

## Questions from Homework

$$c) \quad y = \frac{x^2 + 2x - 3}{x^2 + 1}$$

x-int ( $y=0$ )

$$(x^2+1) \cdot 0 = \frac{x^2 + 2x - 3}{\cancel{x^2+1}} \quad (\cancel{x^2+1})$$

$$0 = x^2 + 2x - 3$$

$$0 = (x+3)(x-1)$$

$$x+3=0 \quad | \quad x-1=0$$

$$x=-3 \quad | \quad x=1$$

$$(-3, 0) \quad (1, 0)$$

y-int ( $x=0$ )

$$y = \frac{(0)^2 + 2(0) - 3}{(0)^2 + 1}$$

$$y = \frac{-3}{1}$$

$$y = -3$$

$$(0, -3)$$

## Questions from Homework

$$\textcircled{1} \text{ e) } y = 3x^2 + 4x - 6$$

$$x\text{-int (} y=0 \text{)}$$

$$0 = 3x^2 + 4x - 6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-6)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{16 + 72}}{6}$$

$$x = \frac{-4 \pm \sqrt{88}}{6}$$

$$x = \frac{-4 \pm \sqrt{2 \cdot 2 \cdot 2 \cdot 11}}{6}$$

$$x = \frac{-4 \pm 2\sqrt{22}}{6}$$

$$x = \frac{-2 \pm \sqrt{22}}{3}$$

$$y\text{ int (} x=0 \text{)}$$

$$y = 3(0)^2 + 4(0) - 6$$

$$y = -6$$

## Questions from Homework

$$\textcircled{a} \quad y = 9x - x^3$$

$$\text{(i) } x\text{-int } (y=0)$$

$$0 = 9x - x^3$$

$$0 = x(9 - x^2)$$

$$0 = x(3-x)(3+x)$$

$$\begin{array}{c|c|c} x=0 & 3-x=0 & 3+x=0 \\ \hline (0,0) & 3=x & x=-3 \\ & (3,0) & (-3,0) \end{array}$$

$$\text{(ii) } y\text{-int } (x=0)$$

$$y = 9(0) - (0)^3$$

$$y = 0 - 0$$

$$y = 0$$

$$(0,0)$$

$$\textcircled{b} \quad y = x^3 - x^2 - x + 1$$

$$x\text{-int } (y=0)$$

$$0 = (x^3 - x^2)(x+1)$$

$$0 = x^2(x-1) - 1(x-1)$$

$$0 = (x-1)(x^2-1)$$

$$0 = (x-1)(x-1)(x+1)$$

$$\begin{array}{c|c} x-1=0 & x+1=0 \\ \hline x=1 & x=-1 \\ (1,0) & (-1,0) \end{array}$$

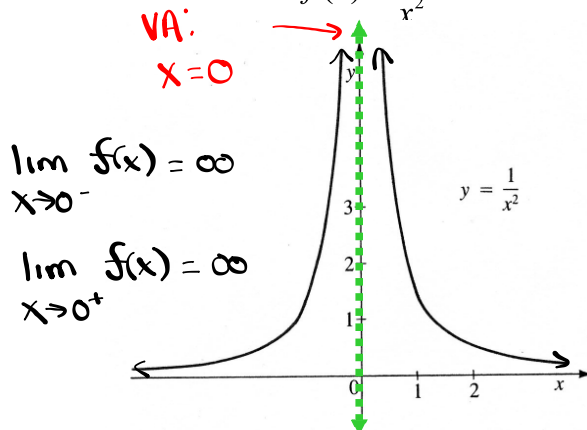
## Curve Sketching

In this chapter we look at further aspects of curves such as vertical and horizontal asymptotes, concavity, and inflections points. Then we use them, together with intervals of increase and decrease and maximum and minimum values, to develop a procedure for curve sketching.

## Vertical Asymptotes

Let us examine the behaviour of the function  $f(x) = \frac{1}{x^2}$  for  $x$  close to 0.

