

Questions from Homework

$$F'(x) = f(x)g'(x) + f'(x)g(x) \quad F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\textcircled{1} \text{ a) } f(x) = \frac{\sqrt{x}}{x^2+1} = \frac{x^{1/2}}{x^2+1} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}(x^2+1) - 2x(x^{1/2})}{(x^2+1)^2}$$

$$f'(x) = \frac{\frac{1}{2}(x^2+1) - 2x^{3/2}}{(x^2+1)^2}$$

$$f'(x) = \frac{\frac{1}{2}x^2 + \frac{1}{2} - 2x^{3/2}}{2\sqrt{x}(x^2+1)^2}$$

$$f'(x) = \frac{x^2+1-4x^{3/2}}{2\sqrt{x}(x^2+1)^2} = \frac{1-3x^{3/2}}{2\sqrt{x}(x^2+1)^2}$$

$$\text{b) } g(x) = \frac{x^3-1}{x^2+x+1}$$

$$g'(x) = \frac{3x^2(x^2+x+1) - (x^3-1)(2x+1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{3x^4 + 3x^3 + 3x^2 - (2x^4 - 2x + x^3 - 1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{(x^2+x+1)^2} \leftarrow \text{expand}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{x^4 + 2x^3 + 3x^2 + 2x + 1} = 1$$

$$\text{c) } f(x) = \frac{1 - \frac{1}{x}}{x+1} = \frac{1-x^{-1}}{x+1}$$

$$f'(x) = \frac{x^2(x+1) - 1(1-x^{-1})}{(x+1)^2}$$

$$f'(x) = \frac{x^3 + x^2 - 1 + x^{-1}}{(x+1)^2}$$

$$f'(x) = \frac{x^3 + 2x^2 - 1}{(x+1)^2}$$

$$f'(x) = \frac{\frac{1}{x^2} + 2 \cdot \frac{x^2}{x} - 1 \cdot x^2}{x^2(x+1)^2} = \frac{1+2x-x^2}{x^2(x+1)^2}$$

$$\text{d) } f(x) = \frac{x - \frac{1}{x}}{x(x+1)} = \frac{x-1}{x^2+x}$$

$$\textcircled{2} \text{ a) } y = \frac{x}{x-2} \rightarrow (4, 2) \quad \begin{matrix} x_1=4 \\ y_1=2 \end{matrix}$$

$y = \frac{x}{x-2}$ $y' = \frac{1(x-2) - (x)}{(x-2)^2}$ $y' = \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$	$\textcircled{b} \text{ Find } y'(4) \text{ "m"}$ $y'(4) = \frac{-2}{(4-2)^2}$ $y'(4) = \frac{-2}{4} = -\frac{1}{2}$	$\textcircled{c} y - y_1 = m(x - x_1)$ $y - 2 = -\frac{1}{2}(x - 4)$ $y - 2 = -\frac{1}{2}x + 2$ $y = -\frac{1}{2}x + 4$ $4 = -\frac{1}{2}x$
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Questions from Homework

$$\textcircled{2} \text{ c) } f(x) = \frac{1}{(x+1)(2x-3)} = \frac{1}{2x^2-3x+2x-3} = \frac{1}{2x^2-x-3} f(x)$$

$$f'(x) = \frac{(0)(2x^2-x-3) - 1(4x-1)}{(2x^2-x-3)^2}$$

$$f'(x) = \frac{0-4x+1}{(2x^2-x-3)^2} = \frac{-4x+1}{(2x^2-x-3)^2}$$

$$\textcircled{3} \text{ c) } y = \frac{1}{x^2+1} \text{ @ } (-2, \frac{1}{5})$$

(i) Find y'

$$y' = \frac{(0)(x^2+1) - 1(2x)}{(x^2+1)^2}$$

$$y' = \frac{-2x}{(x^2+1)^2}$$

(ii) sub in $x = -2$

$$y' = \frac{-2(-2)}{((-2)^2+1)^2}$$

$$y' = \frac{4}{25}$$

$$m = \frac{4}{25}$$

$$x_1 = -2 \quad m = \frac{4}{25}$$

$$y_1 = \frac{1}{5}$$

$$\text{(iii) } y - y_1 = m(x - x_1)$$

$$y - \frac{1}{5} = \frac{4}{25}(x + 2)$$

$$y - \frac{1}{5} = \frac{4x}{25} + \frac{8}{25}$$

$$y = \frac{4x}{25} + \frac{8}{25} + \frac{1}{5}$$

$$y = \frac{4x}{25} + \frac{8}{25} + \frac{5}{25}$$

$$\boxed{y = \frac{4x}{25} + \frac{13}{25}} \quad \checkmark$$

or $25y = 4x + 13$

$$\boxed{0 = 4x - 25y + 13}$$

Questions from Homework

$$\textcircled{6} \quad y = \frac{x^2}{2x+5}$$

horizontal slope = $\frac{0}{1}$

① Find y'

$$y' = \frac{2x(2x+5) - 2(x^2)}{(2x+5)^2}$$

$$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$$

$$y' = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$\frac{0}{1} \rightarrow \frac{2x^2 + 10x}{(2x+5)^2}$$

$$2x^2 + 10x = 0$$

$$2x(x+5) = 0$$

$$\begin{array}{l|l} 2x=0 & x+5=0 \\ x=0 & x=-5 \end{array}$$

Solve for y

$$x=0 \quad y = \frac{(0)^2}{2(0)+5} = 0$$

$$(0, 0)$$

$$x=5 \quad y = \frac{(5)^2}{2(5)+5} = \frac{25}{15} = \frac{5}{3}$$

$$(5, \frac{5}{3})$$

The points are $(0, 0)$ and $(5, \frac{5}{3})$

Questions from Homework

$$\textcircled{4} \quad \left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{[g(a)]^2}$$

