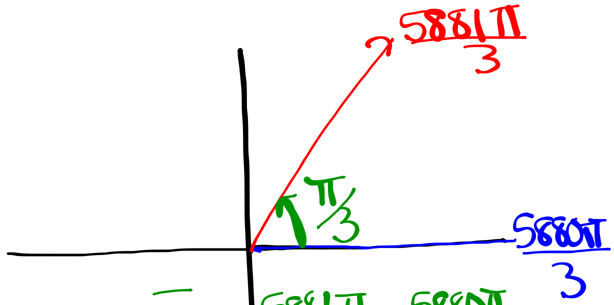
Sketch the following and determine a negative angle co-terminal with:

(i) $\frac{5881\pi}{3}$

$\frac{5880\pi}{3}, \frac{5881\pi}{3}, \frac{5882\pi}{3}$

$1960\pi$




$\bar{\theta} = \frac{5881\pi}{3} - \frac{5880\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

Negative coterminal angle:

$\frac{5881\pi}{3} - \frac{1962\pi}{1}$

$\frac{5881\pi}{3} - \frac{5886\pi}{3}$

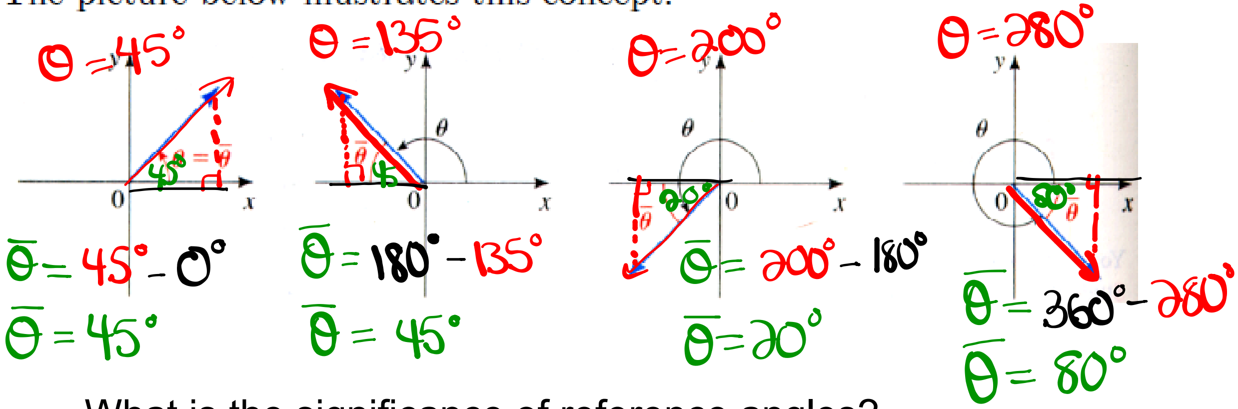
$\frac{-5\pi}{3}$

# Reference Triangles:

**Definition 17** *The reference angle $\bar{\theta}$ of an angle $\theta$ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

arm

The picture below illustrates this concept.



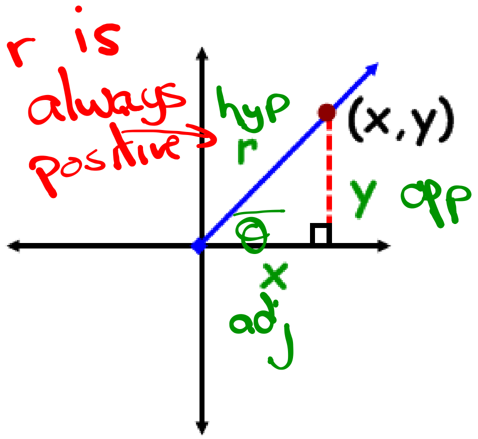

What is the significance of reference angles?

# Angles on the Cartesian Plane

$< 90^\circ$ or $< \pi/2$ or $< 1.57$ rads

- **Reference Angle** ($\theta$) - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.




Notice what will happen if the rotation moves into other quadrants?

