Questions from homework

4. $$\lim_{x\to\infty} \frac{x^2}{2^x} = 0$$

$\frac{x^2}{2^x}$

x	y	
1	1/2	(0.5)
2	4/4	(1)
3	9/8	(1.125)
4	16/16	(1)
5	25/32	(0.78)
6	36/64	(0.56)
7	49/128	(0.38)
8	64/256	(0.25)
⋮	⋮	
∞	0	

5. h) $$\lim_{x\to -\infty} \left(\frac{1}{5}\right)^{2x} = +\infty$$

← negative exponent
take reciprocal of base $\left(\frac{1}{5}\right)^{-\infty} \Rightarrow \left(\frac{5}{1}\right)^{+\infty}$

l) $$\lim_{x\to\infty} \frac{2x}{\sqrt{x^2-2}-3x}$$

Rationalizing does not remove indeterminate form

$$= \lim_{x\to\infty} \frac{2x}{\sqrt{x^2\left(1-\frac{2}{x^2}\right)}-3x}$$

factor out an x^2

$$= \lim_{x\to\infty} \frac{2x}{|x|\sqrt{1-\frac{2}{x^2}}-3x}$$

$$= \lim_{x\to\infty} \frac{2x}{x\sqrt{1-\frac{2}{x^2}}-3x}$$

← Factor out an x

$$= \lim_{x\to\infty} \frac{2\cancel{x}}{\cancel{x}\left(\sqrt{1-\frac{2}{x^2}}-3\right)}$$

$$= \lim_{x\to\infty} \frac{2}{\sqrt{1-\frac{2}{x^2}}-3}$$

Approaches 0

$$= \frac{2}{\sqrt{1}-3}$$

$$= \frac{2}{-2} = -1$$

⑤ q) $\lim\limits_{x\to\infty} \dfrac{\left(2x-\sqrt{4x^2+6x}\right)}{1}$ $\infty-\infty$ Indeterminate

$= \lim\limits_{x\to\infty} \dfrac{\left(2x-\sqrt{4x^2+6x}\right)}{1}\dfrac{\left(2x+\sqrt{4x^2+6x}\right)}{\left(2x+\sqrt{4x^2+6x}\right)}$

$= \lim\limits_{x\to\infty} \dfrac{4x^2 - \overbrace{\left(4x^2+6x\right)}^{-4x^2-6x}}{2x+\sqrt{4x^2+6x}}$

$= \lim\limits_{x\to\infty} \dfrac{-6x}{2x+\sqrt{4x^2+6x}}$ $\frac{-\infty}{\infty}$ Indeterminate

factor out an x^2

$= \lim\limits_{x\to\infty} \dfrac{-6x}{2x+\sqrt{x^2(4+6/x)}}$

$= \lim\limits_{x\to\infty} \dfrac{-6x}{2x+|x|\sqrt{4+6/x}}$

$= \lim\limits_{x\to\infty} \dfrac{-6x}{2x+x\sqrt{4+6/x}}$ Factor out an x

$= \lim\limits_{x\to\infty} \dfrac{-6\cancel{x}}{\cancel{x}\left(2+\sqrt{4+6/x}\right)}$

$= \lim\limits_{x\to\infty} \dfrac{-6}{2+\sqrt{4+6/x}}$ Approach 0

$= \dfrac{-6}{2+\sqrt{4-0}}$

$= \dfrac{-6}{2+2}$

$= \dfrac{-6}{4}$

$= \dfrac{-3}{2}$

8.) Consider the function:

$$f(x) = \begin{cases} b + x, & x \le -1, \\ 1 - x, & -1 < x < 0, \\ \dfrac{1}{1+x^2} & x \ge 0 \end{cases}$$

where b is a parameter.

$\lim_{x \to a} f(x) = f(a)$

$\lim_{x \to -1} f(x) = f(-1)$

a) For what value(s) of b is f continuous at $x = -1$?

