Exploring Continuity

③ a) what is the domain of f

$$f(x) = \begin{cases} x+2 & -\infty < x < -1 \\ 1 & -1 < x < 0 \\ 2x & x \geq 0 \end{cases}$$

$$\boxed{D: \{x \mid x \neq -1,\ x \in \mathbb{R}\}}$$

b) $f(-1) = \text{undefined}$

$$\lim_{x \to -1^-} x+2 = -1+2 = 1$$

$$\lim_{x \to -1^+} 1 = 1$$

Discontinuous @ $x = -1$

$\lim_{x \to -1} f(x) \neq f(-1)$ removable discontinuity

c) $f(0) = 2(0)$
$f(0) = 0$

$$\lim_{x \to 0^-} 1 = 1$$

$$\lim_{x \to 0^+} 2x = 2(0) = 0$$

Discontinuous @ $x = 0$
$\lim_{x \to 0} f(x)$ does not exist or $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

jump discontinuity

Limits at Infinity

What exactly is infinity?

- It is the ***process*** of making a value arbitrarily large or small

$+\infty \longrightarrow$ Positive Infinity...process of becoming arbitrarily large

$-\infty \longrightarrow$ Negative Infinity...process of becoming arbitrarily small

4 Definition Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made as close to L as we like by taking x sufficiently large.

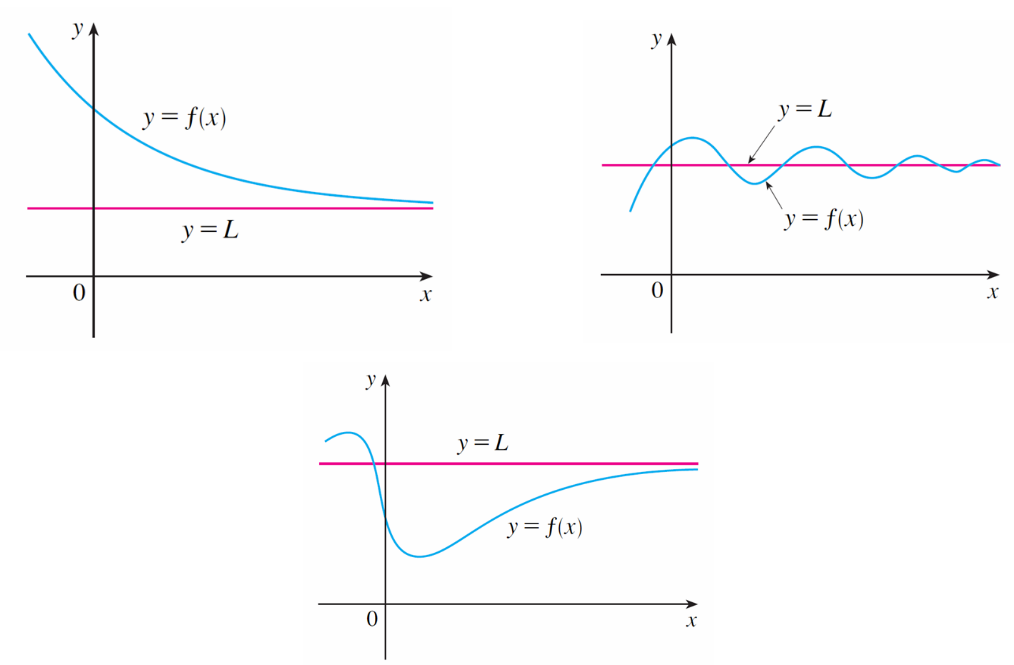


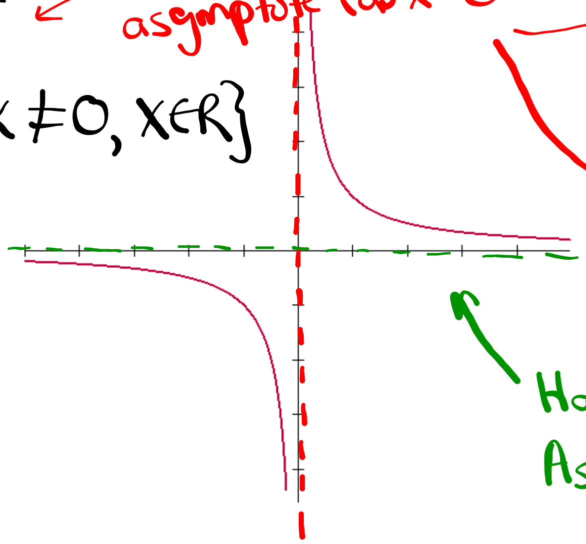
FIGURE 9
Examples illustrating $\lim_{x \to \infty} f(x) = L$

Have a look at these limits... HA

$$\lim_{x\to\infty}\frac{1}{x} = 0 \qquad \lim_{x\to-\infty}\frac{1}{x} = 0$$

$f(x) = \frac{1}{x}$ vertical asymptote @ $x=0$

D: $\{x \mid x \neq 0, x \in \mathbb{R}\}$



$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to 0^+} f(x) = +\infty$

Horizontal Asymptote $y=0$

In general...

