6.6 Vertex Form of a Quadratic Function

$$y = a(x - h)^2 + k$$

Stretch Factor; Horizontal Translation; Vertical Translation

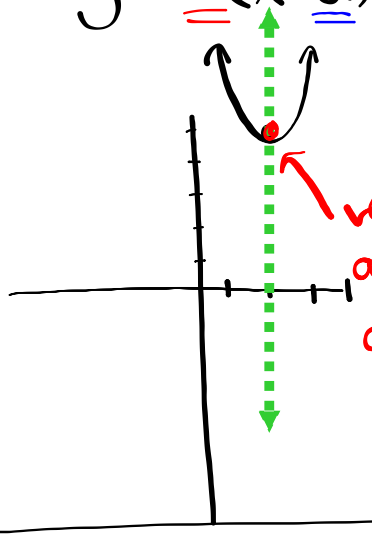
$y = \underline{3}(x - \underline{2})^2 + \underline{5}$

*change sign

$a = 3$ $h = 2$ $k = 5$

$SF = 3$

Vertex: $(2, 5)$ min



vertex is a min

$a > 0$

axis of symmetry: $x = 2$

opens upward

$y = \underline{-5}(x \underline{+6})^2 \underline{-2}$

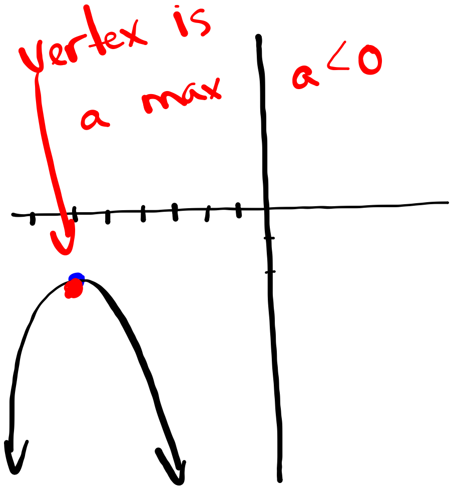
*change sign

$a = -5$ $h = -6$ $k = -2$

$SF = 5$

vertex is a max

$a < 0$



Vertex: $(-6, -2)$ max

axis of symmetry: $x = -6$

opens downward

6.6

REMINDER

The Effect of Parameter "a"...

Consider the graphs of the following functions:

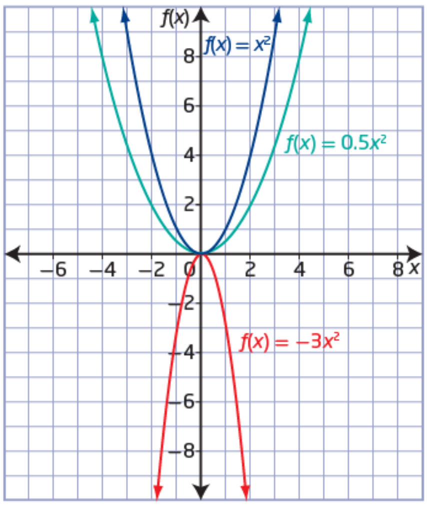
Consider the graphs of the following functions:

$f(x) = x^2$

$f(x) = 0.5x^2$ — The parabola is wider in relation to the y-axis than $f(x) = x^2$ and opens upward.

$f(x) = -3x^2$ — The parabola is narrower in relation to the y-axis than $f(x) = x^2$ and opens downward.

- Parameter a determines the orientation and shape of the parabola.
- Parameter "*a*" is also know as the "stretch factor" and is **ALWAYS POSITIVE.** SF = $|a|$
- When the sign in front of "*a*" is positive, the parabola will open upward and have a minimum point.
- When the sign in front of "*a*" is negative, the parabola will open downward and have a maximum value.
- When $a > 1$, the parabola will be stretched and become narrower.
- When $0 < a < 1$, the parabola will be compressed and become wider.



parabola
The shape of the graph of any quadratic relation.

- The maximum or minimum point on a parabola is otherwise know as the VERTEX.

6.6

The Effect of Parameter " "...