**TRIG RATIOS on the CARTESIAN PLANE**

$\sin\theta = \frac{y}{r} = \frac{o}{h}$ $\csc\theta = \frac{r}{y} = \frac{h}{o}$

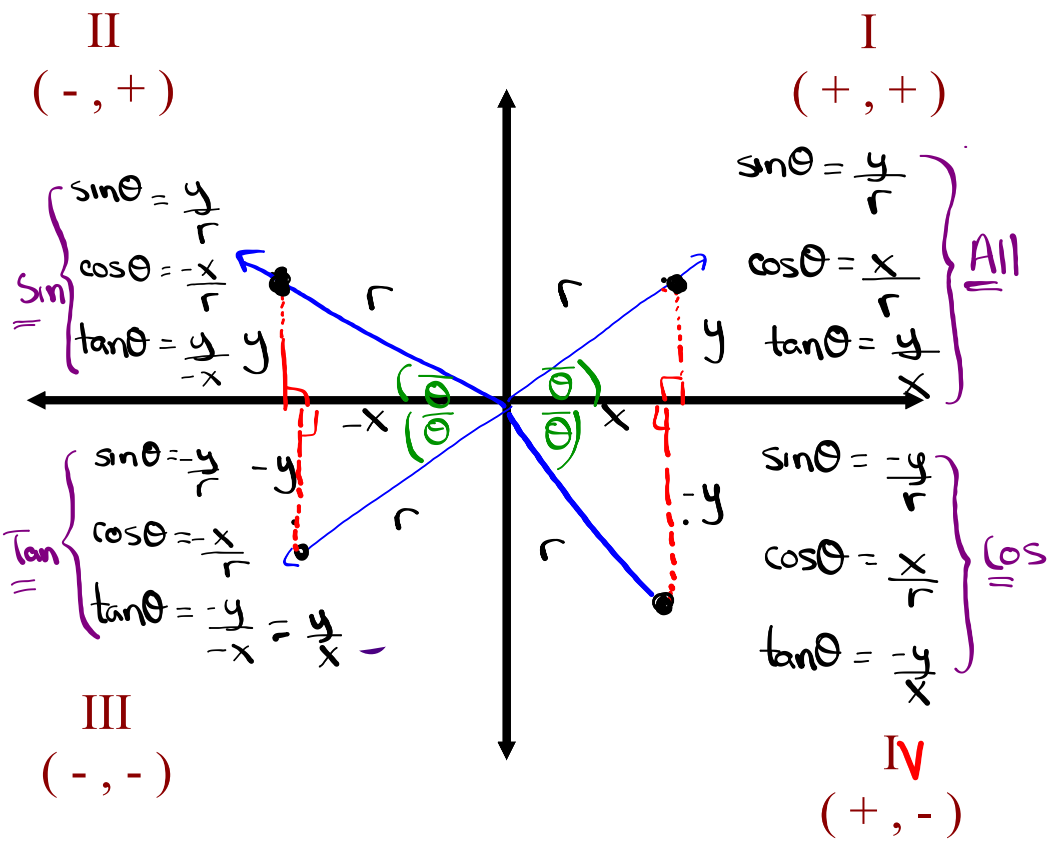
$\cos\theta = \frac{x}{r} = \frac{a}{h}$ $\sec\theta = \frac{r}{x} = \frac{h}{a}$

* $\tan\theta = \frac{y}{x} = \frac{o}{a}$ $\cot\theta = \frac{x}{y} = \frac{a}{o}$

"Primary" "Reciprocal"

# TRIG RATIOS IN ALL 4 QUADRANTS

## What primary trig ratios are POSITIVE in...




## Where is θ if...

Use 4CAST

**cscθ<0**

(sinθ is negative)

| S | A |
|---|---|
| T | C |

In Quad 3 or 4

**sinθ<0 & tanθ<0**

(sinθ is negative + tanθ is negative)

| S | A |
|---|---|
| T | C |

In Quad 4

**cscθ>0 & cotθ<0**

sinθ >0 + tanθ<0

| S | A |
|---|---|
| T | C |

In Quad 2

## Homework

If $\sec\theta = -\sqrt{10}$ *and* $\sin\theta > 0$, determine the value of $\csc\theta = \frac{r}{y}$

Given:

$\sec\theta = \frac{-\sqrt{10}}{1}$

$r = \sqrt{10}$

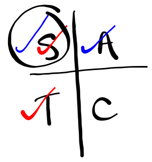
$x = -1$

① Determine what quadrant:

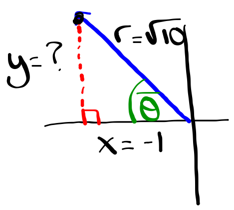
$\sec\theta < 0$ + $\sin\theta > 0$

$\cos\theta < 0$

Quad 2




② Draw a diagram




③ Solve for y:

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

$y = 3$ (Q2)

④ Find csc θ:

$\boxed{\csc\theta = \frac{\sqrt{10}}{3}}$

$r = \sqrt{10}$

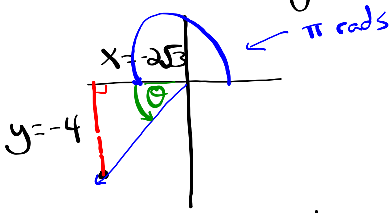
$y = 3$

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$x = -2\sqrt{3}$

$y = -4$

① Draw a diagram:




② Find $\bar{\theta}$:

$\tan\bar{\theta} = \frac{-4}{-2\sqrt{3}}$

$\tan\bar{\theta} = \frac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

use radian mode → $\bar{\theta} = \tan^{-1}(1.1547)$

$\bar{\theta} = 0.86$

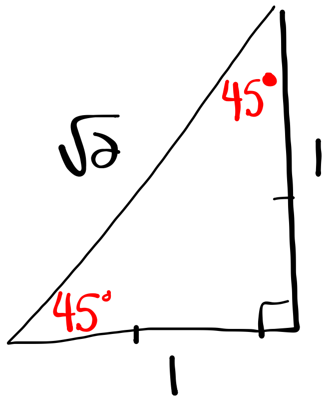
③ Find θ:

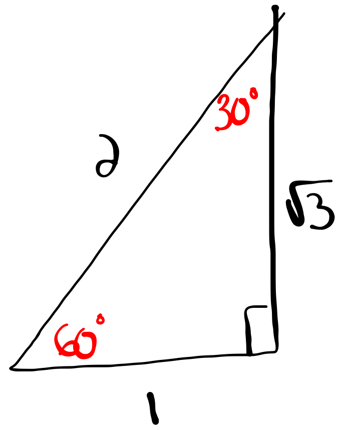
$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

