Questions from homework

④ f) $h(x) = \dfrac{x-1}{x+1}$

$$h'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$h'(x) = \frac{x+1-x+1}{(x+1)^2}$$

$$h'(x) = \frac{2}{(x+1)^2}$$

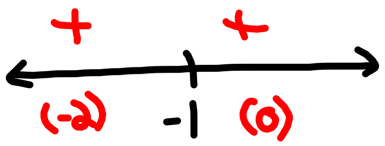
Critical Values:

$(x+1)^2 = 0$

$x+1 = 0$

$x = -1$

Make Number Line:



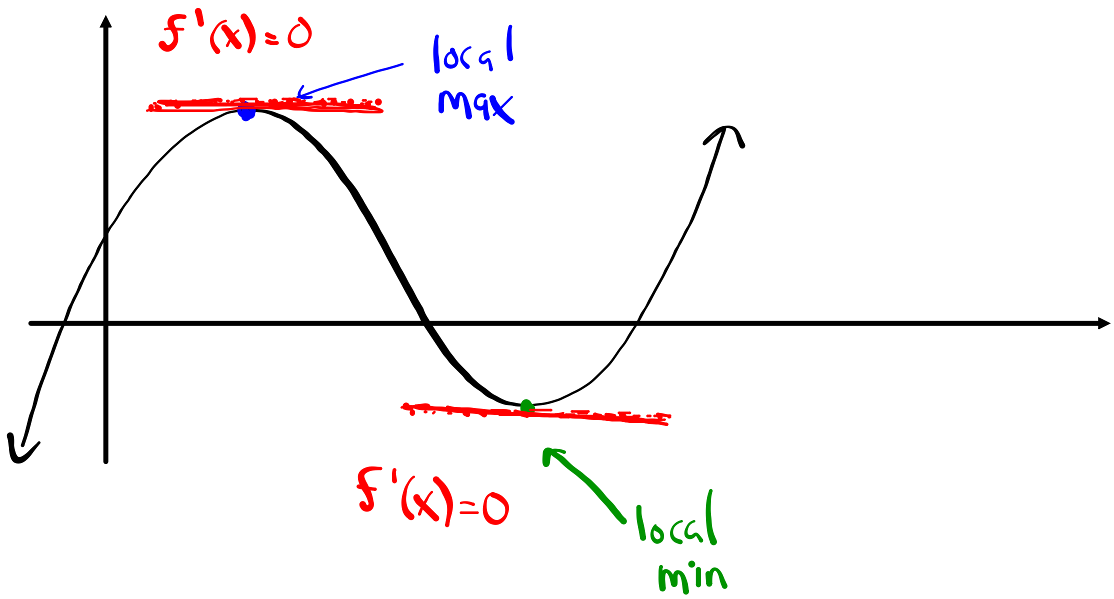
Increasing on $(-\infty, \infty)$

The First Derivative Test

critical value

If f has a local maximum or minimum at c, then c must be a critical value of f (Fermat's Theorem), but not all critical numbers give rise to a maximum or minimum. For instance, recall that 0 is a critical number of the function $y = x^3$ but this function has no maximum or minimum at a critical number.

One way of solving this is suggested by the figure below.
If f is increasing to the left of a critical number c and decreasing to the right of c, then f has a local max at c.
If f is decreasing to the left of a critical number c and increasing to the right of c, then f has a local min at c.



The First Derivative Test

Let c be a critical number of a continuous function f.

1. If $f'(x)$ changes from positive to negative at c, then f has a local max at c.

2. If $f'(x)$ changes from negative to positive at c, then f has a local min at c.

3 If $f'(x)$ does not change signs at c, then f has no max or min at c.

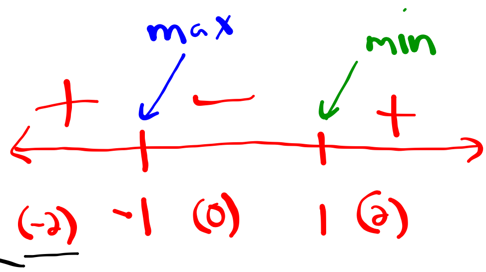
Example 1

Find the local maximum and minimum values of
$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$f'(x) = 3(x^2 - 1)$