Consider the graphs of the following functions:

$f(x) = x^2$ h=0

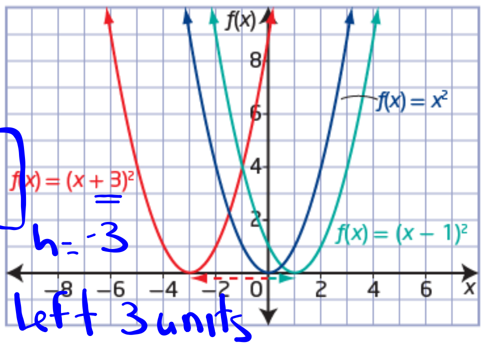
$f(x) = (x - 1)^2$ h=1

$f(x) = (x + 3)^2$ h= -3

Since h = +1, the graph is translated 1 unit right.

Since h = −3, the graph is translated 3 units left.

- Parameter h translates the parabola horizontally h units relative to the graph of $f(x) = x^2$.



h= -3

left 3 units

h=1

right 1 unit

You **must change the siqn** to determine the value of parameter h

After the "sign change":

- When "h" is positive, the parabola will move h units to the right on the x-axis.
- When "h" is negative, the parabola will move "h units to the left on the x-axis.
- Parameter "h" is also the same as the x-cordinate of the vertex.
- The equation of the imaginary line that divides a parabola is half vertically, otherwise know as the AXIS OF SYMMETRY, is written as x = h.

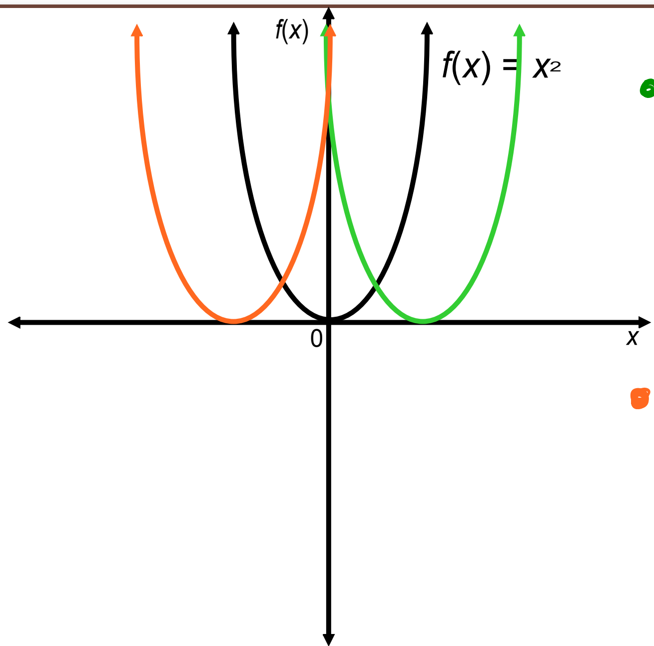
6.6

The Effect of Parameter p in $f(x) = (x - p)^2$ on the Graph of $f(x) = x^2$

Match the functions and the parabolas. Highlight the functions using the coloured boxes to indicate your matching.

$f(x) = (x - 3)^2$

$f(x) = (x + 3)^2$



$f(x) = (x - 3)^2$
$h = 3$
3 units right

$f(x) = (x + 3)^2$
$h = -3$
3 units left

Check answer

6.6

The Effect of Parameter "k"...

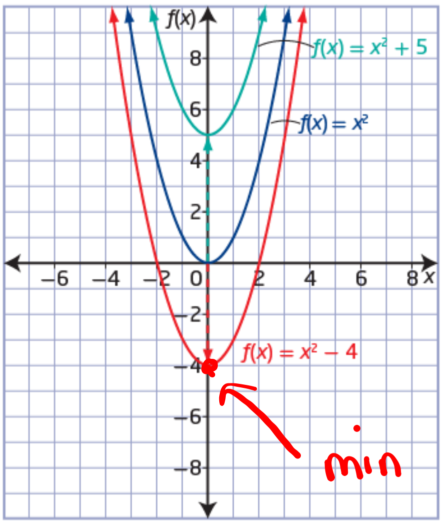
Consider the graphs of the following functions:

$f(x) = x^2$ k=0

$f(x) = x^2 + 5$ k=5 The graph is translated 5 units up.

$f(x) = x^2 - 4$ k=-4 The graph is translated 4 units down.

- Parameter k translates the parabola vertically k units relative to the graph of $f(x) = x^2$.



