# Homework

If $\sec\theta = -\sqrt{10}$ and $\sin\theta > 0$, determine the value of $\csc\theta = \frac{r}{y}$

Given:

$\sec\theta = -\frac{\sqrt{10}}{1}$

$r = \sqrt{10}$

$x = -1$

① Determine what quadrant:

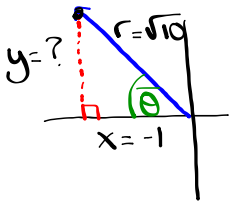
$\sec\theta < 0$ + $\sin\theta > 0$

$\cos\theta < 0$

| S | A |
|---|---|
| T | C |

Quad 2

② Draw a diagram




③ Solve for y:

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

$y = 3$ (Q2)

④ Find $\csc\theta$:

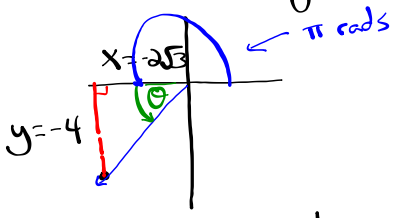
$\csc\theta = \frac{\sqrt{10}}{3}$ $r = \sqrt{10}$ $y = 3$

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$x = -2\sqrt{3}$

$y = -4$

① Draw a diagram:




② Find $\bar{\theta}$:

$\tan\bar{\theta} = \frac{-4}{-2\sqrt{3}}$

$\tan\bar{\theta} = \frac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

use radian mode → $\bar{\theta} = \tan^{-1}(1.1547)$

$\bar{\theta} = 0.86$

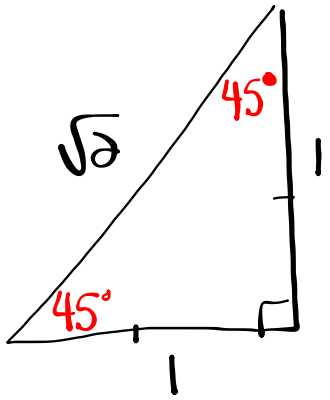
③ Find $\theta$:

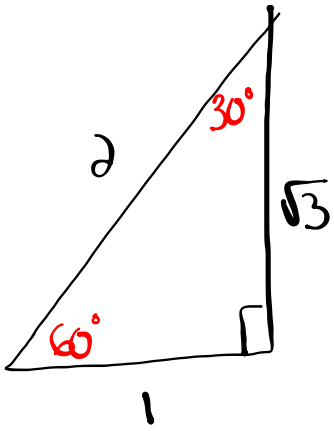
$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