$x$	$f(x) = \frac{1}{x^2}$
$\pm 1$	1
$\pm 0.5$	4
$\pm 0.2$	25
$\pm 0.1$	100
$\pm 0.05$	400
$\pm 0.01$	10000
$\pm 0.001$	1000000



The values in the table and the graph show that the closer we take  $x$  to 0, the larger  $\frac{1}{x^2}$  becomes. In fact, it appears that by taking  $x$  close

enough to 0, we can make  $f(x)$  as large as we like. We indicate this type of behaviour by writing

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

and we say that the line  $x = 0$  is a **vertical asymptote** of  $y = \frac{1}{x^2}$

- To find VA we set the denominator equal to 0 and solve

$$y = \frac{1}{x^2}$$

VA:  $x^2 = 0$

$x = 0$

There is an infinite/essential discontinuity @  $x=0$

- you cannot cut through a vertical asymptote.

$$y = \frac{1}{x^2}$$

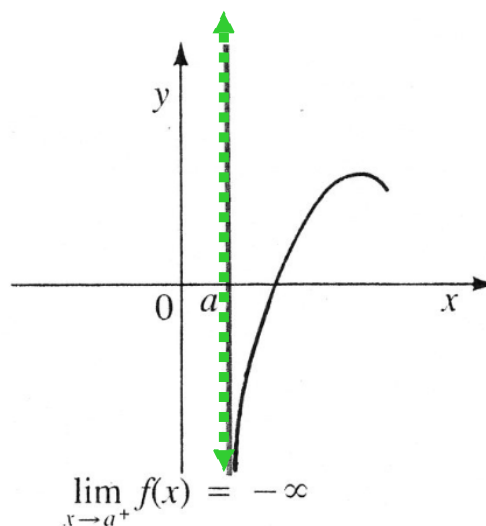
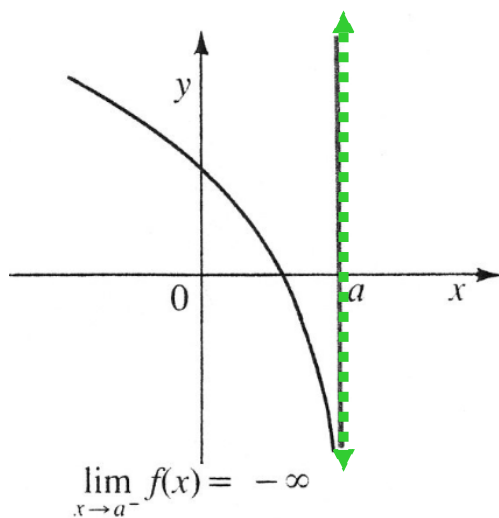
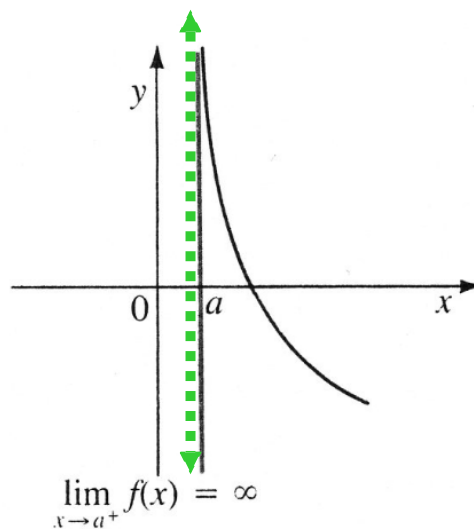
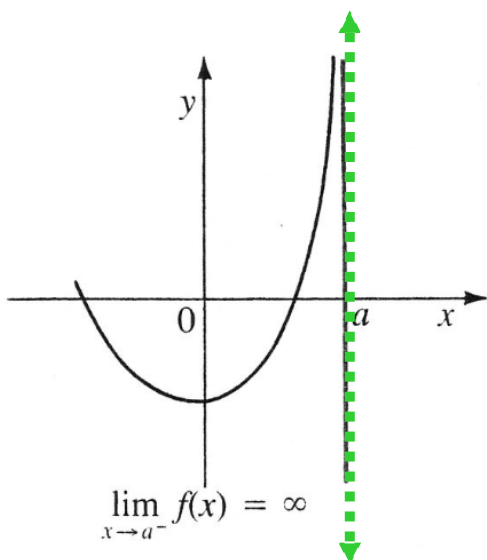
Domain:  $\{x \mid x \neq 0, x \in \mathbb{R}\}$

or  $(-\infty, 0) \cup (0, \infty)$

## Vertical Asymptote

The line  $x = a$  is a vertical asymptote of the graph of a function  $y = f(x)$  if either

$$\lim_{x \rightarrow a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \rightarrow a^+} f(x) = \pm\infty$$



To find the vertical asymptotes of any rational function, we find the values of  $x$  where the denominator is zero and compute the limits of the function from the right and left.