(i) $\lim_{x \to -1^+} 1 - x$

$= 1 - (-1)$

$= 2$

(ii) In order for x to be continuous @ $x = -1$ the $\lim_{x \to -1^-} f(x)$ must equal 2 and $f(-1)$ must equal 2

So $b + x$ must equal 2

$b + (-1) = 2$

$\boxed{b = 3}$ The function is continuous @ $x = -1$ if $b = 3$

b) Find $\lim_{x \to 0} f(x)$ if it exists. Justify your answer.

(i) $\lim_{x \to 0^-} 1 - x$

$= 1 - 0$

$= 1$

(ii) $\lim_{x \to 0^+} \dfrac{1}{1+x^2}$

$= \dfrac{1}{1+0}$

$= 1$

(iii) $\lim_{x \to 0} f(x) = 1$

① a) $\lim_{x\to 0} \frac{(x+2)\frac{2}{\cancel{x+2}} - 1\,(x+2)}{x\,(x+2)}$

$= \lim_{x\to 0} \frac{2-x-2}{x(x+2)}$

$= \lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}(x+2)}$

$= \frac{-1}{2}$

$\lim_{x\to 0} \frac{\frac{2}{x+2} - \frac{(x+2)}{(x+2)}}{x}$

$= \lim_{x\to 0} \frac{\frac{2-x-2}{x+2}}{x}$

$= \lim_{x\to 0} \frac{-\cancel{x}}{x+2} \cdot \frac{1}{\cancel{x}}$

$= \frac{-1}{2}$

b) $\lim_{x\to\infty} \frac{(2-3x^2)^2}{6x^4-7x^2-5}$

$= \lim_{x\to\infty} \frac{(2-3x^2)(2-3x^2)}{6x^4-7x^2-5}$

$= \lim_{x\to\infty} \frac{4-12x^2+9x^4}{6x^4-7x^2-5}$

$= \frac{9}{6} = \frac{3}{2}$

c) $\lim_{x\to 1} \frac{(x+2)^3-27}{x-1}$ ← diff of cubes

$= \lim_{x\to 1} \frac{\overset{x+2-3}{[(x+2)-3]}[(x+2)^2+3(x+2)+9]}{x-1}$

$= \lim_{x\to 1} \frac{[\cancel{x-1}][(x+2)^2+3(x+2)+9]}{\cancel{x-1}}$

$= 9+9+9$

$= 27$

d) $\lim_{x\to 7} \frac{\sqrt{x+9}-4}{(x-7)} \cdot \frac{(\sqrt{x+9}+4)}{(\sqrt{x+9}+4)}$

$= \lim_{x\to 7} \frac{x+9-16}{(x-7)(\sqrt{x+9}+4)}$

$= \lim_{x\to 7} \frac{\cancel{x-7}}{(\cancel{x-7})(\sqrt{x+9}+4)}$

$= \frac{1}{8}$

8 e) $\lim_{x\to-\infty} \frac{\sqrt{9x^2-x}}{x+1}$ ← Factor out an x^2 ← factor out an x

$= \lim_{x\to-\infty} \frac{\sqrt{x^2(9-\frac{1}{x})}}{x(1+\frac{1}{x})}$

$= \lim_{x\to-\infty} \frac{\cancel{|x|}^{-1}\sqrt{9-\frac{1}{x}}}{\cancel{x}(1+\frac{1}{x})}$

$= \lim_{x\to-\infty} -\frac{\sqrt{9-\frac{1}{x}}}{1+\frac{1}{x}}$ ← Approach 0

$= \frac{-\sqrt{9}}{1}$ $= -3$

f) $\lim_{x\to4} \frac{x^2-10x+24}{x^2-16}$ ← Simple trinomial $-6 + -4 = -10$, $-6 \times -4 = 24$ ← diff of squares