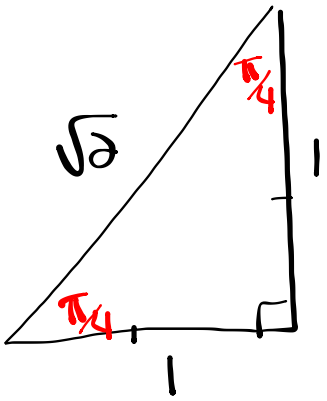
$\theta = 4$ rads

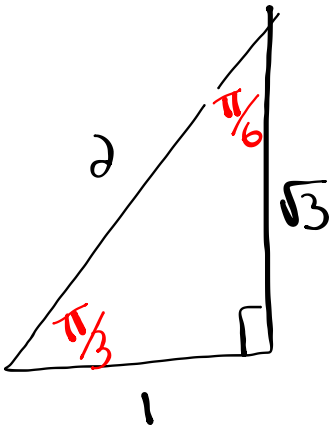
In Degrees



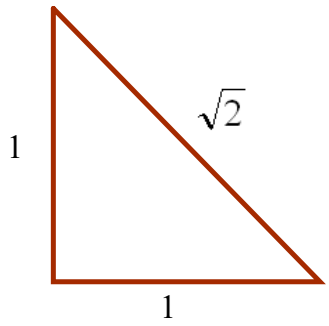



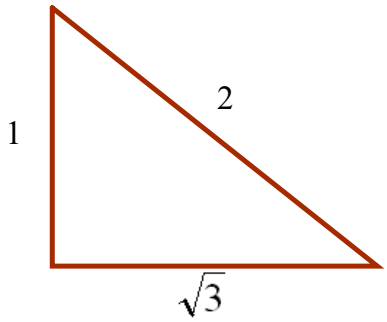

In Radians



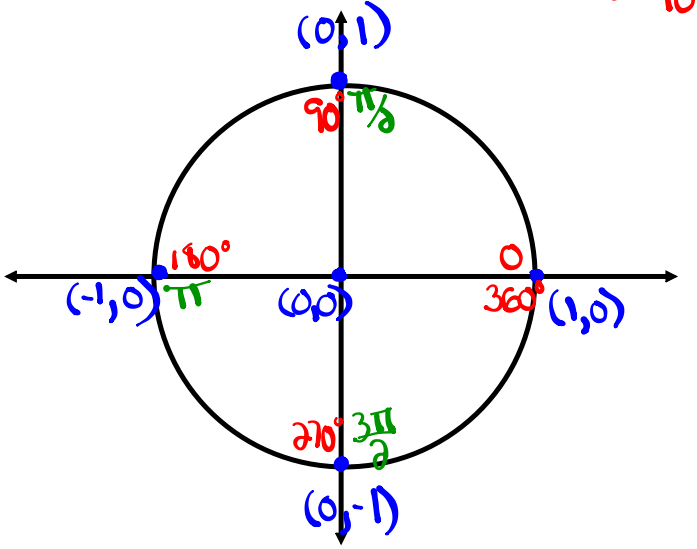



# Special Angles (in radians)



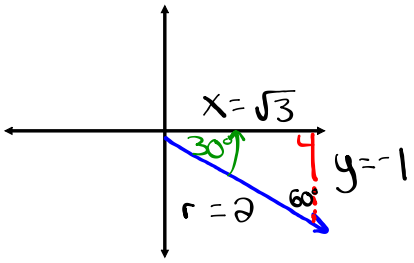

Quadrantal Angles (multiples of 90°)




The Unit Circle
- Center is @ (0,0)
- radius is 1 unit

## Solving Trig Expressions by Sketching Angles

Ex. Evaluate $\sin 690°$




① Sketch the angle.

② Draw ref. triangle.

③ Find $\bar{\theta}$:

$\bar{\theta} = 720° - 690°$

($720°$: x-axis; $\theta$: $690°$)

$\bar{\theta} = 30°$

④ Label triangle.

⑤ Determine the trig ratio:

$\sin 690° = \frac{-1}{2}$ $\quad y=-1,\ r=2$

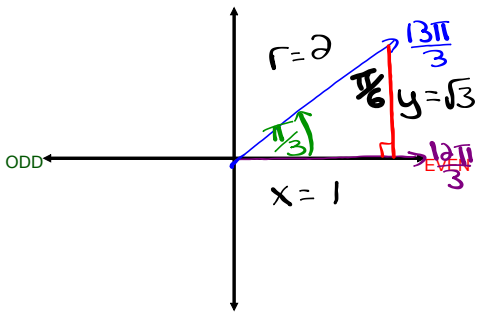
Ex. $\cos \frac{13\pi}{3}$

① Sketch the angle.

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

$4\pi$

Even




② Draw ref. triangle.

③ Find $\bar{\theta}$:

$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

④ Label triangle.

⑤ Determine the trig ratio:

$\cos \frac{13\pi}{3} = \frac{1}{2}$ $\quad x=1,\ r=2$

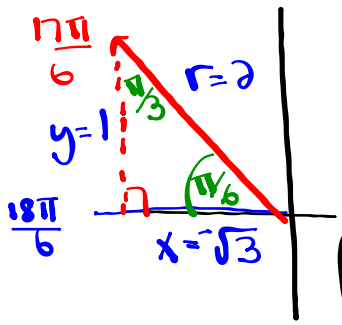
# Homework

## Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan\frac{17\pi}{6} = \frac{-1}{\sqrt{3}}$ 2. $\sin\frac{15\pi}{4} = \frac{-1}{\sqrt{2}}$ 3. $\cos\left(-\frac{21\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