### Example

a) Find the vertical asymptotes of  $y = \frac{x}{x^2 - x - 6} = \frac{x}{(x-3)(x+2)}$

b) Sketch the graph near the asymptotes

a) VA:

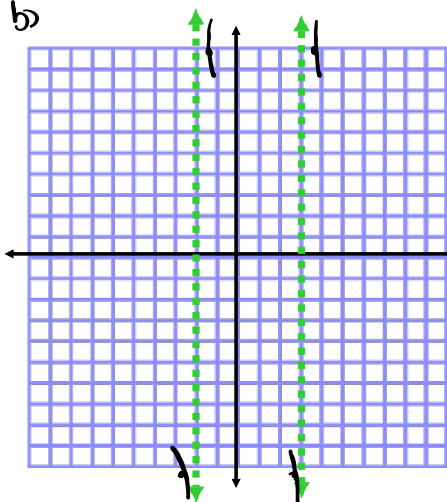
$$x^2 - x - 6 = 0 \quad -3 + 2 = -1$$

$$(x-3)(x+2) = 0 \quad -3 \times 2 = -6$$

$$\begin{array}{l|l} x-3=0 & x+2=0 \\ \hline x=3 & x=-2 \end{array}$$

There are infinite discontinuities @  $x = -2$  and  $x = 3$

b)



Check behaviour near the VA:

$$\lim_{x \rightarrow -2^-} \frac{x}{(x-3)(x+2)} = \frac{(-)}{(-)(-)} = \frac{(-)}{(+)} = -\infty$$

(x = -2.1)

$$\lim_{x \rightarrow -2^+} \frac{x}{(x-3)(x+2)} = \frac{(-)}{(-)(+)} = \frac{(-)}{(-)} = +\infty$$

(x = -1.9)

$$\lim_{x \rightarrow 3^-} \frac{x}{(x-3)(x+2)} = \frac{(+)}{(-)(+)} = \frac{(+)}{(-)} = -\infty$$

(x = 2.9)

$$\lim_{x \rightarrow 3^+} \frac{x}{(x-3)(x+2)} = \frac{(+)}{(+)(+)} = \frac{(+)}{(+)} = +\infty$$

(x = 3.1)

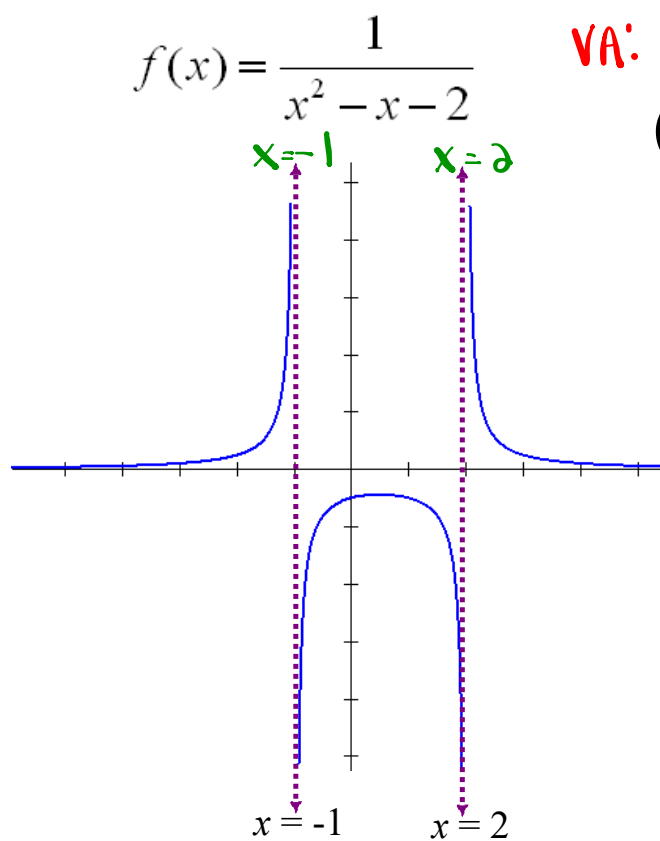
$$y = \frac{x}{(x-3)(x+2)}$$

$$D: \{x \mid x \neq -2, x \neq 3, x \in \mathbb{R}\}$$

or

$$(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$$

Example:



$$\text{VA: } x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x-2=0 \quad | \quad x+1=0$$

$$\underline{x=2} \quad | \quad \underline{x=-1}$$

$$\lim_{x \rightarrow -1^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^-} f(x) = +\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