Given:

$$f(a) = \underline{3}$$

$$f'(a) = \underline{5}$$

$$g(a) = \underline{-1}$$

$$g'(a) = \underline{-4}$$

$$\left(\frac{f}{g}\right)'(a) = \frac{(5)(-1) - (3)(-4)}{[-1]^2} = \frac{-5 + 12}{1} = 7$$

Chain Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product **Work from the outside to the inside**

$$F'(x) = \underbrace{f'(g(x))}_{\text{derivative of outer function}} \times \underbrace{g'(x)}_{\text{derivative of inner function}}$$

Examples:

$$f(x) = (5x^3 + 1)^{10}$$

$$f'(x) = \underbrace{10(5x^3 + 1)^9}_{f'(g(x))} \underbrace{(15x^2)}_{g'(x)}$$

$$f'(x) = \underline{150x^2(5x^3 + 1)^9}$$

$$F(x) = \sqrt{2x^2 + 3}$$

$$F(x) = (2x^2 + 3)^{\frac{1}{2}}$$

$$F'(x) = \underbrace{\frac{1}{2}(2x^2 + 3)^{-\frac{1}{2}}}_{f'(g(x))} \underbrace{(4x)}_{g'(x)}$$

$$F'(x) = \underline{2x(2x^2 + 3)^{-\frac{1}{2}}}$$

$$F'(x) = \frac{2x}{(2x^2 + 3)^{\frac{1}{2}}}$$

$$F'(x) = \frac{2x}{\sqrt{2x^2 + 3}}$$

$$h(x) = \sqrt[3]{5 - 3x^4}$$

$$h(x) = (5 - 3x^4)^{\frac{1}{3}}$$

$$h'(x) = \underbrace{\frac{1}{3}(5 - 3x^4)^{-\frac{2}{3}}}_{f'(g(x))} \underbrace{(-12x^3)}_{g'(x)}$$

$$h'(x) = \underline{-4x^3(5 - 3x^4)^{-\frac{2}{3}}}$$

$$h'(x) = \frac{-4x^3}{(5 - 3x^4)^{\frac{2}{3}}}$$

$$h'(x) = \frac{-4x^3}{\sqrt[3]{(5 - 3x^4)^2}}$$

$$F'(x) = f'(g(x))g'(x)$$

suppose $f(1) = 5$, $f'(1) = 3$, $g(1) = \underline{2}$, $g'(1) = \underline{-1}$
and $f'(2) = \underline{7}$ find $F'(1)$

$$\begin{aligned} F'(1) &= f'(g(1)) \cdot g'(1) \\ &= \underline{f'(2)} \cdot \underline{g'(1)} \\ &= 7 \cdot -1 \\ &= -7 \end{aligned}$$

Homework

$$\textcircled{3} \quad g(x) = (x^3 + x^2 - 2)^{3/4}$$

$$g'(x) = \frac{3}{4}(x^3 + x^2 - 2)^{-1/4} (3x^2 + 2x)$$

$$g'(x) = \frac{3(3x^2 + 2x)}{4(x^3 + x^2 - 2)^{1/4}} = \frac{9x^2 + 6x}{4\sqrt[4]{x^3 + x^2 - 2}}$$

$$\textcircled{7} \quad y = \frac{1}{(x^3 + 2x^2 + 1)^2} = (x^3 + 2x^2 + 1)^{-2}$$

$$y' = -2(x^3 + 2x^2 + 1)^{-3} (3x^2 + 4x)$$

$$y' = \frac{-2(3x^2 + 4x)}{(x^3 + 2x^2 + 1)^3} = \boxed{\frac{-6x^2 - 8x}{(x^3 + 2x^2 + 1)^3}}$$

$$\textcircled{7} \quad y = \frac{1}{(x^3 + 2x^2 + 1)^2} \quad (\text{Using Quotient Rule})$$

$$y' = \frac{0(x^3 + 2x^2 + 1)^2 - 2(x^3 + 2x^2 + 1)(3x^2 + 4x)}{(x^3 + 2x^2 + 1)^4}$$

$$y' = \frac{-2(x^3 + 2x^2 + 1)(3x^2 + 4x)}{(x^3 + 2x^2 + 1)^4} = \boxed{\frac{-6x^2 - 8x}{(x^3 + 2x^2 + 1)^3}}$$

$$\textcircled{6} \quad y = \sqrt{x + \sqrt{x}} = (x + x^{1/2})^{1/2}$$

$$y' = \frac{1}{2}(x + x^{1/2})^{-1/2} (1 + \frac{1}{2}x^{-1/2})$$

$$y' = \frac{1}{2}(x + \sqrt{x})^{-1/2} (1 + \frac{1}{2\sqrt{x}})$$

$$y' = \left(\frac{1}{2\sqrt{x + \sqrt{x}}}\right) \left(\frac{2\sqrt{x}}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\right)$$

$$y' = \left(\frac{1}{2\sqrt{x + \sqrt{x}}}\right) \left(\frac{2\sqrt{x} + 1}{2\sqrt{x}}\right) = \boxed{\frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}}$$

$$g(x) = 9x^{-3}(5x^3 - 1)^6$$

$$g(x) = \frac{(x^2 - 5x + 1)^8}{(1 - x^{-7})^{20}}$$