① $\frac{16\pi}{6}, \boxed{\frac{17\pi}{6}}, \frac{18\pi}{6}$

$3\pi$ (odd)




$\tan\frac{17\pi}{6} = \frac{y}{x}$

$\tan\frac{17\pi}{6} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan\frac{17\pi}{6} = -\frac{\sqrt{3}}{3}$

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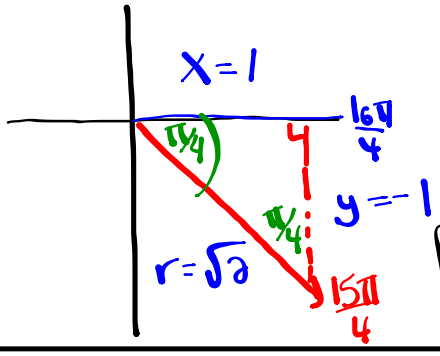
② $\frac{14\pi}{4}, \boxed{\frac{15\pi}{4}}, \frac{16\pi}{4}$

$4\pi$ (Even)




$\sin\frac{15\pi}{4} = \frac{y}{r}$

$\sin\frac{15\pi}{4} = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin\frac{15\pi}{4} = -\frac{\sqrt{2}}{2}$

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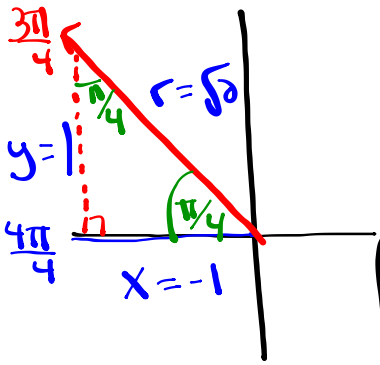
③ $-\frac{21\pi}{4} + \frac{6\pi}{1}$

$-\frac{21\pi}{4} + \frac{24\pi}{4}$

$\frac{3\pi}{4}$

$\frac{2\pi}{4}, \boxed{\frac{3\pi}{4}}, \frac{4\pi}{4}$

$1\pi$ (odd)

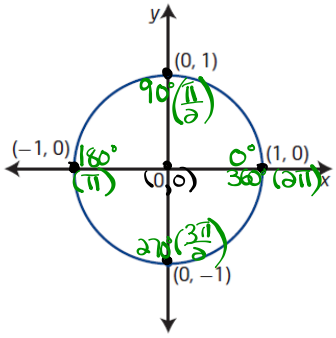



$\cos\left(\frac{-21\pi}{4}\right) = \frac{x}{r}$

$\cos\left(\frac{-21\pi}{4}\right) = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\cos\left(\frac{-21\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

# Unit Circle




**unit circle**

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$ → Ex: $\sin 90° = 1$

$x = 0$
$y = 1$
$r = 1$

$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$ → Ex: $\cos\pi = -1$

$x = -1$
$y = 0$
$r = 1$

$\tan\theta = \frac{y}{x}$ → Ex: $\tan 270° = -\frac{1}{0}$ undefined

$x = 0$
$y = -1$
$r = 1$

(0, -1)

$\csc\theta = \frac{r}{y} = \frac{1}{y}$ → Ex: $\csc 360° = \frac{1}{0}$ undefined

$x = 1$
$y = 0$
$r = 1$

(1, 0)

$\sec\theta = \frac{r}{x} = \frac{1}{x}$ → Ex: $\sec 5\pi = \frac{1}{-1} = -1$

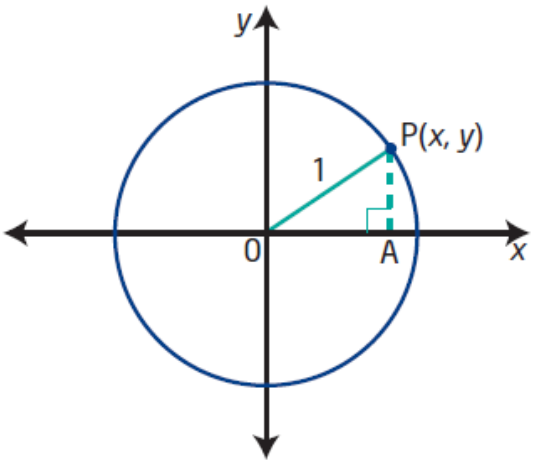
$x = -1$
$y = 0$
$r = 1$

(-1, 0)

$\cot\theta = \frac{x}{y}$ → Ex: $\cot\frac{3\pi}{2} = \frac{0}{-1} = 0$

$x = 0$
$y = -1$
$r = 1$

(0, -1)




$x^2 + y^2 = r^2$ on unit circle $r = 1$

The equation of the unit circle is $x^2 + y^2 = 1$.