- When "k is positive, the parabola will move up "k units on the y-axis.
- When k is negative, the parabola will move down "k units on the y-axis.
- The parameter "k is also the same as the y-coordinate of the vertex.
- * The maximum or minimum point on a parabola, otherwise know as the VERTEX, is written as (h, k)

maximum → if the parabola opens downward

minimum → if the parabola opens upward

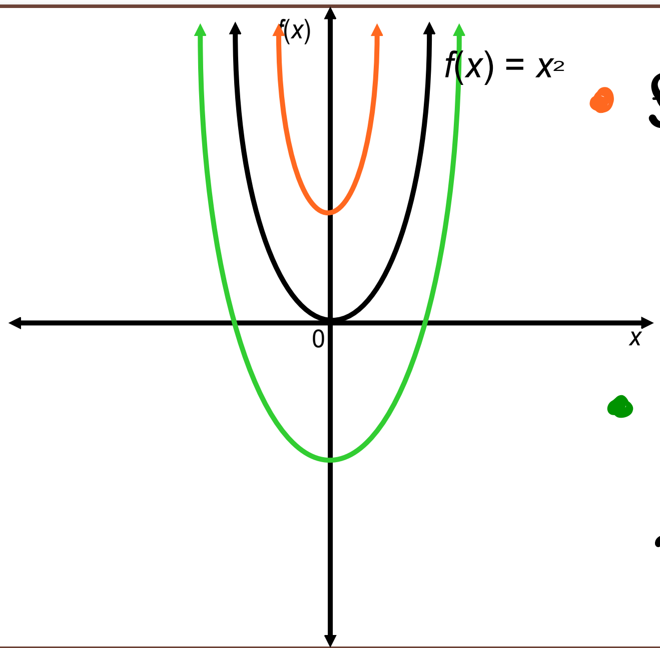
6.6

The Effect of Parameter *q* in $f(x) = x^2 + q$ on the Graph of $f(x) = x^2$

Match the functions and the parabolas. Highlight the functions using the coloured boxes to indicate your matching.

$f(x) = x^2 + 3$

$f(x) = x^2 - 3$



$f(x) = x^2 + 3$
$k = 3$
3 units up

$f(x) = x^2 - 3$
$k = -3$
3 units down

Check answer

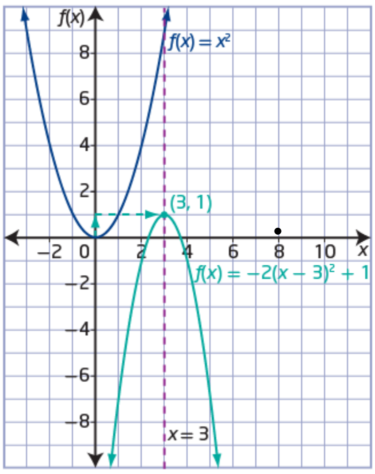
6.6 Combining Transformations

Consider the graphs of the following functions:

$f(x) = x^2$

$f(x) = -2(x - 3)^2 + 1$

$a = -2$ $h = 3$ $k = 1$



Stretch Factor:

$SF = |-2| = 2$

Direction of Opening:

downward $a < 0$

Vertex:

$(3, 1)$

Equation of the Axis of Symmetry:

$x = 3$

Domain:

$\{x \mid x \in R\}$

Range:

$\{y \mid y \leq 1, y \in R\}$

Reflecting

A) Does the value of a in a quadratic function always represent the same characteristic of the parabola, whether the function is written in standard form, factored form, or vertex form? Explain.

B) Neil claims that when you are given the vertex form of a quadratic function, you can determine the domain and range without having to graph the function. Do you agree or disagree? Explain.

C) Which form of the quadratic function—standard, factored, or vertex—would you prefer to start with, if you wanted to sketch the graph of the function? Explain.

Answers

A) Yes. It always represents the direction of opening and narrowness of the parabola.

B) Agree. If I were given $f(x) = 2(x + 3)^2 + 5$, I would know that the graph opens upward because 2 is positive, and that the vertex would be located at $(-3, 5)$. This means that the function contains a minimum value at the vertex. The domain would be $\{x \in \mathrm{R}\}$ and the range would be $\{y \geq 5, y \in \mathrm{R}\}$.

Disagree. If the context of the problem were a "projectile" question, then I would need to know the x-intercepts to state the domain, and I would not be able to state them simply by looking at the equation in vertex form.

C) Standard form would be fine for graphing the function with technology, and would be useful if I needed the y-intercept. I would prefer factored form if I wanted to locate the x-intercepts exactly. I would prefer vertex form if I was most interested in locating the vertex.

APPLY the Math

EXAMPLE 1 Sketching the graph of a quadratic function given in vertex form

Sketch the graph of the following function:

$$y = 2(x - 3)^2 - 4$$

State the domain and range of the function.