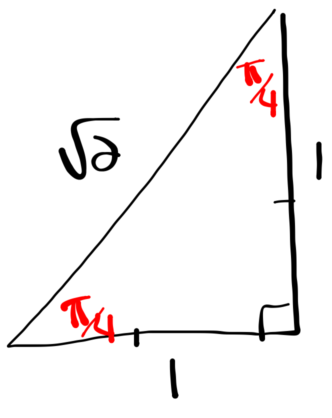
$\boxed{\theta = 4 \text{ rads}}$

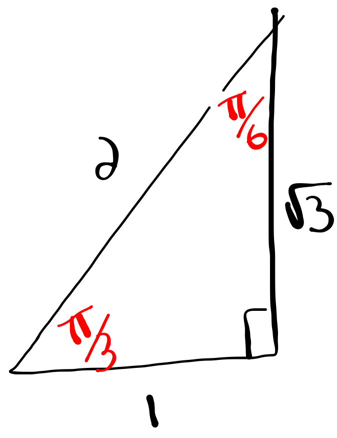
In Degrees



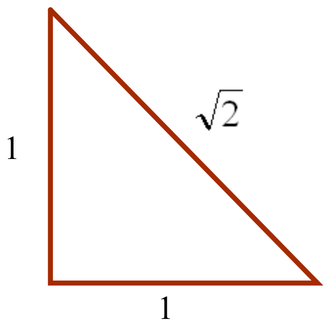



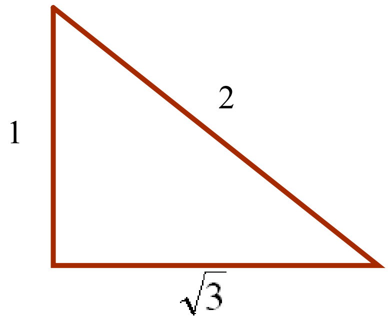

In Radians



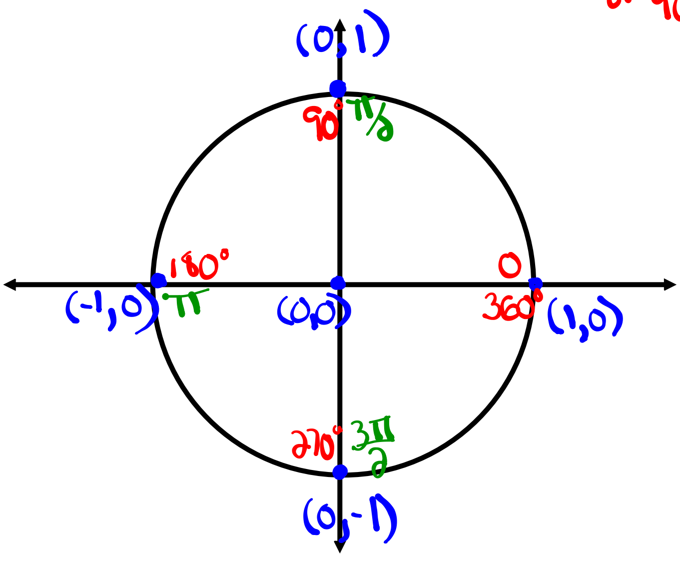



# Special Angles (in radians)




Quadrantal Angles (Multiples of 90°)

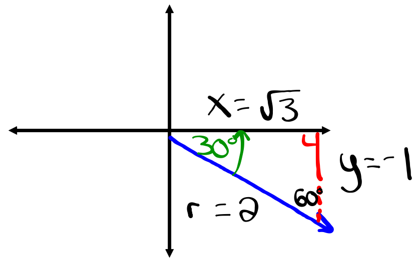



The Unit Circle

- Center is @ (0,0)
- radius is 1 unit

## Solving Trig Expressions by Sketching Angles

Ex. Evaluate $\sin 690°$




① Sketch the angle:

② Draw ref. triangle:

③ Find $\bar{\theta}$:

$\bar{\theta} = 720° - 690°$

↑ x-axis ↑ θ

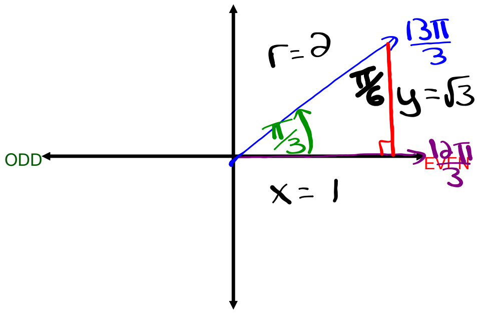
$\bar{\theta} = 30°$

④ Label triangle:

⑤ Determine the trig ratio:

$\sin 690° = \frac{-1}{2}$ $y = -1$, $r = 2$

Ex. $\cos \frac{13\pi}{3}$




① Sketch the angle:

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

$4\pi$

Even

② Draw ref. triangle:

③ Find $\bar{\theta}$:

$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

④ Label triangle:

⑤ Determine the trig ratio:

$\boxed{\cos \frac{13\pi}{3} = \frac{1}{2}}$ $x = 1$, $r = 2$

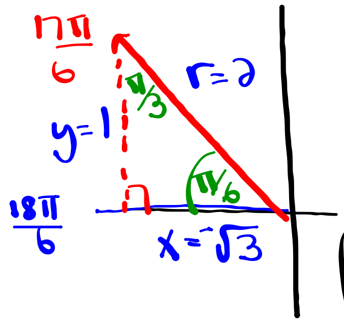
# Homework

Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan\dfrac{17\pi}{6} = -\dfrac{1}{\sqrt{3}}$ 2. $\sin\dfrac{15\pi}{4} = \dfrac{-1}{\sqrt{2}}$ 3. $\cos\left(-\dfrac{21\pi}{4}\right) = \dfrac{-1}{\sqrt{2}}$

① $\dfrac{16\pi}{6}, \boxed{\dfrac{17\pi}{6}}, \dfrac{18\pi}{6}$

$3\pi$ (odd)



