5.4 Optimization Problems I: Creating the Model

A toy company manufactures two types of toy vehicles: **sport-utility vehicles** and **racing cars**.
- The supply of materials is limited, therefore no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- The company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

Create a model that could be used to represent the **cost** of producing the two types of toy vehicles.

Defining Statements:

____

____

____

Restrictions: ____________________

____

Constraints:

____

____

____

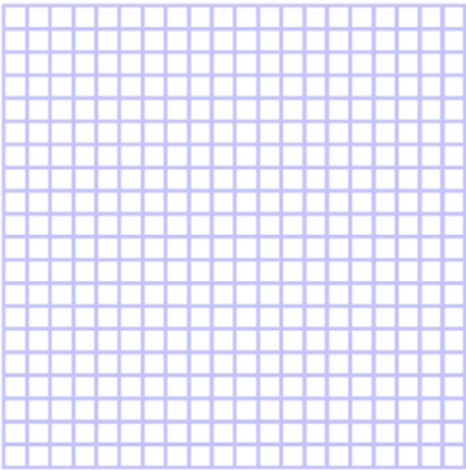
Objective Function: ________________________

Steps to draw the graph:

1.

____

2.



3.

____

Reflecting

Each combination below is a possible solution to the system of linear inequalities:

i) 40 racing cars and 60 sport-utility vehicles
ii) 40 racing cars and 30 sport-utility vehicles
iii) 10 racing cars and 60 sport-utility vehicles
iv) 30 racing cars and 40 sport-utility vehicles

Use your equation from part E to calculate the manufacturing cost for each solution. What do you notice?

Answers

i) $1040
ii) $680
iii) $800
iv) $720

Each combination costs a different amount; the combination of 40 racing cars and 30 sports utility vehicles costs the least of these combinations, and 40 racing cars and 60 sport-utility vehicles costs the most. The other combinations are in between.

APPLY the Math

EXAMPLE 1 Creating a model for an optimization problem with whole-number variables

Three teams are travelling to a basketball tournament in minivans and cars.

- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles. Create a model to represent this situation.

Juanita's Solution

Let x represent the number of minivans.
Let y represent the number of cars.

The two variables in the problem are the number of cars and the number of minivans. The values of these variables are whole numbers.

$x \in W$ and $y \in W$

Constraints:
Number of cars available:
$y \le 12$
Number of minivans available:
$x \le 4$
Number of team members:
$4y + 6x \le 48$

I knew that this is an **optimization problem** because the number of vehicles has to be minimized and maximized.

I wrote three linear inequalities to represent the three limiting conditions, or **constraints**.

The maximum number of team members is the number of teams multiplied by the maximum number of coaches and athletes:
$3(14) + 3(2) = 48$

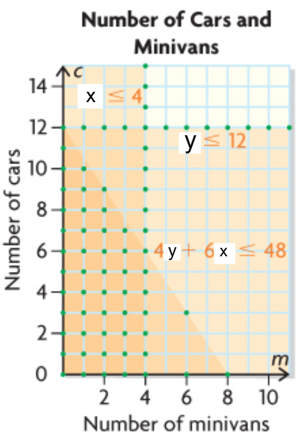
optimization problem
A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

constraint
A limiting condition of the optimization problem being modelled, represented by a linear inequality.

Objective function:
Let V represent the total number of vehicles.
$V = x + y$

I created an equation, called the **objective function**, to represent the relationship between the two variables (number of minivans and number of cars) and the quantity to be minimized and maximized (number of vehicles).

objective function
In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.



I graphed the system of three inequalities.

One of the solutions in the **feasible region** represents the combination of cars and minivans that results in the minimum total number of vehicles and another solution represents the maximum. I think I could use the objective function to determine each point, but I am not certain how yet.

feasible region
The solution region for a system of linear inequalities that is modelling an optimization problem.

In Summary

Key Ideas

- To solve an optimization problem, you need to determine which combination of values of two variables results in a maximum or minimum value of a related quantity.
- When creating a model, the first step is to represent the situation algebraically. An algebraic model includes these parts:
  - a defining statement of the variables used in your model
  - a statement describing the restrictions on the variables
  - a system of linear inequalities that describes the constraints
  - an objective function that shows how the variables are related to the quantity to be optimized
- The second step is to represent the system of linear inequalities graphically.
- In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities.

Need to Know

- You can create a model for an optimization problem by following these steps:

  Step 1. Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, and *greatest* or *least*.

  Step 2. Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.

  Step 3. Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.

  Step 4. Write an objective function to represent the relationship between the variables and the quantity to be optimized.

Assignment: page 248 Questions 1 & 2

5.4 pg. 248 Question 1

Baskets of fruit are being prepared to sell.

- Each basket contains at least 5 apples and at least 6 oranges.
- Apples cost 20 cents each, and oranges cost 35 cents each. The budget allows no more than $7, in total, for the fruit in each basket.