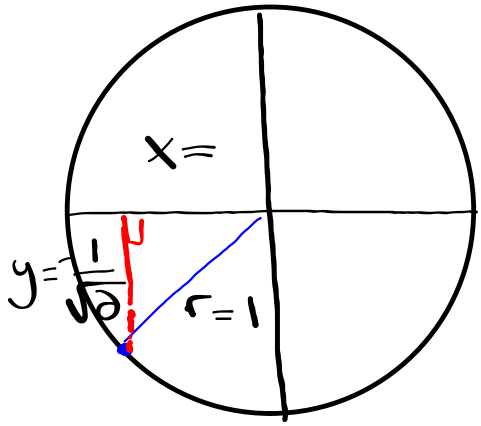
Determine the equation of a circle with centre at the origin and radius 6.

# Problems Involving the Unit Circle:

## Determine Coordinates for Points of the Unit Circle

Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

- the $y$-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III




$x^2 + y^2 = r^2$

$x^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = (1)^2$

$x^2 + \frac{1}{2} = 1$ $-\frac{1}{2}$ $-\frac{1}{2}$

$x^2 = \frac{2}{2} - \frac{1}{2}$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{\sqrt{1}}{\sqrt{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

$x = -\frac{1}{\sqrt{2}}$ Q3

The coordinates are:

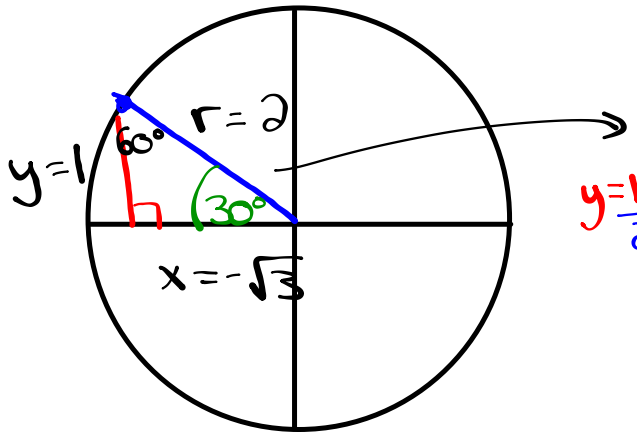
$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ or

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

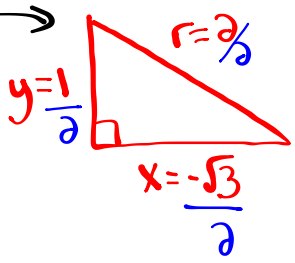
## Problems Involving the Unit Circle:

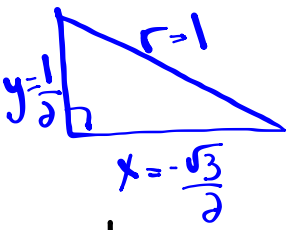
If $P(150°)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of...

$(x, y)$




Scale the diagram so that $r=1$ (unit circle)



