Questions from Homework

④ f) $y = \dfrac{2x^2}{x^2+3x-4} = \dfrac{2x^2}{(x-1)(x+4)}$

① x-int (y=0)

$2x^2 = 0$

$x^2 = 0$

$x = 0$

(0,0)

② y-int (x=0)

$y = \dfrac{2(0)^2}{(0)^2+3(0)-4}$

$y = \dfrac{0}{-4} = 0$

(0,0)

③ VA

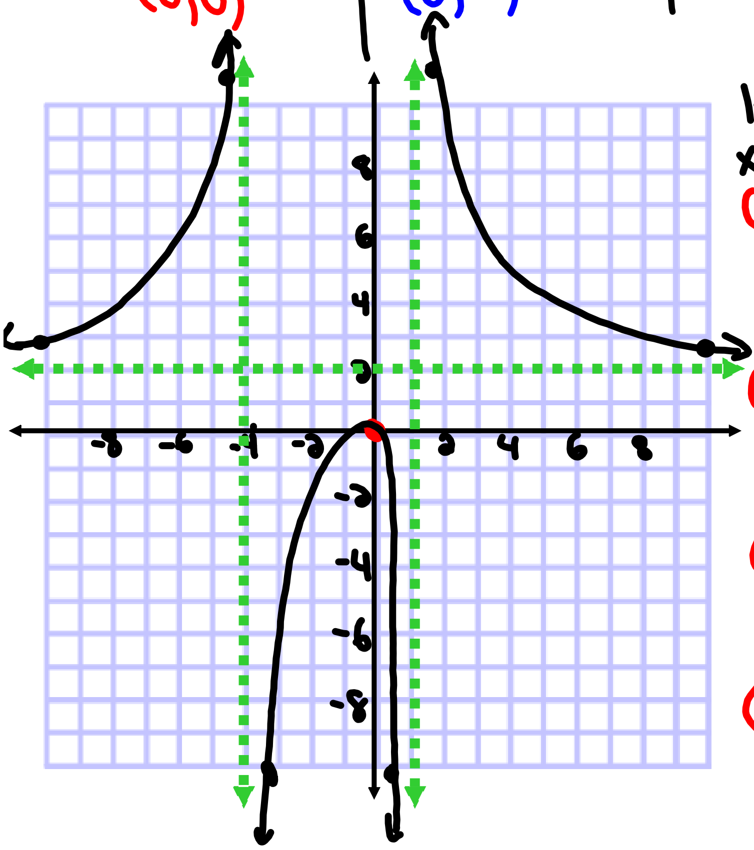
$x-1=0$ | $x+4=0$

$\boxed{x=1}$ | $\boxed{x=-4}$

④ HA:

$\lim\limits_{x\to\infty} \dfrac{2x^2}{x^2+3x-4} = 2$

$\boxed{y=2}$



$\lim\limits_{x\to -4^-} \dfrac{(+)}{(-)(-)} = +\infty$

(x=-4.01)

$\lim\limits_{x\to -4^+} \dfrac{(+)}{(-)(+)} = -\infty$

(x=-3.99)

$\lim\limits_{x\to 1^-} \dfrac{(+)}{(-)(+)} = -\infty$

(x=0.99)

$\lim\limits_{x\to 1^+} \dfrac{(+)}{(+)(+)} = +\infty$

(x=1.01)

Questions from Homework

④ e) $y = \dfrac{x}{x^2-1} = \dfrac{x}{(x-1)(x+1)}$

① x-int $(y=0)$

$(x^2-1)\,0 = \dfrac{x}{\cancel{x^2-1}}\,\cancel{(x^2-1)}$

$0 = x$

$(0,0)$

② y-int $(x=0)$

$y = \dfrac{0}{(0)^2-1} = \dfrac{0}{-1} = 0$

$(0,0)$

③ VA: (set denom $=0$)

$(x-1)(x+1) = 0$ ← factored

$x-1=0$ | $x+1=0$

$\boxed{x=1}$ | $\boxed{x=-1}$

$\lim\limits_{x\to -1^-} \dfrac{(-)}{(-)(-)} = -\infty$

$(x=-1.1)$

$\lim\limits_{x\to -1^+} \dfrac{(-)}{(-)(+)} = +\infty$

$(x=-0.9)$

$\lim\limits_{x\to 1^-} \dfrac{(+)}{(-)(+)} = -\infty$

$(x=0.9)$

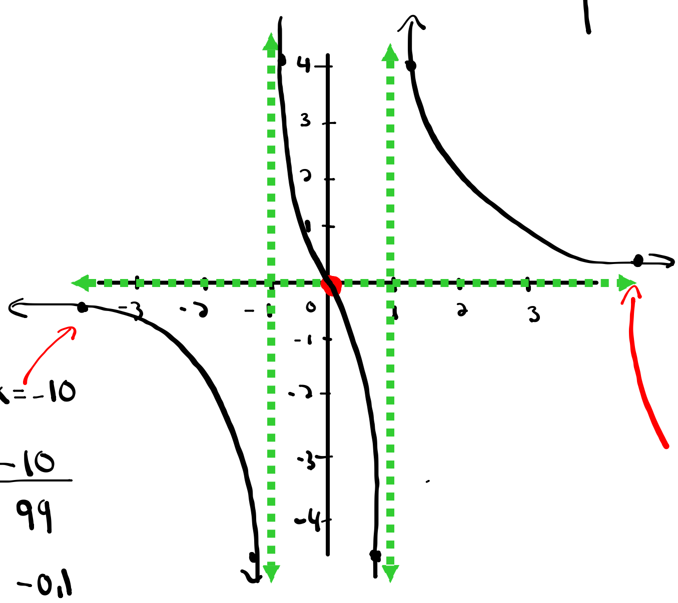
$\lim\limits_{x\to 1^+} \dfrac{(+)}{(+)(+)} = +\infty$