$f'(x) = 3(x+1)(x-1)$



CV: $f'(x) = 0$

$3 \neq 0$ | $x+1=0$ | $x-1=0$
| $x=-1$ | $x=1$

Increasing on $(-\infty,-1) + (1,\infty)$
$x<-1 + x>1$

Decreasing on $(-1,1)$
$-1<x<1$

CV: $x = \pm 1$

max occurs @ $x=-1$

$f(x) = x^3 - 3x + 1$

$f(-1) = (-1)^3 - 3(-1) + 1$

$f(-1) = -1 + 3 + 1$

$f(-1) = 3$

$(-1,3)$ max

min occurs @ $x=1$

$f(x) = x^3 - 3x + 1$

$f(1) = (1)^3 - 3(1) + 1$

$f(1) = 1 - 3 + 1$

$f(1) = -1$

$(1,-1)$ min

Example 2

Find the local maximum and minimum values of
$g(x) = x^4 - 4x^3 - 8x^2 - 1$. Use this information to sketch the graph of g.

$g'(x) = 4x^3 - 12x^2 - 16x$

$g'(x) = 4x(x^2 - 3x - 4)$

$g'(x) = 4x(x-4)(x+1)$

min max min

$- \quad + \quad - \quad +$

$(-2) \quad -1 \quad (-0.5) \quad 0 \quad (1) \quad 4 \quad (5)$

CV: $f'(x) = 0$

$4x = 0$	$x - 4 = 0$	$x + 1$
$x = 0$	$x = 4$	$x = -1$

Increasing on $(-1, 0) + (4, \infty)$

Decreasing on $(-\infty, -1) + (0, 4)$

CV: $x = -1, 0, 4$

$g(x) = x^4 - 4x^3 - 8x^2 - 1$

$g(-1) = (-1)^4 - 4(-1)^3 - 8(-1)^2 - 1$

$g(-1) = 1 + 4 - 8 - 1$

$g(-1) = -4$

local min @ $(-1, -4)$

$g(x) = x^4 - 4x^3 - 8x^2 - 1$

$g(4) = (4)^4 - 4(4)^3 - 8(4)^2 - 1$

$g(4) = 256 - 256 - 128 - 1$

$g(4) = -129$

local min @ $(4, -129)$

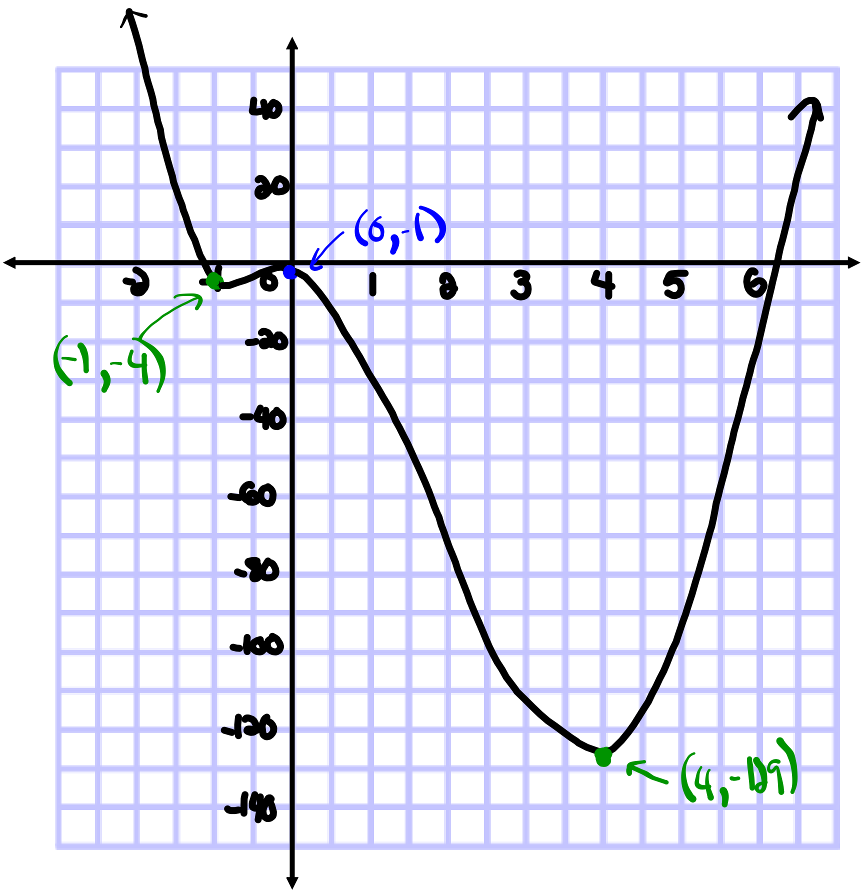
$g(x) = x^4 - 4x^3 - 8x^2 - 1$

$g(0) = (0)^4 - 4(0)^3 - 8(0)^2 - 1$

$g(0) = 0 - 0 - 0 - 1$

$g(0) = -1$

local max @ $(0, -1)$


40
20
(0,-1)
-2
-1
0
1
2
3
4
5
6
(-1,-4)
-20
-40
-60
-80
-100
-120
-140
(4,-129)

The First Derivative Test

