Questions from Homework

③ a) what is the domain of f

$$f(x) = \begin{cases} x+2 & -\infty < x < -1 \\ 1 & -1 < x < 0 \\ 2x & x \geq 0 \end{cases}$$

$$\boxed{D: \{x \mid x \neq -1,\ x \in \mathbb{R}\}}$$

b) $f(-1) =$ undefined

$\lim_{x \to -1^-} x+2 = -1+2 = 1$

$\lim_{x \to -1^+} 1 = 1$

Discontinuous @ $x = -1$

$\lim_{x \to -1} f(x) \neq f(-1)$ removable discontinuity

c) $f(0) = 2(0)$

$f(0) = 0$

$\lim_{x \to 0^-} 1 = 1$

$\lim_{x \to 0^+} 2x = 2(0) = 0$

Discontinuous @ $x = 0$

$\lim_{x \to 0} f(x)$ does not exist or $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

jump discontinuity

Limits at Infinity

What exactly is infinity?

- It is the ***process*** of making a value arbitrarily large or small

$+\infty$ ⟶ Positive Infinity...process of becoming arbitrarily large

$-\infty$ ⟶ Negative Infinity...process of becoming arbitrarily small

4 Definition Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

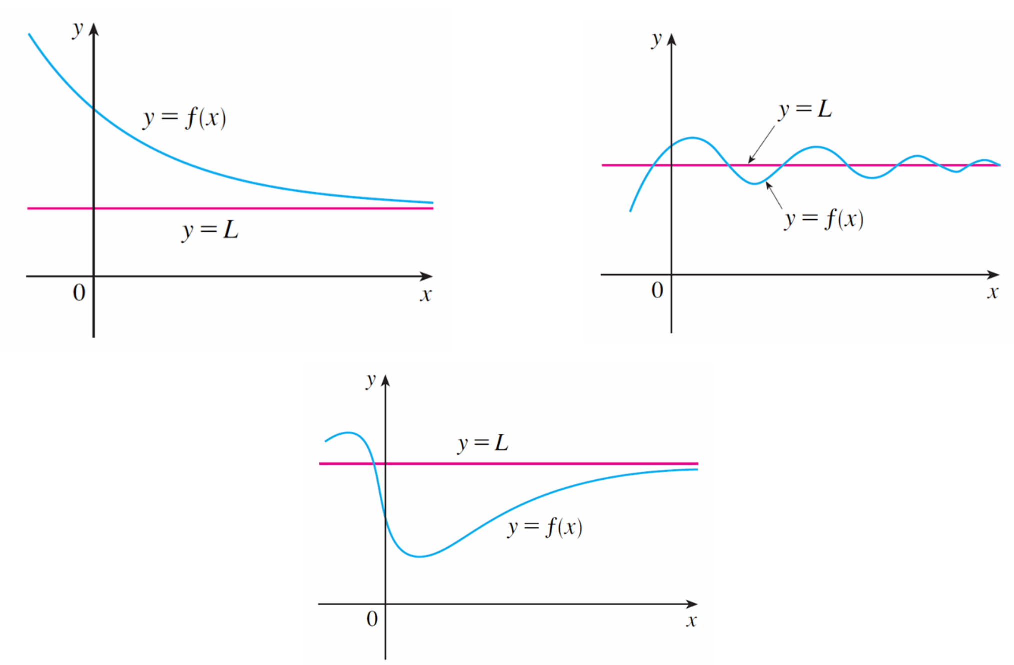


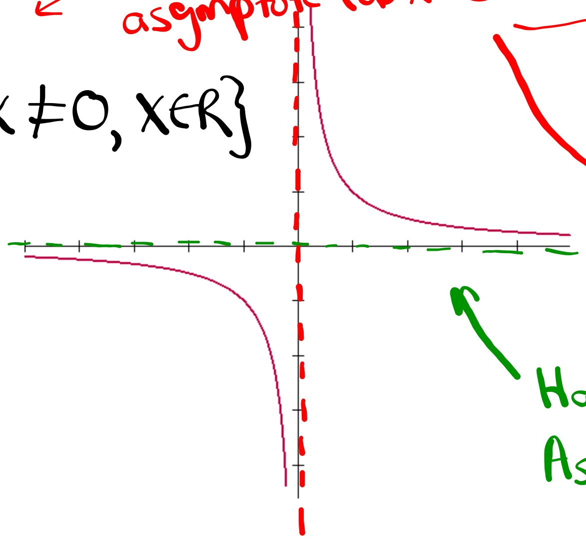
FIGURE 9
Examples illustrating $\lim_{x \to \infty} f(x) = L$

Have a look at these limits... HA

$$\lim_{x\to\infty}\frac{1}{x} = 0 \qquad \lim_{x\to-\infty}\frac{1}{x} = 0$$

$f(x) = \frac{1}{x}$ vertical asymptote @x=0

D: $\{x \mid x \neq 0, x \in \mathbb{R}\}$



$$\lim_{x\to 0^-} f(x) = -\infty$$

$$\lim_{x\to 0^+} f(x) = +\infty$$

Horizontal Asymptote $y=0$

In general...