Samuel's Solution

$y = 2(x - 3)^2 - 4$

Since $a > 0$, the parabola opens upward.

The vertex is at $(3, -4)$.

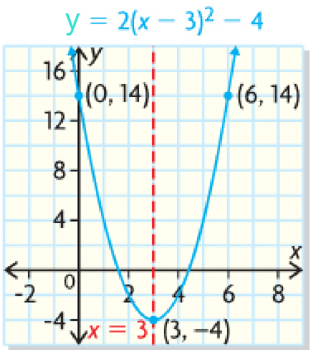
The equation of the axis of symmetry is $x = 3$.

> The function was given in vertex form. I listed the characteristics of the parabola that I could determine from the equation.

$$\begin{aligned} y &= 2(0 - 3)^2 - 4 \\ y &= 2(-3)^2 - 4 \\ y &= 2(9) - 4 \\ y &= 18 - 4 \\ y &= 14 \end{aligned}$$

Point $(0, 14)$ is on the parabola.

> To determine another point on the parabola, I substituted 0 for x.



> I plotted the vertex and the point I had determined, (0, 14). Then I drew the axis of symmetry. I used symmetry to determine the point that is the same horizontal distance from (0, 14) to the axis of symmetry. This point is (6, 14). I connected all three points with a smooth curve.

Domain and range:

$\{(x, y) \mid x \in \mathrm{R}, y \geq -4, y \in \mathrm{R}\}$

EXAMPLE 3 Reasoning about the number of zeros that a quadratic function will have

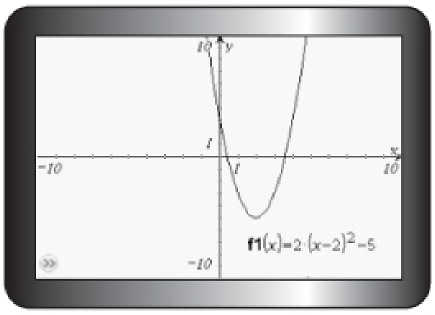
Randy claims that he can predict whether a quadratic function will have zero, one, or two zeros if the function is expressed in vertex form. How can you show that he is correct?

TI-nspire

Eugene's Solution

$f(x) = 2(x - 2)^2 - 5$
Conjecture: two zeros

The vertex of the parabola that is defined by the function is at (2, −5), so the vertex is below the *x*-axis. The parabola must open upward because *a* is positive. Therefore, I should observe two *x*-intercepts when I graph the function.



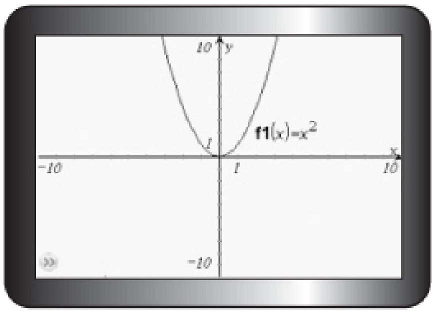
To test my conjecture, I graphed the function on a calculator. I can see two *x*-intercepts on my graph, so the function has two zeros.

The graph supports my conjecture.

$f(x) = x^2$
$f(x) = (x - 0)^2 + 0$
Conjecture: one zero

I decided to use the basic quadratic function, since this provided me with a convenient location for the vertex, (0, 0).

Since the vertex is on the *x*-axis and the parabola opens up, this means that I should observe only one *x*-intercept when I graph the function.

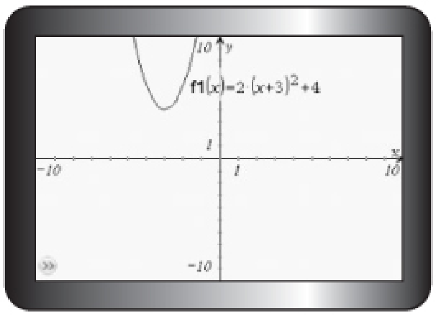


To test my conjecture, I graphed the function on a calculator. Based on my graph, I concluded that the function has only one zero.

The graph supports my conjecture.

$f(x) = 2(x + 3)^2 + 4$
Conjecture: no zeros

The vertex of the parabola that is defined by this function is at (−3, 4), and the parabola opens upward. The vertex lies above the *x*-axis, so I should observe no *x*-intercepts when I graph the function.