(for absolute extreme values)

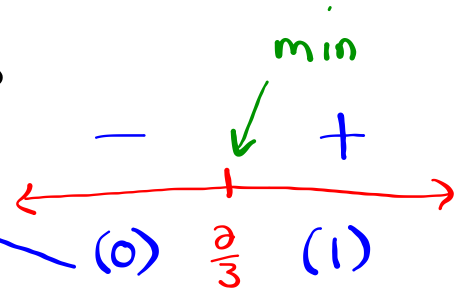
Let c be a critical number of a continuous function f.

1. If $f'(x)$ is positive for all $x < c$ and $f'(x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum value.

2. If $f'(x)$ is negative for all $x < c$ and $f'(x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum value.

Homework

① a) $f(x) = 3x^2 - 4x + 13$



$f'(x) = 6x - 4$

$0 = 6x - 4$

$\frac{4}{6} = \frac{6x}{6}$

$\frac{2}{3} = x$

CV: $x = \frac{2}{3}$

Increasing on $(\frac{2}{3}, \infty)$

Decreasing on $(-\infty, \frac{2}{3})$

min @ $x = \frac{2}{3}$

$f(x) = 3x^2 - 4x + 13$

$f(\frac{2}{3}) = 3(\frac{2}{3})^2 - 4(\frac{2}{3}) + 13$

$f(\frac{2}{3}) = {}^{1}\cancel{3}\left(\frac{4}{\cancel{9}\,3}\right) - \frac{8}{3} + 13$

$f(\frac{2}{3}) = \frac{4}{3} - \frac{8}{3} + \frac{39}{3}$

$f(\frac{2}{3}) = \frac{35}{3}$

$(\frac{2}{3}, \frac{35}{3})$

2 e) $h(x) = x^4 - 8x^2 + 6$

$h'(x) = 4x^3 - 16x$

$h'(x) = 4x(x^2 - 4)$

$h'(x) = 4x(x+2)(x-2)$

$0 = 4x(x+2)(x-2)$

$4x = 0$	$x+2=0$	$x-2=0$
$x = 0$	$x = -2$	$x = 2$

min max min

$-\ +\ -\ +$

$(-3)\ -2\ (-1)\ 0\ (1)\ 2\ (3)$

Increasing on $(-2, 0)$ and $(2, \infty)$

Decreasing on $(-\infty, -2)$ and $(0, 2)$

$h(x) = x^4 - 8x^2 + 6$

$h(-2) = (-2)^4 - 8(-2)^2 + 6$

$h(-2) = 16 - 32 + 6$

$h(-2) = -10$

$(-2, -10)$ min

$h(x) = x^4 - 8x^2 + 6$

$h(0) = (0)^4 - 8(0)^2 + 6$

$h(0) = 6$

$(0, 6)$ max

$h(x) = x^4 - 8x^2 + 6$

$h(2) = (2)^4 - 8(2)^2 + 6$

$h(2) = 16 - 32 + 6$

$h(2) = -10$

$(2, -10)$ min