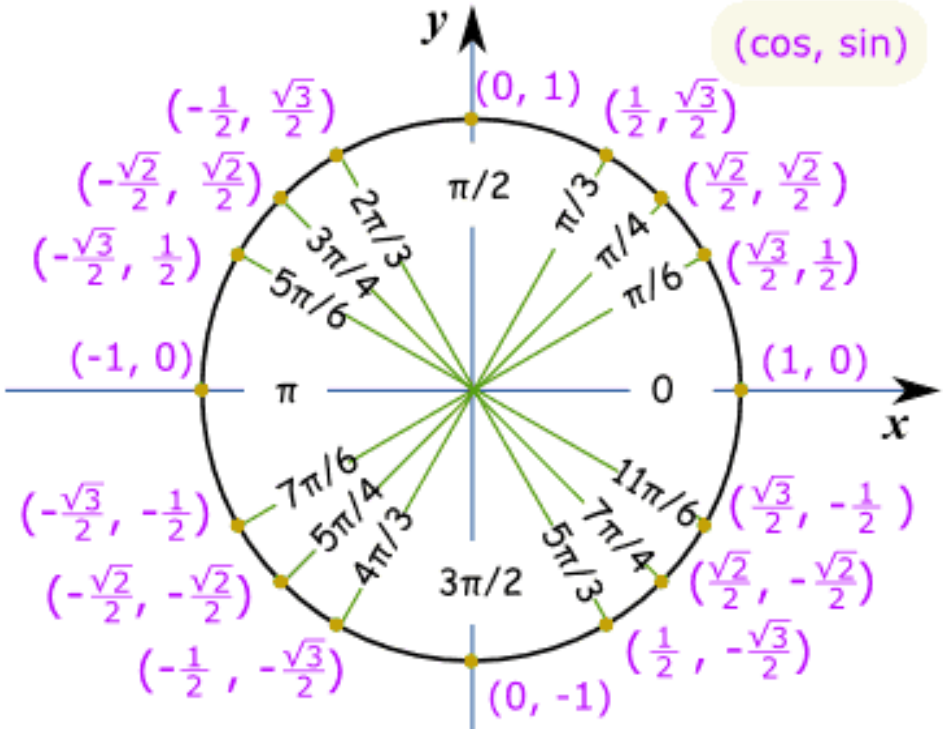




$\bar{\theta} = 180° - 150°$

$\bar{\theta} = 30°$

coordinates are $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

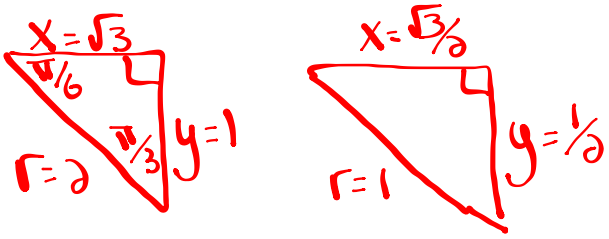
# Unit Circle of Special Angles in Radians



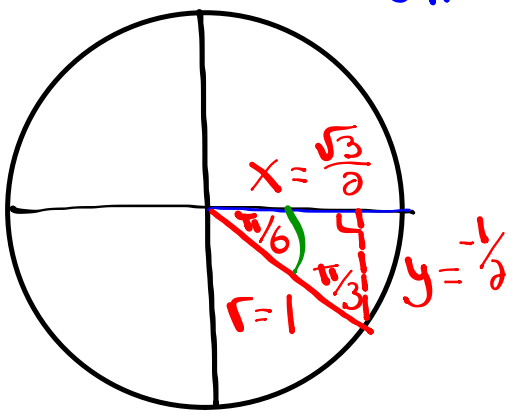
# Questions from Homework

① c) $\frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}$

Scale to fit unit circle




$2\pi$




Coordinates are $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\bar{\theta} = \frac{12\pi}{6} - \frac{11\pi}{6}$

$\bar{\theta} = \frac{\pi}{6}$

# Questions from Homework

③ If $\csc\theta = -\frac{\sqrt{10}}{3}$ and $\tan\theta > 0$ determine the value of the 5 remaining trig ratios as radicals in simplest form.