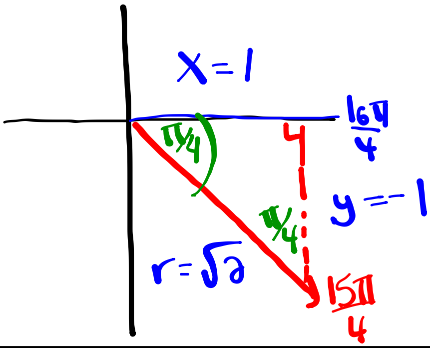

$\tan\dfrac{17\pi}{6} = \dfrac{y}{x}$

$\tan\dfrac{17\pi}{6} = -\dfrac{1\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$

$\boxed{\tan\dfrac{17\pi}{6} = -\dfrac{\sqrt{3}}{3}}$

② $\dfrac{14\pi}{4}, \boxed{\dfrac{15\pi}{4}}, \dfrac{16\pi}{4}$

$4\pi$ (Even)




$\sin\dfrac{15\pi}{4} = \dfrac{y}{r}$

$\sin\dfrac{15\pi}{4} = \dfrac{-1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$

$\boxed{\sin\dfrac{15\pi}{4} = -\dfrac{\sqrt{2}}{2}}$

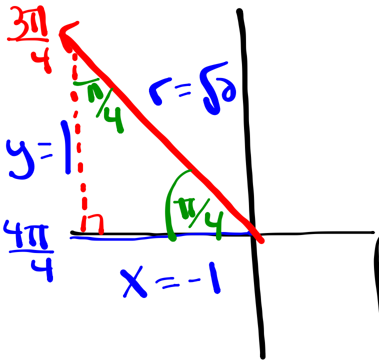
③ $-\dfrac{21\pi}{4} + \dfrac{6\pi}{1}$

$-\dfrac{21\pi}{4} + \dfrac{24\pi}{4}$

$\dfrac{3\pi}{4}$

$\dfrac{2\pi}{4}, \boxed{\dfrac{3\pi}{4}}, \dfrac{4\pi}{4}$

$1\pi$ (odd)

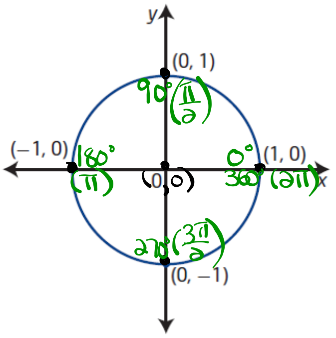



$\cos\left(\dfrac{-21\pi}{4}\right) = \dfrac{x}{r}$

$\cos\left(\dfrac{-21\pi}{4}\right) = \dfrac{-1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$

$\boxed{\cos\left(\dfrac{-21\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}}$

# Unit Circle




**unit circle**

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y \rightarrow$ Ex: $\sin 90° = 1$

$x = 0$
$y = 1$
$r = 1$

$\cos\theta = \frac{x}{r} = \frac{x}{1} = x \rightarrow$ Ex: $\cos\pi = -1$

$x = -1$
$y = 0$
$r = 1$

$\tan\theta = \frac{y}{x} \longrightarrow$ Ex: $\tan 270° = \frac{-1}{0}$ undefined

$x = 0$
$y = -1$
$r = 1$

(0, -1)

$\csc\theta = \frac{r}{y} = \frac{1}{y} \rightarrow$ Ex: $\csc 360° = \frac{1}{0}$ undefined

$x = 1$
$y = 0$
$r = 1$

(1, 0)

$\sec\theta = \frac{r}{x} = \frac{1}{x} \rightarrow$ Ex: $\sec 5\pi = \frac{1}{-1} = -1$

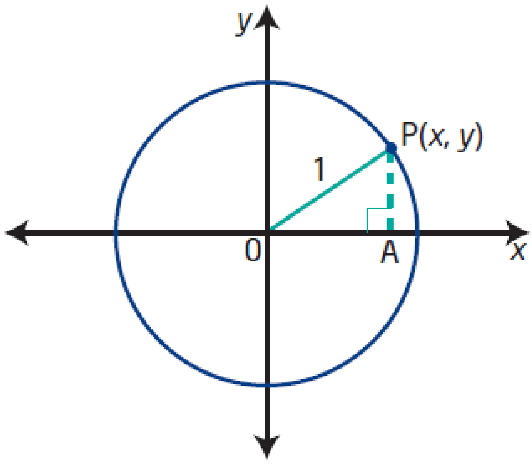
$x = -1$
$y = 0$
$r = 1$

(-1, 0)