$(x=1.1)$

④ HA: $(\lim\limits_{x\to\infty})$

$\lim\limits_{x\to\infty} \dfrac{x}{x^2-1} = 0$ ← unfactored

$\boxed{y=0}$



if $x = -10$

$y = \dfrac{-10}{99}$

$y \doteq -0.1$

if $x = 10$

$y = \dfrac{x}{x^2-1}$

$y = \dfrac{10}{99} \doteq 0.1$

17) i) $f(x)=\dfrac{2x^3-18x}{x^3-x^2-2x}=\dfrac{2x(x^2-9)}{x(x^2-x-2)}$

$=\dfrac{2\cancel{x}(x-3)(x+3)}{\cancel{x}(x-2)(x+1)}=\dfrac{2(x-3)(x+3)}{(x-2)(x+1)}$

(i) VA: (zeros of the denominator)

$(x-2)(x+1)=0$

$x-2=0$ | $x+1=0$

$\boxed{x=2}$ | $\boxed{x=-1}$

Infinite discontinuity @ $x=-1$ and $x=2$

(ii) HA: calculate the limit as x approaches infinity.

$\lim\limits_{x\to\infty}\dfrac{2x^3-18x}{x^3-x^2-2x}$

$=\lim\limits_{x\to\infty}\dfrac{\frac{2x^3}{x^3}-\frac{18x}{x^3}}{\frac{x^3}{x^3}-\frac{x^2}{x^3}-\frac{2x}{x^3}}$

$=\lim\limits_{x\to\infty}\dfrac{2-\frac{18}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}}$ approach 0

$=\dfrac{2-0}{1-0-0}$

$=2$ $\boxed{\text{HA @ } y=2}$

(iii) Point of discontinuity/Hole:

(Take the factor that completely divided away and set it equal to 0, then solve)

$\boxed{x=0}$

removable discontinuity

to figure out the height of the hole

$f(x)=\dfrac{2(x-3)(x+3)}{(x-2)(x+1)}$

$f(0)=\dfrac{2(0-3)(0+3)}{(0-2)(0+1)}$

$f(0)=\dfrac{2(-3)(3)}{(-2)(1)}$

$f(0)=9$

$(0,9)$

open dot

17) j $f(x) = \dfrac{6x^3 - 30x^2 - 84x}{2x^3 + 3x^2 + x}$

$f(x) = \dfrac{6x(x^2 - 5x - 14)}{x(2x^2 + 3x + 1)}$

$2 + -7 = -5$
$2 \times -7 = -14$

$1 + 2 = 3$
$1 \times 2 = 2$

$f(x) = \dfrac{6x(x+2)(x-7)}{x(x+\frac{1}{2})(x+\frac{2}{2})}$

$f(x) = \dfrac{6\cancel{x}(x+2)(x-7)}{\cancel{x}(2x+1)(x+1)} = \dfrac{6(x+2)(x-7)}{(2x+1)(x+1)}$

i) VA: (Zeros of denominator)

$(2x+1)(x+1) = 0$

$2x + 1 = 0$ | $x + 1 = 0$

$2x = -1$ | $\boxed{x = -1}$

$\boxed{x = -\frac{1}{2}}$

ii) HA:

$\lim_{x\to\infty} \dfrac{6x^3 - 30x^2 - 84x}{2x^3 + 3x^2 + x}$

$\lim_{x\to\infty} \dfrac{\frac{6\cancel{x^3}}{\cancel{x^3}} - \frac{30\cancel{x^2}}{x^{\cancel{3}}} - \frac{84\cancel{x}}{x^{\cancel{3}2}}}{\frac{2\cancel{x^3}}{\cancel{x^3}} + \frac{3\cancel{x^2}}{x^{\cancel{3}1}} + \frac{\cancel{x}}{x^{\cancel{3}2}}}$

$\lim_{x\to\infty} \dfrac{6 - \frac{30}{x} - \frac{84}{x^2}}{2 + \frac{3}{x} + \frac{1}{x^2}}$

$= \dfrac{6 - 0 - 0}{2 + 0 + 0}$

$= \dfrac{6}{2}$

$= 3$

$\boxed{\text{HA @ } y = 3}$

iii) Point of discontinuity or a hole

$x = 0$

To find height @ $x = 0$

$f(x) = \dfrac{6(x+2)(x-7)}{(2x+1)(x+1)}$ use simplified

$f(0) = \dfrac{6(0+2)(0-7)}{(2(0)+1)(0+1)}$

$f(0) = \dfrac{-84}{1}$

$f(0) = -84$

$(0, -84)$

Curve Sketching

In this chapter we look at further aspects of curves such as vertical and horizontal asymptotes, concavity, and inflections points. Then we use them, together with intervals of increase and decrease and maximum and minimum values, to develop a procedure for curve sketching.

Slant Asymptotes

For rational functions, slant asymptotes occur when the degree of the numerator is one more than the degree of the denominator and can be found by division.

Example

Find the slant asymptote of the curve $y = \dfrac{2x^3 - 3x^2 + x - 3}{x^2 + 1}$

Example

Find the slant asymptote of the curve $y = \dfrac{1 + x - x^2}{x - 1}$

Homework