To test my conjecture, I graphed the function on a calculator. I concluded that the function has no zeros.

The graph supports my conjecture.

In Summary

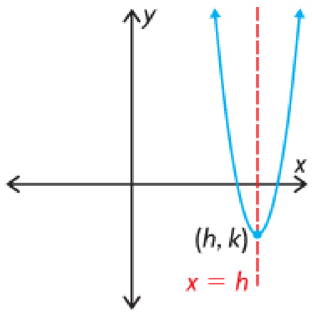
Key Idea

- The vertex form of the equation of a quadratic function is written as follows:
$$y = a(x - h)^2 + k$$
The graph of the function can be sketched more easily using this form.

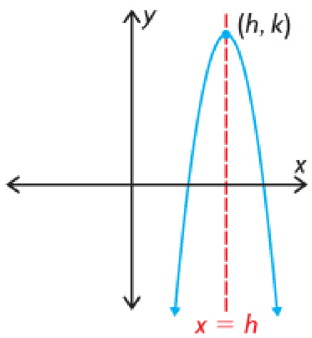
Need to Know

- A quadratic function that is written in vertex form,
$$y = a(x - h)^2 + k$$
has the following characteristics:
  - The vertex of the parabola has the coordinates (h, k).
  - The equation of the axis of symmetry of the parabola is $x = h$.
  - The parabola opens upward when $a > 0$, and the function has a minimum value of k when $x = h$.
  - The parabola opens downward when $a < 0$, and the function has a maximum value of k when $x = h$.

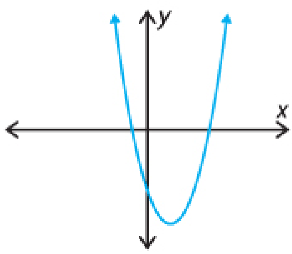
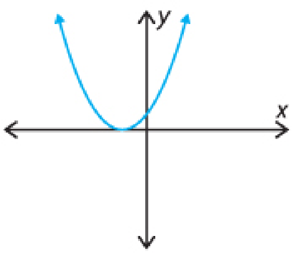
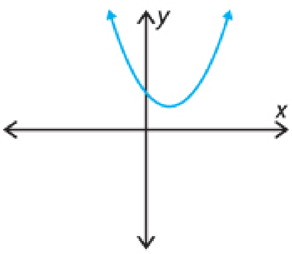
$y = a(x - h)^2 + k$
$a > 0$



$y = a(x - h)^2 + k$
$a < 0$



- A parabola may have zero, one, or two x-intercepts, depending on the location of the vertex and the direction in which the parabola opens. By examining the vertex form of the quadratic function, it is possible to determine the number of zeros, and therefore the number of x-intercepts.

Two x-intercepts	One x-intercept	No x-intercepts
		

Assignment: pages 335 - 337

Questions 1, 2(ac), 3, 4, 5, 8abc

SOLUTIONS => 6.6 Vertex Form of a Quadratic Function.

1. For each quadratic function below, identify the following:

i) the direction in which the parabola opens.
ii) the coordinates of the vertex
iii) the equation of the axis of symmetry.

a) $f(x) = (x-3)^2 + 7$

i) Opens Upward
ii) Vertex: $(3, 7)$
iii) Axis of Symmetry: $x = 3$

b) $m(x) = -2(x+7)^2 - 3$

i) Opens Downward
ii) Vertex $(-7, -3)$
iii) Axis of Symmetry: $x = -7$

c) $g(x) = 7(x-2)^2 - 9$

i) Opens Upward
ii) Vertex $(2, -9)$
iii) Axis of Symmetry: $x = 2$

d) $n(x) = \frac{1}{2}(x+1)^2 + 10$

i) Opens Upward
ii) Vertex $(-1, 10)$
iii) Axis of Symmetry: $x = -1$

e) $r(x) = -2x^2 + 5$

$r(x) = -2(x - 0)^2 + 5$

i) Opens Downward

ii) Vertex $(0, 5)$

iii) Axis of Symmetry: $x = 0$

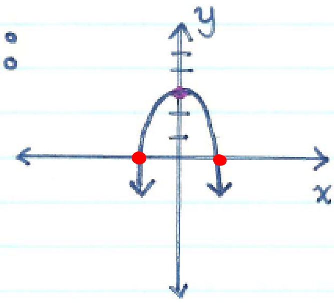
2. Predict which of the following functions have a minimum value. Also predict the number of x-intercepts that each function has. Test your predictions by sketching the graph of each function.