$\cot\theta = \frac{x}{y} \rightarrow$ Ex: $\cot \frac{3\pi}{2} = \frac{0}{-1} = 0$

$x = 0$
$y = -1$
$r = 1$

(0, -1)




$x^2 + y^2 = r^2$ on unit circle $r = 1$

$\boxed{\text{The equation of the unit circle is } x^2 + y^2 = 1.}$

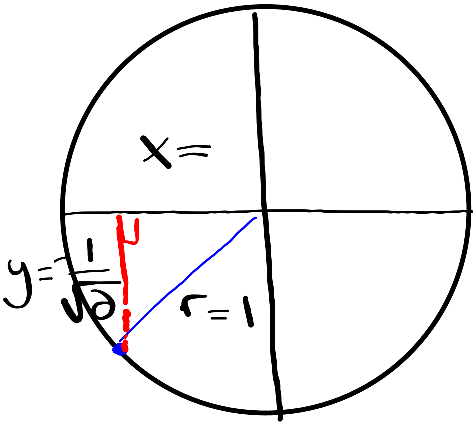
Determine the equation of a circle with centre at the origin and radius 6.

# Problems Involving the Unit Circle:

## Determine Coordinates for Points of the Unit Circle

Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

- the $y$-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III




$x^2+y^2=r^2$

$x^2+\left(-\frac{1}{\sqrt{2}}\right)^2=(1)^2$

$x^2+\frac{1}{2}-\frac{1}{2}=1-\frac{1}{2}$

$x^2=\frac{2}{2}-\frac{1}{2}$

$x^2=\frac{1}{2}$

$x=\pm\sqrt{1/2}$

$x=\pm\sqrt{\frac{1}{2}}$

$x=\pm\frac{\sqrt{1}}{\sqrt{2}}$

$\boxed{x=-\frac{1}{\sqrt{2}}}$ Q3

The coordinates are:

$\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ or

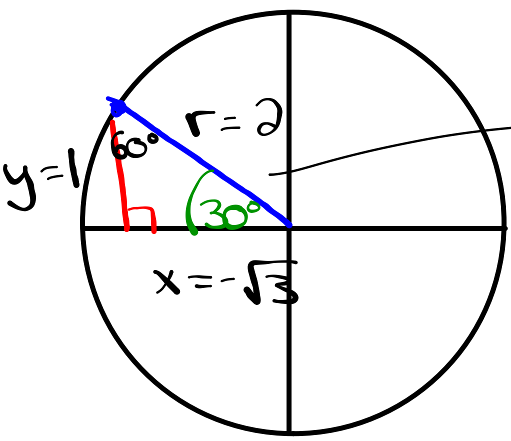
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$

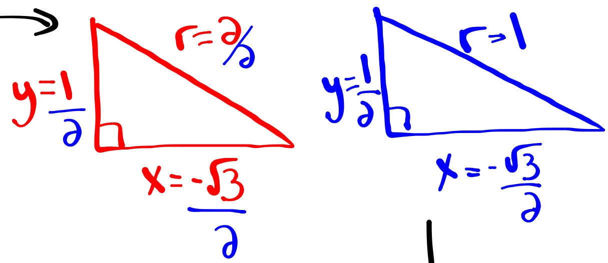
## Problems Involving the Unit Circle:

If $P(150°)$ is the point at which the terminal arm of an angle $\theta$ in standard position intersects the unit circle, determine the exact coordinates of...

