Questions from Homework

① g) $y = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}} = x^{-1/2}(x+1) = x^{1/2} + x^{-1/2}$

$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$

k) $u(t) = a + \frac{b}{t} + \frac{c}{t^2} = a + bt^{-1} + ct^{-2}$

$u'(t) = -bt^{-2} - 2ct^{-3} = \frac{-b}{t^2} - \frac{2c}{t^3}$

l) $v(r) = \sqrt{r}(2+3r) = r^{1/2}(2+3r) = 2r^{1/2} + 3r^{3/2}$

$v'(r) = r^{-1/2} + \frac{9}{2}r^{1/2} = \frac{1}{\sqrt{r}} + \frac{9\sqrt{r}}{2}$

Questions from Homework

③ c) $y = x + \frac{6}{x}$; $(2, 5)$

$x_1 = 2$
$y_1 = 5$

$m = -\frac{1}{2}$

(i) $y = x + 6x^{-1}$

$y' = 1 - 6x^{-2}$

$y' = 1 - \frac{6}{x^2}$

(ii) $y' = 1 - \frac{6}{(2)^2}$

$y' = 1 - \frac{3}{2}$

$y' = \frac{2}{2} - \frac{3}{2}$

$y' = -\frac{1}{2}$

$m = -\frac{1}{2}$

(iii) $y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 2)$

$y - 5 = -\frac{1}{2}x + 1$

$\boxed{y = -\frac{1}{2}x + 6}$

$2y = -x + 12$

$x + 2y - 12 = 0$

Questions from Homework

④ $y = 2\sqrt{x}$ $(9, 6)$

$x_1 = 9$ $y_1 = 6$

(i) Differentiate

$y = 2\sqrt{x}$

$y = 2x^{1/2}$

$y' = 1x^{-1/2}$

$y' = \frac{1}{x^{1/2}}$

$y' = \frac{1}{\sqrt{x}}$

(ii) Sub in x-value

$y' = \frac{1}{\sqrt{9}}$

$y' = \frac{1}{3}$

$m = \frac{1}{3}$

(iii) $y - y_1 = m(x - x_1)$

$y - 6 = \frac{1}{3}(x - 9)$

$y - 6 = \frac{1}{3}x - 3$

$y = \frac{1}{3}x - 3 + 6$

$\boxed{y = \frac{1}{3}x + 3}$

$0 = \frac{1}{3}x - y + 3$

or $\boxed{0 = x - 3y + 9}$

Questions from Homework

⑦ At what point on the parabola $y = 3x^2$ is the slope of the tangent line equal to 24
m

① $y = 3x^2$
$y' = 6x$

② $y' = 6x$
$24 = 6x$
$4 = x$

③ $y = 3x^2$
$y = 3(4)^2$
$y = 3(16)$
$y = 48$

④ $(4, 48)$

Questions from Homework

(x,y)

⑧ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to $6x - y = 4$

same slope.

(i) Find slope of

$6x - y = 4$

$\frac{-y}{-1} = \frac{-6x}{-1} + \frac{4}{-1}$

$y = 6x - 4$

$(y = mx + b)$

$\boxed{m = 6}$

(ii) Find y' (slope of tangent)

$y = x\sqrt{x} = x \cdot x^{1/2} = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2} \leftarrow m$

(iii) set slopes equal

$\cancel{2} \cdot \frac{3\sqrt{x}}{\cancel{2}} = 6 \cdot 2$

$\frac{3\sqrt{x}}{3} = \frac{12}{3}$

$\sqrt{x} = 4$

$(x^{1/2})^2 = (4)^2$

$\boxed{x = 16}$

(iv) Solve for y

$y = x\sqrt{x}$

$y = (16)\sqrt{(16)}$

$y = 16(4)$

$\boxed{y = 64}$

(16, 64) is the point

Warm Up

Differentiate the following:

$$f(x) = -4x^2 - 5x(x^3+7)^2 + 2\sqrt[5]{x^9} - \frac{5}{x^{10}} + \frac{7x^2}{\sqrt{x}} - e^6$$

$$f(x) = -4x^2 - 5x(x^6+14x^3+49) + 2x^{9/5} - 5x^{-10} + 7x^2(x^{-1/2})$$

$$f(x) = -4x^2 - 5x^7 - 70x^4 - 245x + 2x^{9/5} - 5x^{-10} + 7x^{3/2}$$

$$f'(x) = -8x - 35x^6 - 280x^3 - 245 + \frac{18}{5}x^{4/5} + 50x^{-11} + \frac{21}{2}x^{1/2}$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$ = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is:*** ***the derivative of the first function times the second function, plus the first function times the derivative of the second function***

