7e2 finalt.mp4

7s7e3 finalt.mp4

7s7e4 finalt.mp4