7 If n is a positive integer, then

$$\lim_{x\to\infty}\frac{1}{x^n} = 0 \qquad \lim_{x\to-\infty}\frac{1}{x^n} = 0$$

Calculating limits at infinity without using a graph

• Rational Functions

***Note:* If every term in a rational expression is divided by the same value, the rational expression will still be equal to it's original value**

$$\frac{12+8}{6-2}$$

Divde the numerator and denominator by 2 →

This will be important when evaluating limits for rational functions approaching infinity...

Look at the following example:

approach 0

$$\lim_{x\to\infty}\frac{3x^2-x-2}{5x^2+4x+1} = \lim_{x\to\infty}\frac{\dfrac{3x^2-x-2}{x^2}}{\dfrac{5x^2+4x+1}{x^2}} = \lim_{x\to\infty}\frac{3-\dfrac{1}{x}-\dfrac{2}{x^2}}{5+\dfrac{4}{x}+\dfrac{1}{x^2}}$$

Divide every term by the HIGHEST power that is present in denominator of the rational expression once they are expanded

$$=\frac{3-0-0}{5+0+0}$$

$$=\frac{3}{5}$$

This graph below validates our solution:

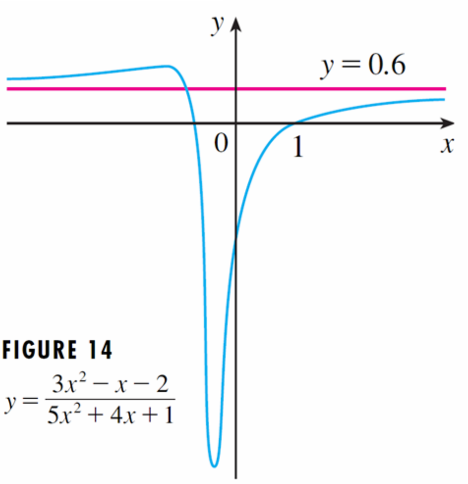


FIGURE 14

$y=\dfrac{3x^2-x-2}{5x^2+4x+1}$

- Remember

If the highest degree is in the denominator then the *Limit* will be equal to 0

If the highest degree is in the numerator then the *Limit* will not exist. Determine if it's $\pm\infty$

If the degree is the same in the numerator and denominator then the *Limit* will be equal to the coefficients in front of the highest degree. (Quotient of leading coefficients)

Evaluate the following limit:

$$\lim_{n\to\infty}\frac{n^2-n}{2n^2+1}$$

$$=\frac{1}{2}$$

$$\lim_{n\to\infty}\frac{\frac{n^2-n}{n^2}}{\frac{2n^2+1}{n^2}}$$

$$=\lim_{n\to\infty}\frac{1-\frac{1}{n}}{2+\frac{1}{n^2}}$$

$$=\frac{1-0}{2+0}$$

$$=\frac{1}{2}$$

$$\lim_{n\to\infty}\frac{1-n^5}{1+2n^5}$$

$$=\frac{-1}{2}$$

$$\lim_{n\to\infty}4n=+\infty \qquad \lim_{n\to-\infty}4n=-\infty$$

$$\lim_{x\to\infty}-4x=-\infty \qquad \lim_{x\to-\infty}-4x=+\infty$$

$$\lim_{x\to\infty}\frac{-3(x^2-4)^2}{3-5x^2}$$

$$=\lim_{x\to\infty}\frac{-3(x^4-8x^2+16)}{3-5x^2}$$

$$=\lim_{x\to\infty}\frac{-3x^4+24x^2-48}{3-5x^2}$$

$$=\lim_{x\to\infty}\frac{\frac{-3x^4+24x^2-48}{x^2}}{\frac{3-5x^2}{x^2}}$$

$$=\lim_{x\to\infty}\frac{-3x^2+24-\frac{48}{x^2}}{\frac{3}{x^2}-5}$$ approaching 0

