Questions from Homework

Find the derivative of each function.

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. $f(x) = 8x^2 - 10$

   $f'(x) = 16x$

2. $f(x) = 2x^2 + 14x - 7$

   $f'(x) = 4x + 14$

3. $f(x) = x^3$

   $f'(x) = 3x^2$

4. $f(x) = \dfrac{x+4}{2x+3}$ $\quad f'(x) = \dfrac{-5}{(2x+3)^2}$

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

① Find $f(x+h)$

② Fill in formula

③ Solve

① $f(x) = \boxed{8x^2 - 10}$

(1) $f(x+h) = 8(x+h)^2 - 10$

$f(x+h) = 8(x+h)(x+h) - 10$

$f(x+h) = 8(x^2 + 2xh + h^2) - 10$

$f(x+h) = \boxed{8x^2 + 16xh + 8h^2 - 10}$

(2) $f'(x) = \lim_{h \to 0} \dfrac{8x^2 + 16xh + 8h^2 - 10 - (8x^2 - 10)}{h}$

(3) $f'(x) = \lim_{h \to 0} \dfrac{\cancel{8x^2} + 16xh + 8h^2 \cancel{-10} \cancel{-8x^2} \cancel{+10}}{h}$

$f'(x) = \lim_{h \to 0} \dfrac{16xh + 8h^2}{h}$

$f'(x) = \lim_{h \to 0} \dfrac{\cancel{h}(16x + 8h)}{\cancel{h}}$

$f'(x) = 16x + 8(0)$

$\boxed{f'(x) = 16x}$

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

① Find $f(x+h)$

② Fill in formula

③ Solve.

2. $f(x) = \boxed{2x^2 + 14x - 7}$

① $f(x+h) = 2(x+h)^2 + 14(x+h) - 7$

$= 2(x^2 + 2xh + h^2) + 14x + 14h - 7$

$= 2x^2 + 4xh + 2h^2 + 14x + 14h - 7$

(2) $f'(x) = \lim\limits_{h \to 0} \dfrac{2x^2 + 4xh + 2h^2 + 14x + 14h - 7 - (2x^2 + 14x - 7)}{h}$

(3) $= \lim\limits_{h \to 0} \dfrac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+14x} + 14h \cancel{-7} \cancel{-2x^2} \cancel{-14x} \cancel{+7}}{h}$

$= \lim\limits_{h \to 0} \dfrac{4xh + 2h^2 + 14h}{h}$ common factor

$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(4x + 2h + 14)}{\cancel{h}}$

$= 4x + 14$

$\boxed{f'(x) = 4x + 14}$

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

① Find $f(x+h)$
② Fill in formula
③ Solve.

③ $f(x) = x^3$

$f(x+h) = (x+h)^3$
$= (x+h)(x+h)(x+h)$
$= (x+h)(x^2+2xh+h^2)$
$= x^3+2x^2h+xh^2+x^2h+2xh^2+h^3$
$= x^3+3x^2h+3xh^2+h^3$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3 - \cancel{x^3}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{3x^2h+3xh^2+h^3}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}} = 3x^2$$

$$\boxed{f'(x) = 3x^2}$$

$f(x) = x^3$ $\qquad$ $f(x+h) = (x+h)^3$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)-x][(x+h)^2+x(x+h)+x^2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[(x+h)^2+x(x+h)+x^2]}{\cancel{h}} = x^2+x^2+x^2$$

$$\boxed{f'(x) = 3x^2}$$

Remember!

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

① Find $f(x+h)$
② Fill in formula
③ Solve.

④ $f(x) = \boxed{\dfrac{x+4}{2x+3}}$

(1) $f(x+h) = \dfrac{(x+h)+4}{2(x+h)+3}$

$f(x+h) = \boxed{\dfrac{x+h+4}{2x+2h+3}}$

(2) $f'(x) = \lim\limits_{h \to 0} \dfrac{\cancel{(2x+3)}\cancel{(2x+2h+3)}\dfrac{x+h+4}{\cancel{2x+2h+3}} - \dfrac{x+4}{\cancel{2x+3}}\cancel{(2x+3)}(2x+2h+3)}{h\,(2x+3)(2x+2h+3)}$

$f'(x) = \lim\limits_{h \to 0} \dfrac{(2x+3)(x+h+4) - (x+4)(2x+2h+3)}{h(2x+3)(2x+2h+3)}$

$-\cancel{2x^2} - \cancel{2xh} - \cancel{3x} - \cancel{8x} - 8h - \cancel{12}$

$f'(x) = \lim\limits_{h \to 0} \dfrac{\cancel{2x^2} + \cancel{2xh} + \cancel{8x} + \cancel{3x} + 3h + \cancel{12} - (2x^2 + 2xh + 3x + 8x + 8h + 12)}{h(2x+3)(2x+2h+3)}$

$f'(x) = \lim\limits_{h \to 0} \dfrac{-5\cancel{h}}{\cancel{h}(2x+3)(2x+2h+3)}$

$f'(x) = \dfrac{-5}{(2x+3)(2x+2(0)+3)}$

$$\boxed{f'(x) = \frac{-5}{(2x+3)^2}}$$

Remember!

Try this one!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiate the following function using the *Limit Definition of the Derivative*

$$f(x) = \sqrt{x+3}$$

$$f(x+h) = \sqrt{(x+h)+3} = \sqrt{x+h+3}$$

$$f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{x+h+3} - \sqrt{x+3}\right)}{h} \cdot \frac{\left(\sqrt{x+h+3} + \sqrt{x+3}\right)}{\left(\sqrt{x+h+3} + \sqrt{x+3}\right)}$$

$$= \lim_{h \to 0} \frac{x+h+3-(x+3)}{h\left(\sqrt{x+h+3} + \sqrt{x+3}\right)}$$

Homework