7 If n is a positive integer, then

$$\lim_{x\to\infty}\frac{1}{x^n} = 0 \qquad \lim_{x\to-\infty}\frac{1}{x^n} = 0$$

Calculating limits at infinity without using a graph

• Rational Functions

***Note:** If every term in a rational expression is divided by the same value, the rational expression will still be equal to it's original value*

$$\frac{12+8}{6-2}$$

Divde the numerator and denominator by 2 →

This will be important when evaluating limits for rational functions approaching infinity...

Look at the following example:

approach 0

$$\boxed{\lim_{x\to\infty}\frac{3x^2-x-2}{5x^2+4x+1}} = \lim_{x\to\infty}\frac{\dfrac{3x^2-x-2}{x^2}}{\dfrac{5x^2+4x+1}{x^2}} = \lim_{x\to\infty}\frac{3-\dfrac{1}{x}-\dfrac{2}{x^2}}{5+\dfrac{4}{x}+\dfrac{1}{x^2}}$$

Divide every term by the HIGHEST power that is present in denominator of the rational expression once they are expanded

$$=\frac{3-0-0}{5+0+0}$$

$$=\frac{3}{5}$$

This graph below validates our solution:

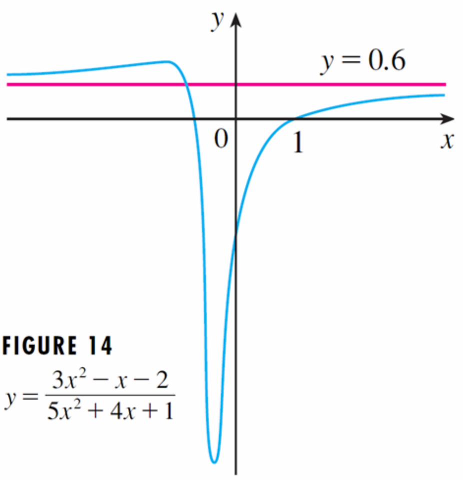


FIGURE 14
$y=\dfrac{3x^2-x-2}{5x^2+4x+1}$

- Remember

If the highest degree is in the denominator then the *Limit* will be equal to 0

If the highest degree is in the numerator then the *Limit* will not exist. Determine if it's $\pm\infty$

If the degree is the same in the numerator and denominator then the *Limit* will be equal to the coefficients in front of the highest degree. (Quotient of leading coefficients)

Evaluate the following limit:

$$\lim_{n\to\infty} \frac{n^2-n}{2n^2+1}$$

$$= \frac{1}{2}$$

$$\lim_{n\to\infty} \frac{\frac{n^2-n}{n^2}}{\frac{2n^2+1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{1-\frac{1}{n}}{2+\frac{1}{n^2}}$$

$$= \frac{1-0}{2+0} = \frac{1}{2}$$

$$\lim_{n\to\infty} \frac{1-n^5}{1+2n^5}$$

$$= \frac{-1}{2}$$

$$\lim_{n\to\infty} 4n = +\infty \qquad \lim_{n\to-\infty} 4n = -\infty$$

$$\lim_{x\to\infty} -4x = -\infty \qquad \lim_{x\to-\infty} -4x = +\infty$$

$$\lim_{x\to\infty} \frac{-3(x^2-4)^2}{3-5x^2}$$

$$= \lim_{x\to\infty} \frac{-3(x^4-8x^2+16)}{3-5x^2}$$

$$= \lim_{x\to\infty} \frac{-3x^4+24x^2-48}{3-5x^2}$$

$$= \lim_{x\to\infty} \frac{\frac{-3x^4+24x^2-48}{x^2}}{\frac{3-5x^2}{x^2}}$$

$$= \lim_{x\to\infty} \frac{-3x^2+24-\frac{48}{x^2}}{\frac{3}{x^2}-5}$$

approaching 0

$$= \frac{-3x^2+24-0}{0-5}$$

$$= \frac{-3x^2+24}{-5}$$

A huge negative

$$= +\infty$$

Negative divided by negative

Homework

#1, 3, 4, 5

Determinate-Indeterminate Forms Table	
Indeterminate Forms	Determinate Forms
$0/0$	$\infty + \infty = \infty$
$\pm\infty/\pm\infty$	$-\infty - \infty = -\infty$
$\infty - \infty$	$0^{\infty} = 0$
$0(\infty)$	$0^{-\infty} = \infty$
0^0	$(\infty) \cdot (\infty) = \infty$
1^{∞}	
∞^0	