Given:

$\csc\theta = -\frac{\sqrt{10}}{3} = \frac{h}{o} = \frac{r}{y}$

$r = \sqrt{10}$ (r is always positive)

$y = -3$

① Determine what quadrant

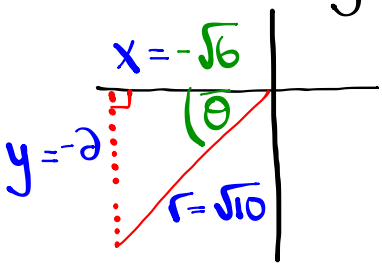
~~$\csc < 0$~~ + ~~$\tan\theta > 0$~~

S | A

T | C

Quad 3

② Draw a diagram:




④ Find the 5 trig ratios:

$\sin\theta = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10} = -\frac{\sqrt{10}}{5}$

$\cos\theta = \frac{-\sqrt{6}}{\sqrt{10}} = \frac{-\sqrt{60}}{10} = \frac{-2\sqrt{15}}{10} = -\frac{\sqrt{15}}{5}$

$\tan\theta = \frac{-3}{-\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

$\sec\theta = \frac{\sqrt{10}}{\sqrt{6}} = \frac{\sqrt{60}}{6} = \frac{2\sqrt{15}}{6} = \frac{\sqrt{15}}{3}$

$\cot\theta = \frac{-\sqrt{6}}{-3} = \frac{\sqrt{6}}{3}$

③ Find the missing side:

$x^2 + y^2 = r^2$

$x^2 + (-3)^2 = (\sqrt{10})^2$

$x^2 + 9 = 10$

$x^2 = 1$

$x = \pm 1$

$x = -1$ (Quad 3)

## Questions from Homework

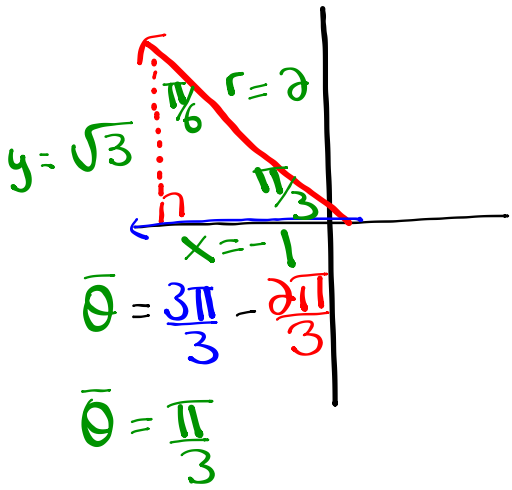
④ b) $\sec\frac{2\pi}{3} = \frac{r}{x}$

$\frac{1\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}$

$1\pi$ (odd)

$\sec\frac{2\pi}{3} = \frac{2}{-1}$

$\sec\frac{2\pi}{3} = -2$



Evaluate without the use of a calculator:

$$\sin\frac{9\pi}{2} - \cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$

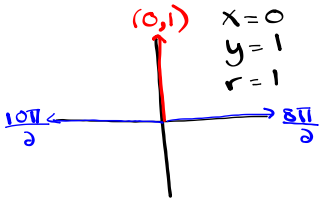
(i) $\sin\frac{9\pi}{2} = \frac{y}{r}$

$\frac{8\pi}{2}, \frac{9\pi}{2}, \frac{10\pi}{2}$

$4\pi \quad 5\pi$

$\sin\frac{9\pi}{2} = \frac{1}{1}$

$\sin\frac{9\pi}{2} = 1$

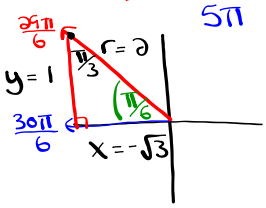



(ii) $\cos\left(\frac{29\pi}{6}\right) = \frac{x}{r}$

$\frac{28\pi}{6}, \frac{29\pi}{6}, \frac{30\pi}{6}$

$5\pi$

$\cos\left(\frac{29\pi}{6}\right) = -\frac{\sqrt{3}}{2}$



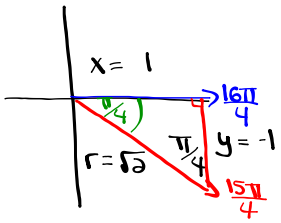

(iii) $\tan\left(\frac{15\pi}{4}\right) = \frac{y}{x}$

$\frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}$

$4\pi$

$\tan\left(\frac{15\pi}{4}\right) = -\frac{1}{1}$

$\tan\left(\frac{15\pi}{4}\right) = -1$




$\sin\frac{9\pi}{2} - \cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$

$(1) - \left(-\frac{\sqrt{3}}{2}\right)^2(-1)$

$1 - \left(\frac{3}{4}\right)(-1)$

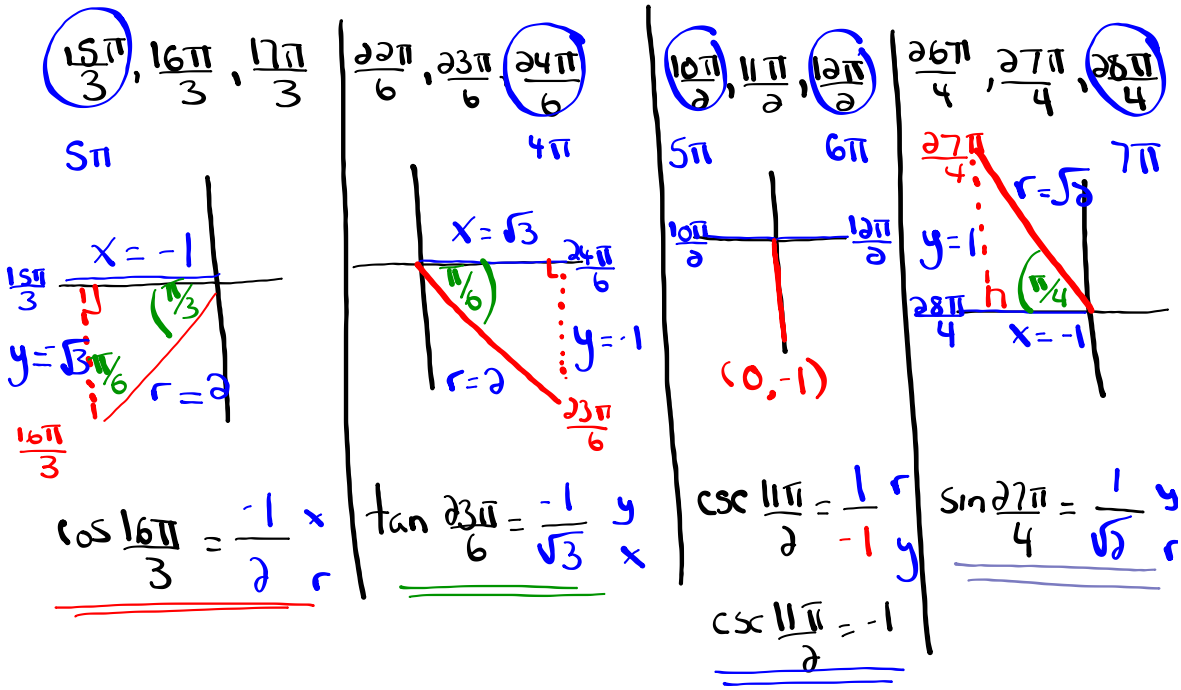
$1 + \frac{3}{4}$

$\frac{4}{4} + \frac{3}{4}$

$\frac{7}{4}$

Evaluate without the use of a calculator:

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right)+\csc\left(\frac{11\pi}{2}\right)+\sin^2\left(\frac{27\pi}{4}\right)$$




$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right)+\csc\left(\frac{11\pi}{2}\right)+\sin^2\left(\frac{27\pi}{4}\right)$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)^2+(-1)+\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\left(-\frac{1}{2}\right)\left(\frac{1}{3}\right)-1+\frac{1}{2}$$

$$-\frac{1}{6}-\frac{6}{6}+\frac{3}{6}$$

$$\frac{-4}{6}$$

$$-\frac{2}{3}$$

# Homework:

Worksheet - Sketching Angles in Radians.doc

Solutions...

1. $-\frac{5}{3}$

2. $\frac{-\sqrt{6}}{3}$

3. $-2-\sqrt{3}$

4. $\frac{-5}{3}$

5. $\frac{4+3\sqrt{3}}{6}$

6. $\frac{-10}{3}$

7. $0$

8. $\frac{3+3\sqrt{3}}{-2}$

## Attachments

Worksheet - Sketching Angles in Radians.doc