Slant Asymptotes:

② d) $y = \frac{(x-1)^3}{x^2} = \frac{(x-1)(x-1)(x-1)}{x^2} = \frac{x^3-3x^2+3x-1}{x^2}$

(i) x int (y=0)

$x^2 \cdot 0 = \frac{(x-1)^3}{\cancel{x^2}} \cdot \cancel{x^2}$

$0 = (x-1)^3$

$0 = x-1$

$1 = x$

$(1,0)$

(ii) y-int (x=0)

$y = \frac{(0-1)^3}{0^2} = \frac{-1}{0}$

undefined

No y-int

(iii) VA: $x^2 = 0$

$\boxed{x = 0}$

$\lim_{x \to 0^-} \frac{(-)}{(+)} = -\infty$

$(x = -0.1)$

$\lim_{x \to 0^+} \frac{(-)}{(+)} = -\infty$

$(x = 0.1)$

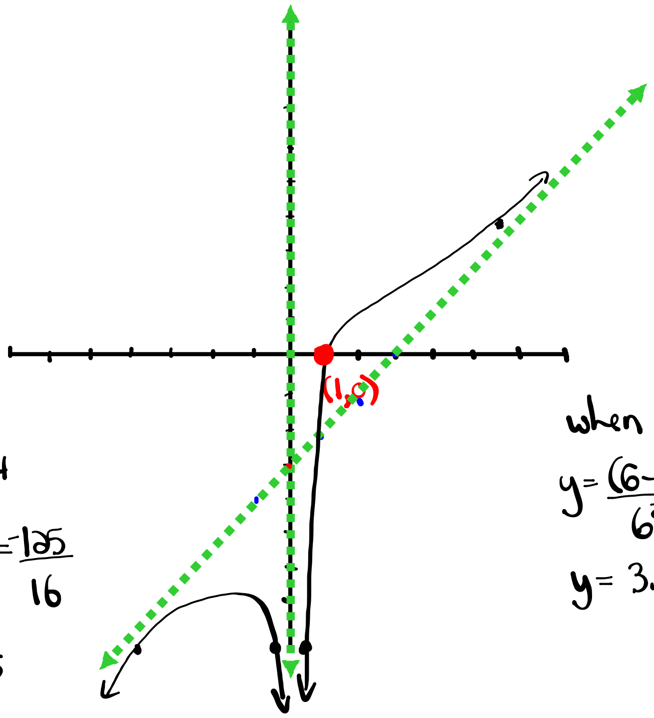
(iv) SA:

$$\begin{array}{r|l} & x - 3 \\ x^2 & x^3 - 3x^2 + 3x - 1 \\ & -(x^3) \\ \hline & -3x^2 + 3x - 1 \\ & -(-3x^2) \\ \hline & 3x - 1 \ R \end{array}$$

$\boxed{y = x - 3}$

$m = \frac{1}{1} \frac{\text{rise}}{\text{run}}$

$b = -3$ y-int



when $x = -4$

$y = \frac{(-4-1)^3}{(-4)^2} = \frac{-125}{16}$

$y = -7.8125$

when $x = 6$

$y = \frac{(6-1)^3}{6^2} = \frac{125}{36}$

$y = 3.47$

Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \leq 0$$

where does this function have y values that are less than or equal to zero

$y = x^3 - 3x^2 - 4x + 12$ (factor)