Create a model that could be used to determine the combination of apples and oranges that will result in the maximum number of pieces of fruit in a basket.

Defining Statements:

Let x = the # of apples.
Let y = the # of oranges
• Let F = the total # of fruit

Restrictions:

$x \mid x \in W$
$y \mid y \in W$
} not going to use decimals, fractions or negatives.

Constraints:

• $x \geq 5$ (Quick Graph) → Vertical
• $y \geq 6$ (Quick Graph) → Horizontal
• $0.2x + 0.35y \leq 7$

Objective Function: $F = x + y$ (Do not Graph)

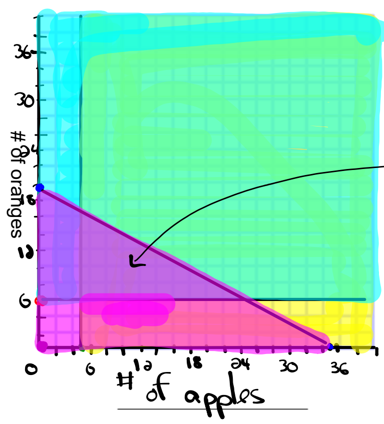
Steps to draw the graph:

$x \geq 5$	$y \geq 6$	$0.2x + 0.35y \leq 7$	
x-int	y-int	x-int (y = 0)	y-int (x = 0)
$x = 5$	$y = 6$	$0.2x + 0.35y = 7$	$0.2x + 0.35y = 7$
(5,0)	(0,6)	$0.2x = 7$	$0.35y = 7$
vertical	Horizontal	$x = 35$	$y = 20$
		(35,0)	(0,20)

• Plot Points:

• Boundary Lines:
$x \geq 5$ (Solid)
$y \geq 6$ (Solid)
$0.2x + 0.35y \leq 7$ (Solid)

total # of fruit



Solution set should be stipled

Test (0,0)

$x \geq 5$	$y \geq 6$	$0.2x + 0.35y \leq 7$
$0 \geq 5$	$0 \geq 6$	$0.2(0) + 0.35(0) \leq 7$
False	False	$0 \leq 7$
Shade where (0,0) isn't	Shade where (0,0) isn't	True
		Shade where (0,0) is

5.4 pg. 248 Question 2

A fast-food concession stand sells hot dogs and hamburgers.

- Daily sales can be as high as 300 hamburgers and hot dogs combined.
- The stand has room to stock no more than 200 hot dogs and no more than 150 hamburgers.
- Hot dogs are sold for \$3.25, and hamburgers are sold for \$4.75.

Create a model that could be used to determine the combination of hamburgers and hot dogs that will result in maximum sales.

Defining Statements:

Let x = the # of hot dogs

Let y = the # of hamburgers

Let S = the total sales

Restrictions: $x \mid x \in W$

$y \mid y \in W$

Constraints:

- $x + y \leq 300$
- $x \leq 200$ (Quick Graph)
- $y \leq 150$ (Quick Graph)

Objective Function: $S = 3.25x + 4.75y$ (Do not Graph)

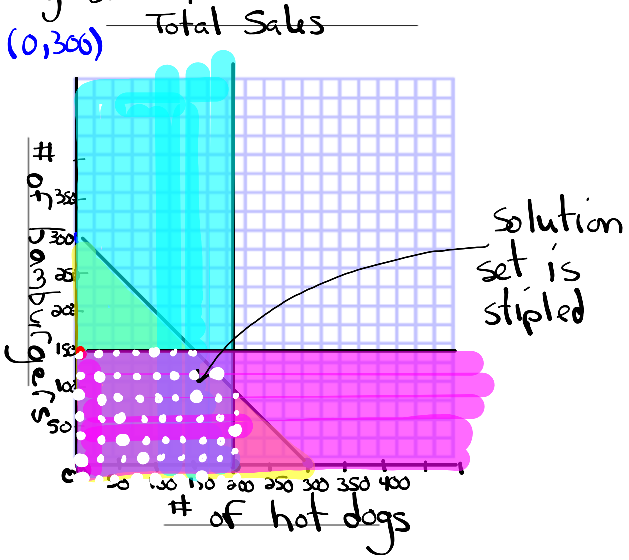
Steps to draw the graph:

$x+y \leq 300$		$x \leq 200$	$y \leq 150$
x-int (y=0)	y-int (x=0)	$x = 200$	$y = 150$
$x + y = 300$	$x + y = 300$	vertical Line	horizontal Line
$x = 300$	$y = 300$		
(300, 0)	(0, 300)		

$x+y \leq 300$ (Solid Boundary)

$x \leq 200$ (Solid Boundary)

$y \leq 150$ (Solid Boundary)



Test (0,0)

$x+y \leq 300$	$x \leq 200$	$y \leq 150$
$0+0 \leq 300$	$0 \leq 200$	$0 \leq 150$
$0 \leq 300$	TRUE	TRUE
TRUE		

Attachments

6Ws4e1.mp4

6Ws4e2.mp4