$= \lim_{x\to4} \frac{(x-6)\cancel{(x-4)}}{(x+4)\cancel{(x-4)}}$

$= \frac{-2}{8} = -\frac{1}{4}$

g) $\lim_{h\to0} \frac{(h-2)^2-4}{h}$ ← diff of squares

$= \lim_{h\to0} \frac{[(h-2)+2][(h-2)-2]}{h}$

$= \lim_{h\to0} \frac{\cancel{h}(h-4)}{\cancel{h}}$

$= -4$

h) $\lim_{x\to3} \frac{\sqrt{x^2-8}}{x^2-x-4}$

$= \frac{\sqrt{9-8}}{9-3-4}$

$= \frac{1}{2}$

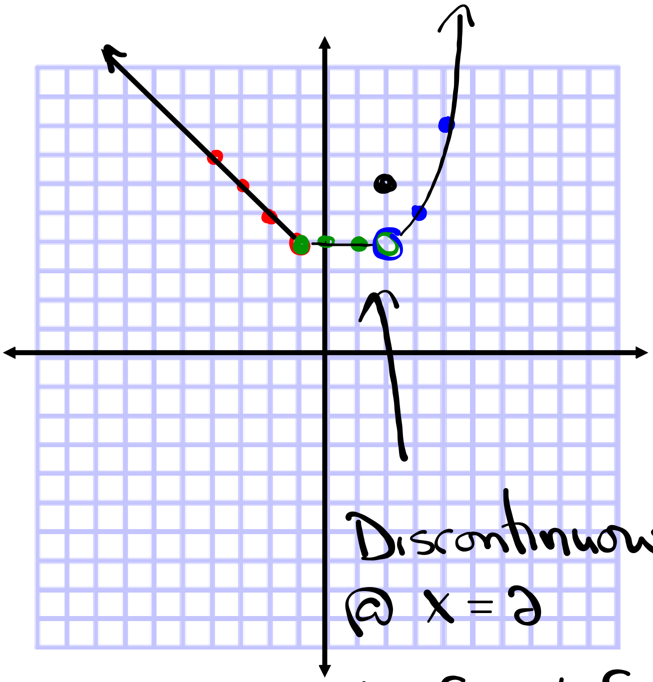
$$f(x) = \begin{cases} 3-x, & x<-1 \\ 4, & -1 \le x < 2 \\ 6, & x=2 \\ (x-2)^2+4, & x>2 \end{cases}$$

$3-x$

x	y
-1	4
-2	5
-3	6
-4	7
⋮	⋮

4

x	y
-1	4
0	4
1	4
2	4



6

x	y
2	6

$(x-2)^2+4$

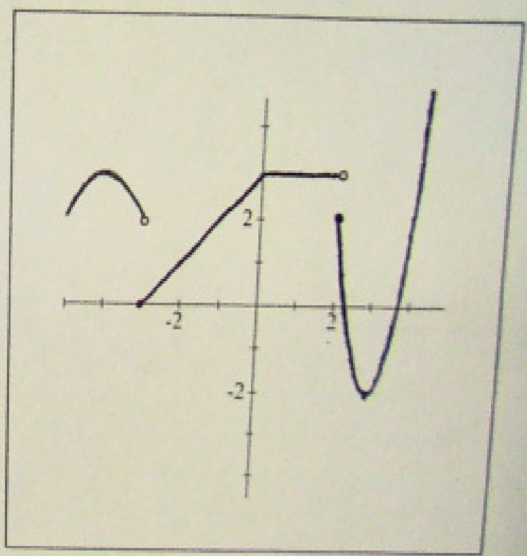
x	y
2	4
3	5
4	8
5	13
⋮	⋮

Discontinuous @ $x=2$

$\lim_{x\to 2} f(x) \neq f(2)$

This is a removable discontinuity

3. The following is a graph of $f(x)$:



Evaluate each of the following:

(a) $\lim_{x \to -3^+} f(x) =$ 0 (b) $\lim_{x \to -3^-} f(x) =$ 2

(c) $f(-3) =$ 0 (d) $\lim_{x \to 2^-} f(x) =$ 3

(e) $\lim_{x \to 2^+} f(x) =$ 2 (f) $f(2) =$ 2 closed dot

(g) $\lim_{x \to 3} f(x) =$ -2 (h) $f(3) =$ -2

i) $\lim_{x \to -3} f(x) =$ DNE