$y = x^2(x-3) - 4(x-3)$

$y = (x-3)(x^2-4)$

$y = (x-3)(x-2)(x+2)$

$-\quad +\quad -\quad +$

$(-3)\ \ -2\ \ (0)\ \ 2\ \ (2.5)\ \ 3\ \ (4)$

Find x-int $(y=0)$

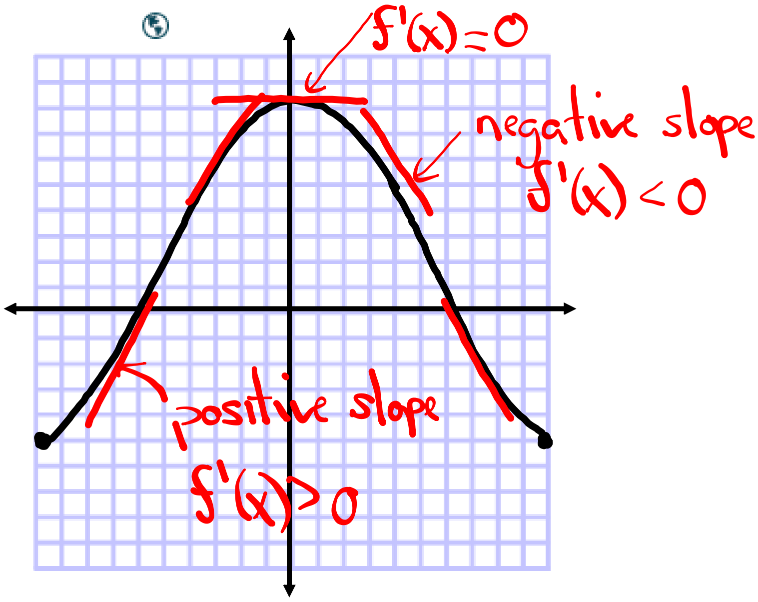
$0 = (x-3)(x-2)(x+2)$

$x-3=0$	$x-2=0$	$x+2=0$
$x=3$	$x=2$	$x=-2$

$x \in (-\infty, -2] \cup [2, 3]$

Increasing and Decreasing Functions

slope of tangent line $= f'(x)$



Test for Increasing and Decreasing Functions

1. If $f'(x) > 0$ for all x in an interval I, then f is increasing on I. $f'(x)$ is positive

2. If $f'(x) < 0$ for all x in an interval I, then f is decreasing on I $f'(x)$ is negative

Example 1

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

$f(x) = 1 - 5x + 4x^2$

$f'(x) = -5 + 8x$

local min

$-$ $+$

(0) $\frac{5}{8}$ (1)

CV: $f'(x) = 0$ or undefind

$0 = -5 + 8x$

$5 = 8x$

$\frac{5}{8} = x$

CV: $x = \frac{5}{8} = 0.625$

Thus f will be increasing on the interval $\left(\frac{5}{8}, \infty\right)$

Similarly,

Thus f will be decreasing on the interval $\left(-\infty, \frac{5}{8}\right)$

Example 2

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

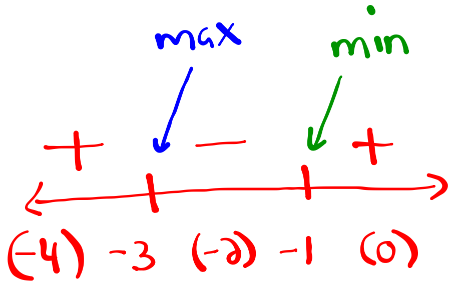
Solution

$y = x^3 + 6x^2 + 9x + 2$

$y' = 3x^2 + 12x + 9$

$y' = 3(x^2 + 4x + 3)$

$y' = 3(x+3)(x+1)$



CV: $y' = 0$ or undefined

Increasing on $(-\infty, -3) + (-1, \infty)$

$0 = 3(x+3)(x+1)$

$3 \neq 0$ | $x+3=0$, $x=-3$ | $x+1=0$, $x=-1$

CV: $x = -3, -1$

Example 3

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

Solution

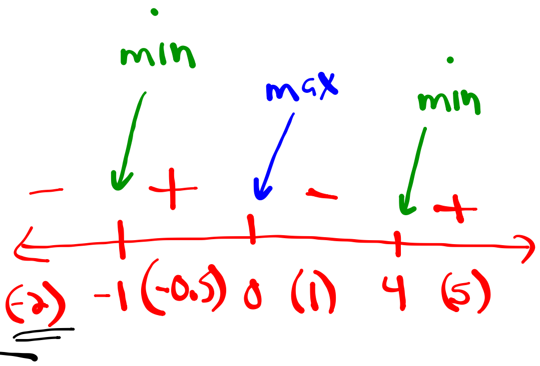
First we compute the derivative and factor it:

$f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x-4)(x+1)$



cv: $f'(x) = 0$ or undefined

$0 = 4x(x-4)(x+1)$

$4x = 0 \quad | \quad x - 4 = 0 \quad | \quad x + 1 = 0$

$x = 0 \quad | \quad x = 4 \quad | \quad x = -1$

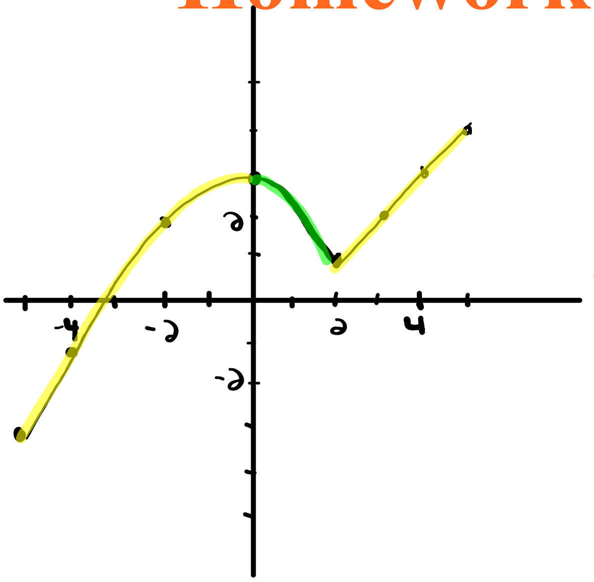
cv: $x = -1, 0, 4$

Increasing on $(-1, 0)$ + $(4, \infty)$

Decreasing on $(-\infty, -1)$ + $(0, 4)$

Homework

1



Increasing on:

(-5,0) + (2,5)

$-5<x<0$ $2<x<5$

Decreasing on:

(0,2)

$0<x<2$

Homework

$f'(x)>0 \rightarrow$ increasing
$f'(x)<0 \rightarrow$ decreasing

② b) $f(x)=x^4$

$f'(x)=4x^3$

CV: $f'(x)=0$ or undefined

$0=4x^3$

$0=x^3$

$0=x$

CV: $x=0$

min

$-$ $+$

(-1) 0 (1)

decreasing on $(-\infty,0)$

$\boxed{\text{increasing on } (0,\infty)}$

④ g) $y=x\sqrt{4-x}=\underbrace{x}_{f(x)}\underbrace{(4-x)^{1/2}}_{g(x)}$

$f'(x)g(x)+f(x)g'(x)$

$y'=1(4-x)^{1/2}+x\left(\frac{1}{2}\right)(4-x)^{-1/2}(-1)$

$y'=(4-x)^{1/2}-\frac{x}{2}(4-x)^{-1/2}$

$\frac{(4-x)^{1/2}}{(4-x)^{-1/2}}=(4-x)^{1/2-(-1/2)}$

$y'=(4-x)^{-1/2}\left[(4-x)-\frac{x}{2}\right]$

$y'=(4-x)^{-1/2}\left(\frac{8}{2}-\frac{2x}{2}-\frac{x}{2}\right)$

$y'=(4-x)^{-1/2}\left(\frac{8-3x}{2}\right)$

$y'=\frac{8-3x}{2(4-x)^{1/2}}$

max neither

$+$ $-$ und.

(0) $\frac{8}{3}$ (3) 4 (5)

$\frac{(+)}{(+)}$ $\frac{(-)}{(+)}$ $\frac{(-)}{\text{und}}$

CV: $y'=0$ or undefined

$8-3x=0$	$2(4-x)^{1/2}=0$
$8=3x$	$(4-x)^{1/2}=0$
$8/3=x$	$4-x=0$
$x\doteq 2.\overline{6}$	$4=x$

Increasing on $(-\infty, 8/3)$

Decreasing on $(8/3, 4)$

CV: $x=\frac{8}{3}, 4$

$x^2=9$

$x=\pm 3$

② f) $y = x^5 + 8x^3 + x$

$y' = 5x^4 + 24x^2 + 1$ ← always positive

Increasing on $(-\infty, \infty)$

④e) $h(x) = \underbrace{x^3}_{f(x)}\underbrace{(x-1)^4}_{g(x)}$

$h'(x) = 3x^2(x-1)^4 + x^3(4)(x-1)^3(1)$

$h'(x) = 3x^2(x-1)^4 + 4x^3(x-1)^3$

$3x-3 \quad +4x$

$h'(x) = x^2(x-1)^3\left[3(x-1)+4x\right]$

$h'(x) = x^2(x-1)^3(7x-3)$

$0 = x^2(x-1)^3(7x-3)$

$$\begin{array}{l|l|l} x^2=0 & (x-1)^3=0 & 7x-3=0 \\ x=0 & x-1=0 & 7x=3 \\ & x=1 & x=\frac{3}{7} \end{array}$$

$$\begin{array}{ccccccc} + & & + & & - & & + \\ & 0 & & \frac{3}{7} & & 1 & \\ (-1) & & \left(\frac{2}{7}\right) & & \left(\frac{4}{7}\right) & & (2) \\ (+)(-)(-) & & (+)(-)(-) & & (+)(-)(+) & & (+)(+)(+) \end{array}$$

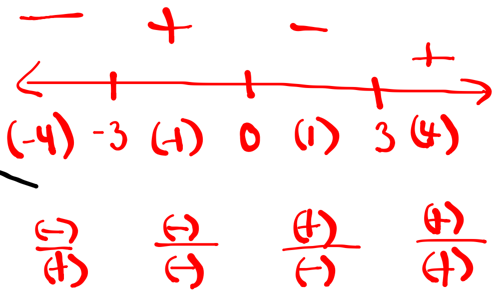
Increasing on $(-\infty, \frac{3}{7})$ and $(1, \infty)$

Decreasing on $(\frac{3}{7}, 1)$

④ h) $y = (x^2 - 9)^{2/3}$

$y' = \frac{2}{3}(x^2-9)^{-1/3}(2x)$

$y' = \frac{4x}{3(x^2-9)^{1/3}}$



CV: $y' = 0$ or undefined

$4x = 0$

$x = 0$

$3(x^2-9)^{1/3} = 0$

$(x^2-9)^{1/3} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

Increasing $(-3,0)$ + $(3,\infty)$

$-3 < x < 0$ + $x > 3$

Decreasing on $(-\infty,-3)$ + $(0,3)$

$x < -3$ + $0 < x < 3$

CV: $x = -3, 0, 3$