{a & c}

a) $f(x) = -x^2 + 3$
$f(x) = -(x-0)^2 + 3$
Vertex: $(0,3)$

↳ Opens Downward ⇒ Maximum
↳ 2 x-intercepts

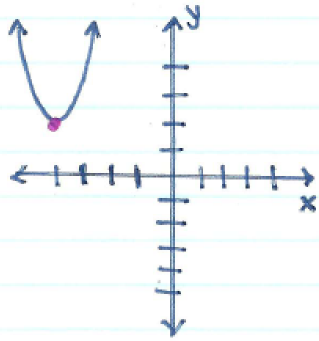
Sketch:



c) $m(x) = (x+4)^2 + 2$
Vertex: $(-4,2)$

↳ Opens Upward ⇒ Minimum
↳ No x-intercepts

Sketch:



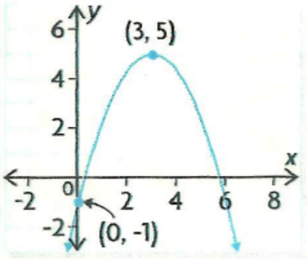
3. Determine the value of a, if point (-1,4) is on the quadratic function:

$$f(x) = a(x+2)^2 + 7$$

$(-1, 4)$
x y

$$y = a(x+2)^2 + 7$$
$$4 = a(-1+2)^2 + 7$$
$$4 = a(1)^2 + 7$$
$$4 = a(1) + 7$$
$$4 - 7 = 1a$$
$$\frac{-3}{1} = \frac{1a}{1}$$
$$-3 = a$$

4. Which equation represents the graph? Justify your decision.



* Opens Downward

** Vertex (3,5)

*** c = -1 (0,-1)

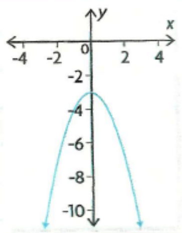
A. $y = -\frac{2}{3}x^2 + 5$

$\boxed{\text{C.}}$ $y = -\frac{2}{3}(x-3)^2 + 5$

B. $y = -(x-3)^2 + 5$

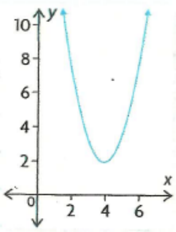
D. $y = \frac{2}{3}(x-3)^2 + 5$

5. Match each equation with its corresponding graph. Explain your reasoning.

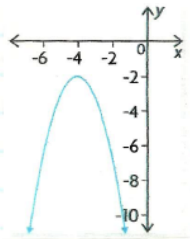
i) 

Match: c) $y = -x^2 - 3$

Vertex $(0, -3)$; Opens Downward

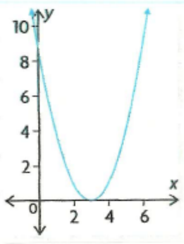
ii) 

Match: d) $y = (x-4)^2 + 2$

iii) 

Match: b) $-(x+4)^2-2$

Vertex $(-4,-2)$; Opens Downward

iv) 

Match: a) $y=(x-3)^2$

Vertex $(3,0)$: Opens Upward

8. Marleen and Candice are both 6 ft tall, and they play on the same college volleyball team. In a game, Candice set up Marleen with an outside high ball for an attack hit. Using a video of the game, their coach determined that the height of the ball above the court, in feet, on its path from Candice to Marleen could be defined by the function

$$h(x) = -0.03(x-9)^2 + 8$$

where x is the horizontal distance, measured in feet, from one edge of the court.

a) Determine the axis of symmetry of the parabola.

Vertex $(\underline{\underline{9}}, 8)$ Axis of Symmetry $\boxed{x = 9}$

b) Marleen hit the ball at its highest point. How high above the court was the ball when she hit it?

The ball was 8 ft above the court when she hit it.

c) How high was the ball when Candice set it, if she was 2 ft from the edge of the court?

$h = -0.03(x-9)^2 + 8$
$h = -0.03(2-9)^2 + 8$
$h = -0.03(-7)^2 + 8$
$h = -0.03(49) + 8$
$h = -1.47 + 8$
$h = 6.53$ ft

The ball was 6.53 ft high when Candice set it.

7s6e1 final.mp4

7s6e2 final.mp4

7s6e3 final.mp4

7s6e4 final.mp4

fm7s6-p9.tns

FM11-7s6-ahk.gsp

FM11-7s6.gsp