$$=\frac{-3x^2+24-0}{0-5}$$

$$=\frac{-3x^2+24}{-5}$$ A huge negative

$$=+\infty$$ Negative divided by negative

Homework

#1, 3, 4, 5

- Factor
- Rationalize
- Expand
- Common Denominator

Determinate-Indeterminate Forms Table	
Indeterminate Forms	Determinate Forms
✓ $0/0$	$\infty + \infty = \infty$
✓ $\pm\infty / \pm\infty$	$-\infty - \infty = -\infty$
✓ $\infty - \infty$	$0^{\infty} = 0$
$0(\infty)$	$0^{-\infty} = \infty$
0^0	$(\infty) \cdot (\infty) = \infty$
1^{∞}	
∞^0	

Questions from Homework

⑤ b) $\lim\limits_{x \to -\infty} \dfrac{x^5 - x^2}{x^3 - 2x}$

$= \lim\limits_{x \to -\infty} \dfrac{\frac{x^5}{x^3} - \frac{x^2}{x^3}}{\frac{x^3}{x^3} - \frac{2x}{x^3}}$

$= \lim\limits_{x \to -\infty} \dfrac{x^2 - \frac{1}{x}}{1 - \frac{2}{x^2}}$ Approaching 0

$= \lim\limits_{x \to -\infty} \dfrac{x^2}{1}$ when square a negative you get a positive

$= +\infty$

Questions from Homework

5 f) $\displaystyle\lim_{x\to\infty}\left(\frac{1}{8}\right)^x + \frac{x^3-4x^2-5x}{4x^3+3x} - 7$

$$= \underbrace{\lim_{x\to\infty}\left(\frac{1}{8}\right)^x} + \underbrace{\lim_{x\to\infty}\frac{x^3-4x^2-5x}{4x^3+3x}} - \underbrace{\lim_{x\to\infty} 7}$$

$$= 0 + \frac{1}{4} - 7$$

$$= \frac{1}{4} - \frac{28}{4}$$

$$= \frac{-27}{4}$$

g) $\displaystyle\lim_{x\to\infty}\left(\frac{7}{3}\right)^{-x} + \frac{4x^2-5x}{2x^2+1} - 9$

$$= \underbrace{\lim_{x\to\infty}\left(\frac{3}{7}\right)^x} + \underbrace{\lim_{x\to\infty}\frac{4x^2-5x}{2x^2+1}} - \underbrace{\lim_{x\to\infty} 9}$$

$$= 0 + 2 - 9$$

$$= -7$$

Try These ones on your own!

by definition: $\sqrt{x^2} = |x| = \begin{cases} |x| & x<0 \\ x & x\geq 0 \end{cases}$

Indeterminate $\frac{\infty}{\infty}$ $\frac{-\infty}{\infty}$

Rationalizing does not work

$$\lim_{x\to\infty} \frac{4x+3}{\sqrt{4x^2+9}}$$

$$= \lim_{x\to\infty} \frac{x(4+\frac{3}{x})}{\sqrt{x^2(4+\frac{9}{x^2})}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}(4+\frac{3}{x})}{\cancel{|x|}\sqrt{4+\frac{9}{x^2}}}$$

x values are getting infinitely large (+)

$$= \lim_{x\to\infty} \frac{4+\frac{3}{x}}{\sqrt{4+\frac{9}{x^2}}}$$

approach 0

$$= \frac{4+0}{\sqrt{4+0}}$$

$$= \frac{4}{2}$$

$$= 2$$

$$\lim_{x\to-\infty} \frac{4x+3}{\sqrt{4x^2+9}}$$

$$= \lim_{x\to-\infty} \frac{x(4+\frac{3}{x})}{\sqrt{x^2(4+\frac{9}{x^2})}}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x}(4+\frac{3}{x})}{\cancel{|x|}\sqrt{4+\frac{9}{x^2}}}$$

x values are getting infinitely small (-)

$$= \lim_{x\to-\infty} \frac{-(4+\frac{3}{x})}{\sqrt{4+\frac{9}{x^2}}}$$

approach 0

$$= \frac{-(4+0)}{\sqrt{4+0}}$$

$$= \frac{-4}{2}$$

$$= -2$$

What about the following limits:

$$\lim_{x\to-\infty} \frac{\sqrt{x+16x^2}}{x+1} = -4$$

$$\lim_{x\to\infty} \frac{\sqrt{x+16x^2}}{x+1} = 4$$

Homework