Use limits to examine the behaviour of the function near the asymptotes



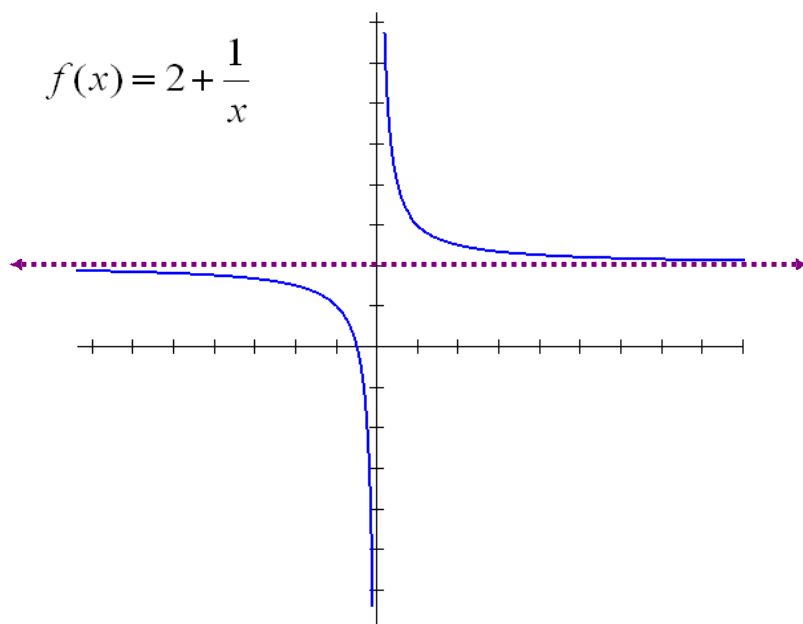
# Homework

# Asymptotes

## Horizontal Asymptote

The line  $y = b$  is a horizontal asymptote of the graph of a function  $y = f(x)$  if either

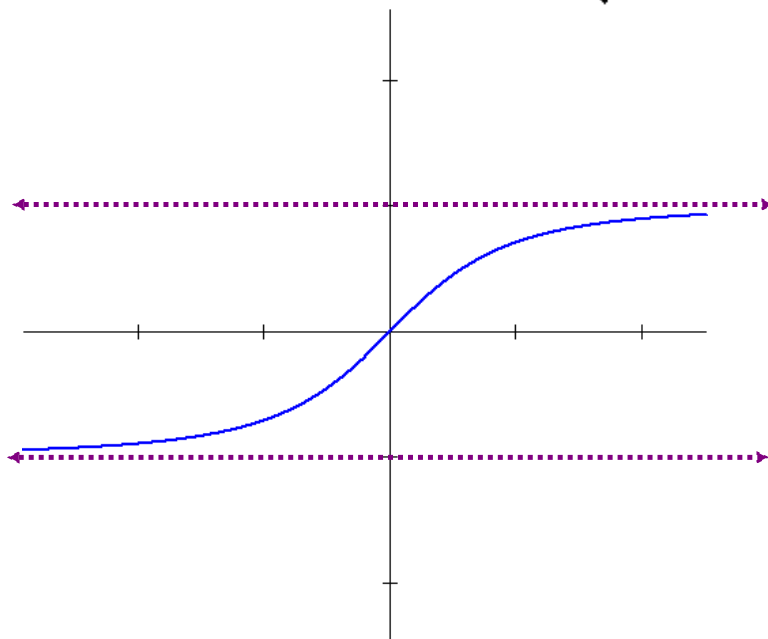
$$\lim_{x \rightarrow \infty} f(x) = b \quad \text{or} \quad \lim_{x \rightarrow -\infty} f(x) = b$$



Examine the limits of  $f(x)$  as  $x$  approaches  $\pm \infty$

There can be more than one horizontal asymptote.

Examine the function  $f(x) = \frac{x}{\sqrt{x^2 + 1}}$



Examine the limits of  $f(x)$  as  $x$  approaches  $\pm \infty$

Sketch the following function:

$$f(x) = \frac{8(x-2)}{x^2} \quad f'(x) = \frac{-8(x-4)}{x^3} \quad f''(x) = \frac{16(x-6)}{x^4}$$

Be sure to examine...

- Intercepts
- Asymptotes (*vertical and horizontal*)
- Regions of increase/decrease
- Local extrema
- Regions where concave up/down
- Inflection points

