Slant Asymptotes:

2 d) $y = \frac{(x-1)^3}{x^2} = \frac{(x-1)(x-1)(x-1)}{x^2} = \frac{x^3-3x^2+3x-1}{x^2}$

(i) x int (y=0)

$x^2 \cdot 0 = \frac{(x-1)^3}{\cancel{x^2}} \cdot \cancel{x^2}$

$0 = (x-1)^3$

$0 = x-1$

$1 = x$

(1,0)

(ii) y-int (x=0)

$y = \frac{(0-1)^3}{0^2} = \frac{-1}{0}$

undefined

No y-int

(iii) VA: $x^2 = 0$

$\boxed{x = 0}$

$\lim_{x \to 0^-} \frac{(-)}{(+)} = -\infty$

$(x=-0.1)$

$\lim_{x \to 0^+} \frac{(-)}{(+)} = -\infty$

$(x=0.1)$

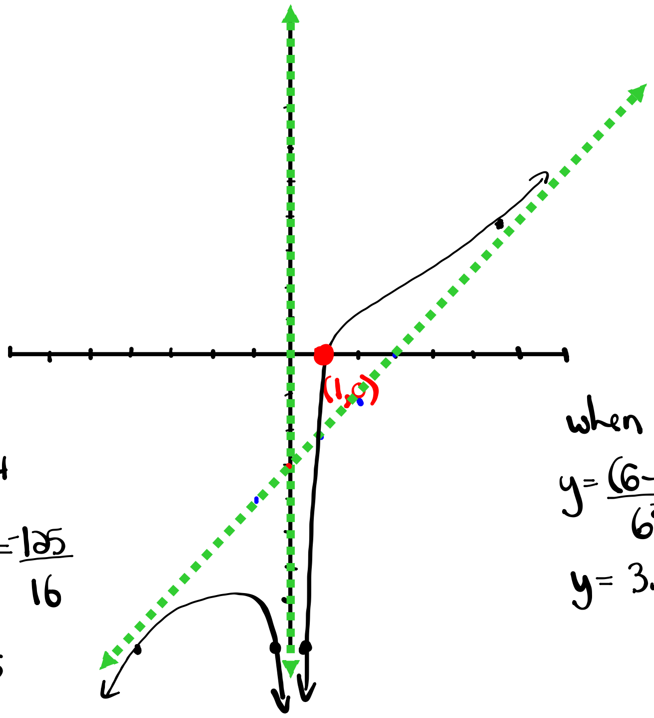
(iv) SA:

$$\begin{array}{r} x - 3 \\ x^2 \overline{\big)\, x^3 - 3x^2 + 3x - 1} \\ -(x^3) \\ \hline -3x^2 + 3x - 1 \\ -(-3x^2) \\ \hline 3x - 1 \ R \end{array}$$

$\boxed{y = x-3}$

$m = \frac{1}{1} \frac{\text{rise}}{\text{run}}$

$b = -3$ y-int



when $x=-4$

$y = \frac{(-4-1)^3}{(-4)^2} = \frac{-125}{16}$

$y = -7.8125$

when $x=6$

$y = \frac{(6-1)^3}{6^2} = \frac{125}{36}$

$y = 3.47$

Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \le 0$$

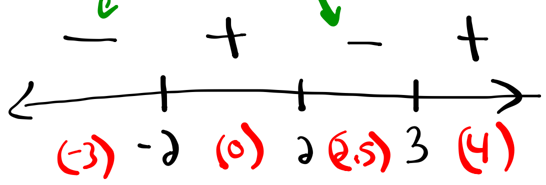
where does this function have y values that are less than or equal to zero

$y = x^3 - 3x^2 - 4x + 12$ (factor)

$y = x^2(x-3) - 4(x-3)$

$y = (x-3)(x^2-4)$

$y = (x-3)(x-2)(x+2)$



Find x int (y=0)