$(x, y)$

Scale the diagram so that $r = 1$ (unit circle)



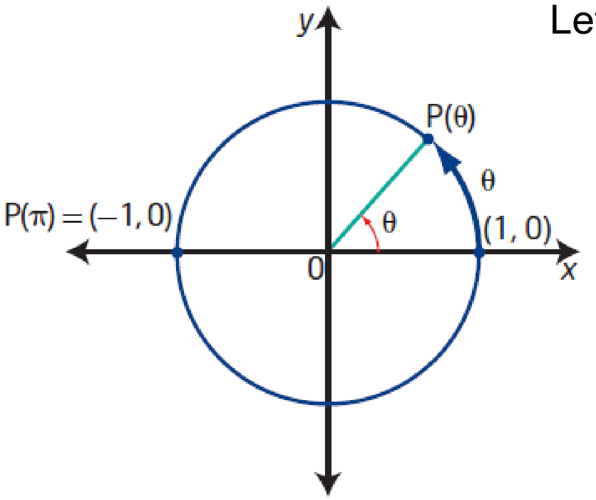




$\boxed{\text{coordinates are } \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)}$

$\bar{\theta} = 180° - 150°$

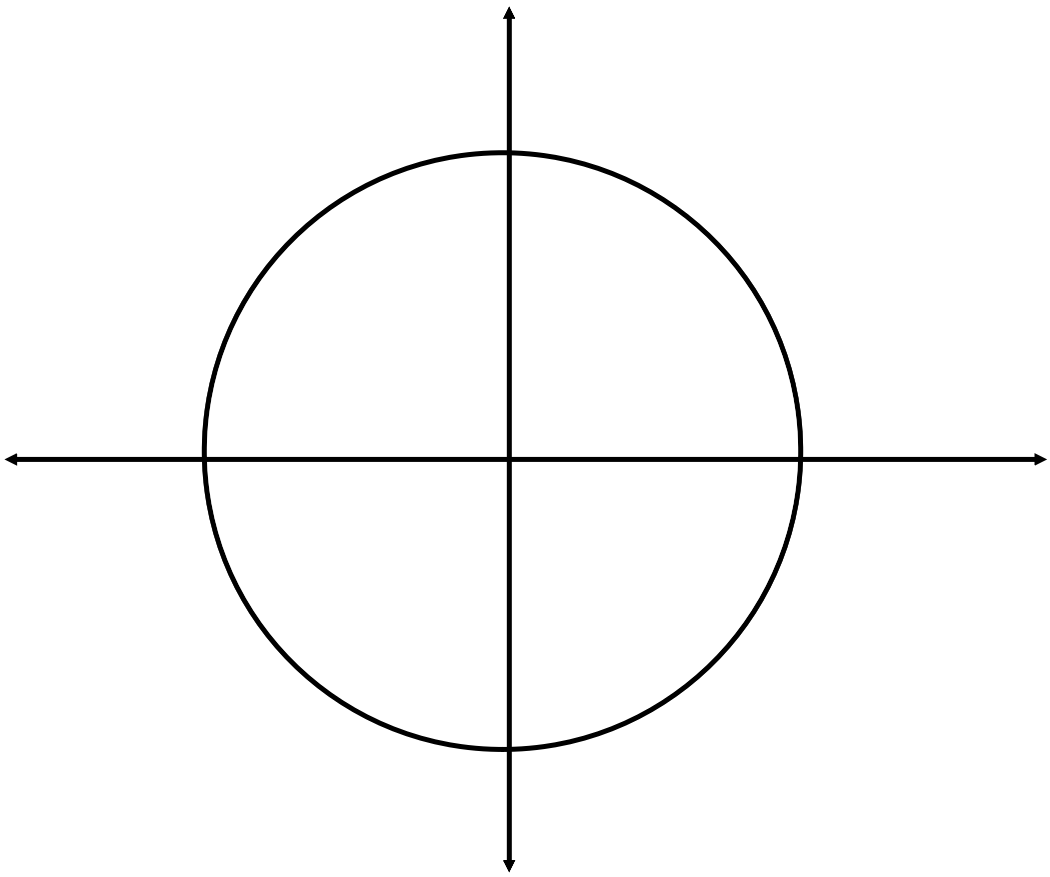
$\bar{\theta} = 30°$

# Special Angles on the Unit Circle:

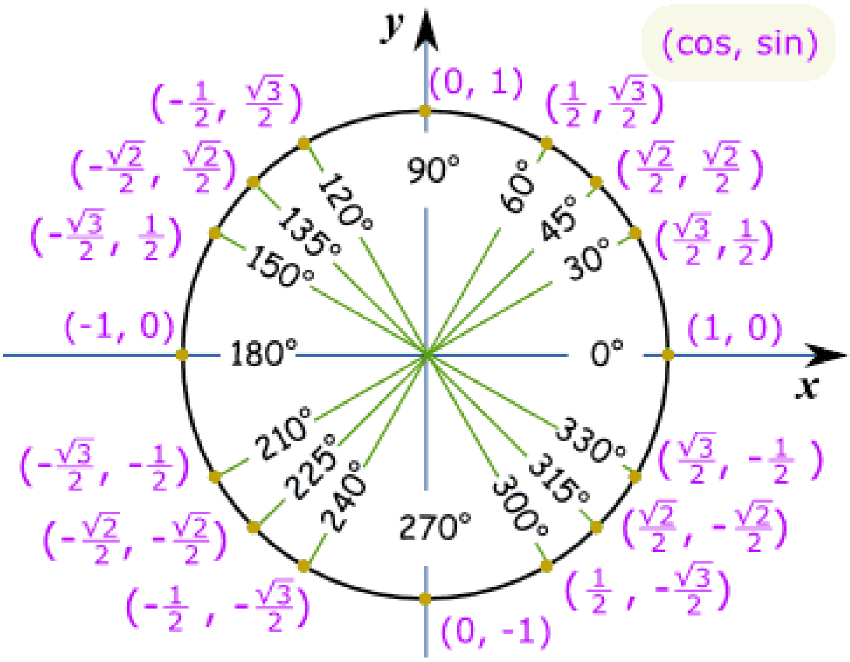



Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles for all multiples of $\pi/4$ between 0 and $2\pi$

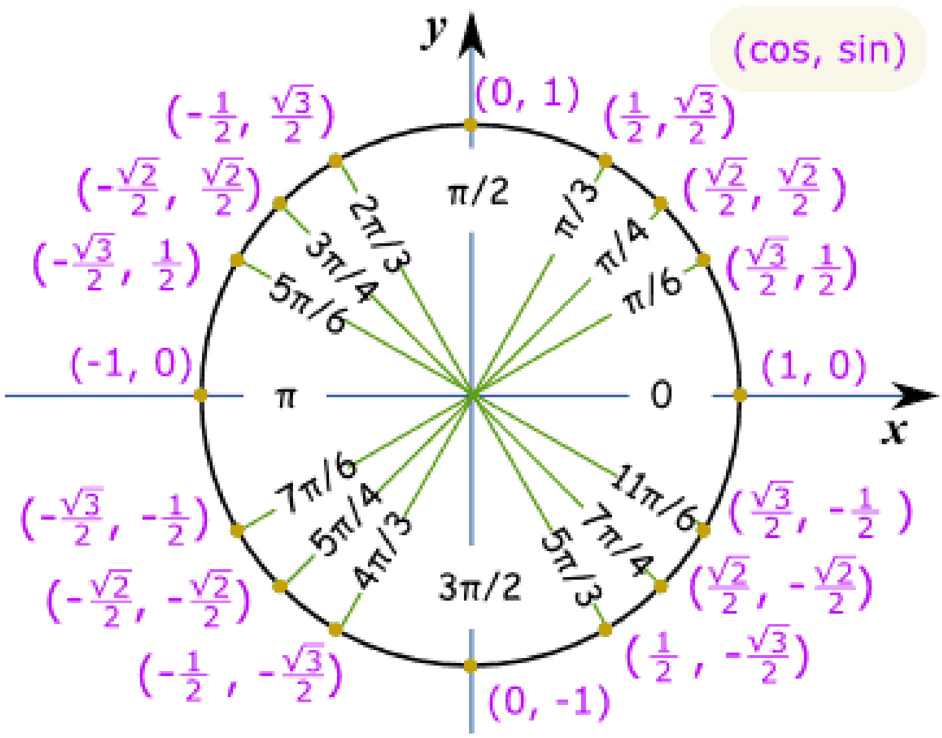


Unit Circle of Special Angles in Degrees




This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



Finish worksheet

## Attachments

Worksheet - Sketching Angles in Radians.doc