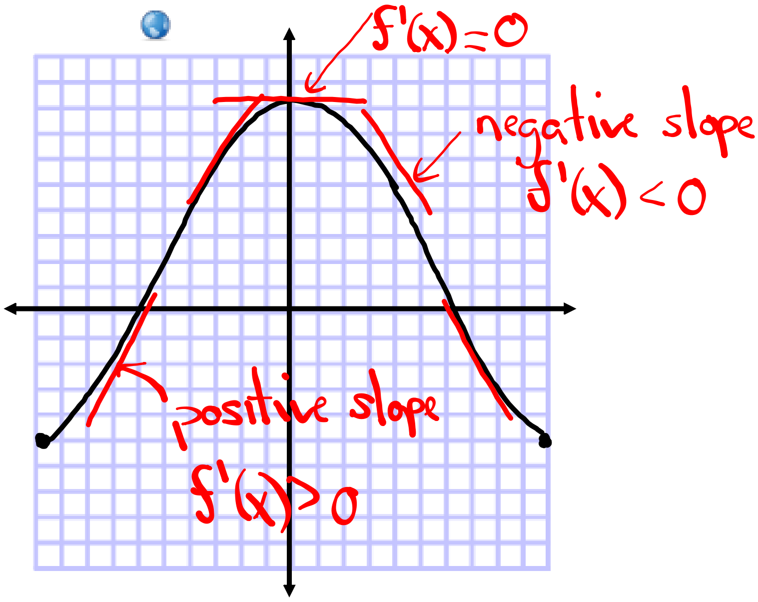
$0 = (x-3)(x-2)(x+2)$

$x-3=0 \;|\; x-2=0 \;|\; x+2=0$

$x=3 \;|\; x=2 \;|\; x=-2$

$x \in (-\infty, -2] \cup [2, 3]$

Increasing and Decreasing Functions



$\boxed{\text{slope of tangent line} = f'(x)}$

Test for Increasing and Decreasing Functions

1. If $f'(x) > 0$ for all x in an interval I, then f is increasing on I. f'(x) is positive

2. If $f'(x) < 0$ for all x in an interval I, then f is decreasing on I f'(x) is negative

Example 1

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

$f'(x) = 1 - 5x + 4x^2$

$f'(x) = -5 + 8x$

$-$ $+$

(0) $5/8$ (1)

CV: $f'(x) = 0$ or undefined

$0 = -5 + 8x$

$5 = 8x$

$5/8 = x$

CV: $x = \frac{5}{8} = 0.625$

Thus f will be increasing on the interval $\left(\frac{5}{8}, \infty\right)$

Similarly,

Thus f will be decreasing on the interval $\left(-\infty, \frac{5}{8}\right)$

Example 2

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

Solution

$y = x^3 + 6x^2 + 9x + 2$

$y' = 3x^2 + 12x + 9$

$y' = 3(x^2 + 4x + 3)$

$y' = 3(x+3)(x+1)$

+   −   +

(−4)   −3   (−2)   −1   (0)

Increasing on $(-\infty, -3) + (-1, \infty)$

CV: $y' = 0$ or undefined

$0 = 3(x+3)(x+1)$

$3 \neq 0$	$x+3=0$	$x+1=0$
	$x=-3$	$x=-1$

CV: $x = -3, -1$

Example 3

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

Solution

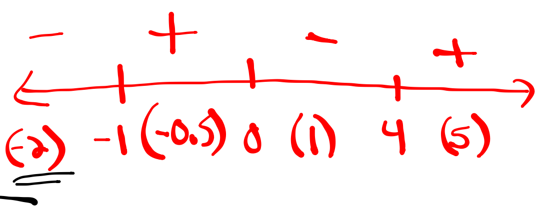
First we compute the derivative and factor it:

$$f(x) = x^4 - 4x^3 - 8x^2 - 1$$

$$f'(x) = 4x^3 - 12x^2 - 16x$$

$$f'(x) = 4x(x^2 - 3x - 4)$$

$$f'(x) = 4x(x-4)(x+1)$$



cv: $f'(x) = 0$ or undefined

$$0 = 4x(x-4)(x+1)$$

$4x = 0$	$x - 4 = 0$	$x + 1 = 0$
$x = 0$	$x = 4$	$x = -1$

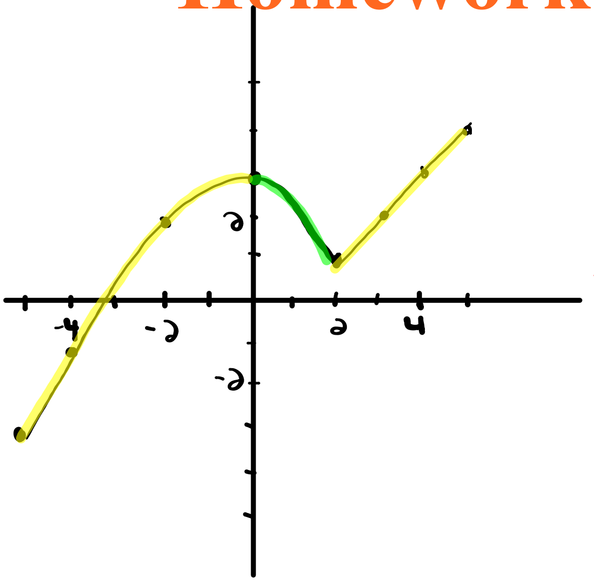
cv: $x = -1, 0, 4$

Increasing on $(-1,0) + (4,\infty)$

Decreasing on $(-\infty,-1) + (0,4)$

Homework

1



Increasing on:

(-5,0) + (2,5)

$-5<x<0 \quad 2<x<5$

Decreasing on:

(0,2)

$0<x<2$

Homework

$f'(x) > 0 \rightarrow$ increasing
$f'(x) < 0 \rightarrow$ decreasing

② b) $f(x) = x^4$

$f'(x) = 4x^3$

− +
(−1) 0 (1)

CV: $f'(x) = 0$ or undefined

$0 = 4x^3$

$0 = x^3$

$0 = x$

CV: $x = 0$

decreasing on $(-\infty, 0)$
$\boxed{\text{increasing on } (0, \infty)}$

④ g) $y = x\sqrt{4-x} = \underbrace{x}_{f(x)}\underbrace{(4-x)^{1/2}}_{g(x)}$ $f'(x)g(x) + f(x)g'(x)$

$y' = 1(4-x)^{1/2} + x\left(\frac{1}{2}\right)(4-x)^{-1/2}(-1)$

$y' = (4-x)^{1/2} - \frac{x}{2}(4-x)^{-1/2}$ $\frac{(4-x)^{1/2}}{(4-x)^{-1/2}} = (4-x)^{1/2-(-1/2)}$

$y' = (4-x)^{-1/2}\left[(4-x) - \frac{x}{2}\right]$

$y' = (4-x)^{-1/2}\left(\frac{8}{2} - \frac{2x}{2} - \frac{x}{2}\right)$

$y' = (4-x)^{-1/2}\left(\frac{8-3x}{2}\right)$

$y' = \frac{8-3x}{2(4-x)^{1/2}}$

+ − und.
(0) $\frac{8}{3}$ (3) 4 (5)
$\frac{(+)}{(+)}$ $\frac{(-)}{(+)}$ $\frac{(-)}{\text{und}}$

CV: $y' = 0$ or undefined

$8 - 3x = 0$	$2(4-x)^{1/2} = 0$
$8 = 3x$	$(4-x)^{1/2} = 0$
$\frac{8}{3} = x$	$4 - x = 0$
$x \doteq 2.\overline{6}$	$4 = x$

Increasing on $(-\infty, \frac{8}{3})$
Decreasing on $(\frac{8}{3}, 4)$

CV: $x = \frac{8}{3}, 4$

$x^2 = 9$
$x = \pm 3$

④ h) $y = (x^2-9)^{2/3}$

$$y' = \frac{2}{3}(x^2-9)^{-1/3}(2x)$$

$$y' = \frac{4x}{3(x^2-9)^{1/3}}$$

$-\quad +\quad -\quad +$

$(-4)\quad -3\quad (-1)\quad 0\quad (1)\quad 3\quad (4)$

$\frac{(-)}{(+)}\quad \frac{(-)}{(-)}\quad \frac{(+)}{(-)}\quad \frac{(+)}{(+)}$

CV: $y' = 0$ or undefined

$4x = 0$

$x = 0$

$3(x^2-9)^{1/3} = 0$

$(x^2-9)^{1/3} = 0$

$x^2-9 = 0$

$x^2 = 9$

$x = \pm 3$

Increasing $(-3,0)$ + $(3,\infty)$

$-3 < x < 0$ + $x > 3$

Decreasing on $(-\infty,-3)$ + $(0,3)$

$x < -3$ + $0 < x < 3$

CV: $x = -3, 0, 3$