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$

Examples:

$y = f(x)g(x)$

$y' = f'(x)g(x) + f(x)g'(x)$

$$y = (2x^3 + 5)(3x^2 - x)$$

first second

$y' = 6x^2(3x^2 - x) + (2x^3 + 5)(6x - 1)$

$y' = 18x^4 - 6x^3 + 12x^4 - 2x^3 + 30x - 5$

$\boxed{y' = 30x^4 - 8x^3 + 30x - 5}$

$$f(x) = \sqrt{x}(2 - 3x) = x^{1/2}(2 - 3x)$$

$f'(x) = \frac{1}{2}x^{-1/2}(2 - 3x) + x^{1/2}(-3)$

$f'(x) = \frac{1}{2x^{1/2}}(2 - 3x) - 3x^{1/2}$

$f'(x) = \frac{2 - 3x}{2x^{1/2}} - \frac{3x^{1/2}}{1}$ CD: $2x^{1/2}$

$f'(x) = \frac{2 - 3x}{2x^{1/2}} - \frac{6x}{2x^{1/2}} = \boxed{\frac{2 - 9x}{2\sqrt{x}}}$

Examples:

$$f(x)=(7x^3-x^2+5)(x^9+3x-5)$$

$$f'(x)=(21x^2-2x)(x^9+3x-5)+(7x^3-x^2+5)(9x^8+3)$$

$$h(t)=(t^3-5t)(6\sqrt{t}-t^{-5})$$

$$h'(t)=(3t^2-5)(6\sqrt{t}-t^{-5})+(t^3-5t)(3t^{-1/2}+5t^{-6})$$

Homework

$$(f\cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

① a) $y = (2x-1)(x^2+1)$

$$y' = 2(x^2+1) + (2x-1)(2x)$$

f) $f(t) = \sqrt[3]{t}\,(1-t) = t^{1/3}(1-t)$

$$f'(t) = \frac{1}{3}t^{-2/3}(1-t) + t^{1/3}(-1)$$

g) $F(y) = \sqrt{y}\,(y - 2\sqrt{y} + 2) = \underbrace{y^{1/2}}_{f(x)}\underbrace{(y - 2y^{1/2} + 2)}_{g(x)}$

$$F'(y) = \frac{1}{2}y^{-1/2}(y - 2y^{1/2} + 2) + y^{1/2}(1 - y^{-1/2})$$

$$= \frac{1}{2\sqrt{y}}(y - 2\sqrt{y} + 2) + \sqrt{y}\left(1 - \frac{1}{\sqrt{y}}\right)$$

$$(f\cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

① d) $y = \underbrace{(x^3+x^2+1)}_{f(x)}\underbrace{(x^2+2)}_{g(x)}$

$$y' = (3x^2+2x)(x^2+2) + (x^3+x^2+1)(2x)$$

f) $f(t) = \sqrt[3]{t}\,(1-t) = \underbrace{t^{1/3}}_{f(x)}\underbrace{(1-t)}_{g(x)}$

$$f'(t) = \frac{1}{3}t^{-2/3}(1-t) + t^{1/3}(-1)$$

$$f'(t) = \frac{1}{3t^{2/3}}(1-t) - t^{1/3}$$

$$f'(t) = \frac{1-t}{3t^{2/3}} - \frac{t^{1/3}}{1}$$

CD: $3t^{2/3}$

$$f'(t) = \frac{1-t}{3t^{2/3}} - \frac{3t}{3t^{2/3}} = \frac{1-t-3t}{3t^{2/3}} = \frac{1-4t}{3\sqrt[3]{t^2}}$$

② b) $y = x^{-2}(x^3 - 3x^2 + 6)$

$y' = -2x^{-3}(x^3 - 3x^2 + 6) + x^{-2}(3x^2 - 6x)$

$y' = -2 + 6x^{-1} - 12x^{-3} + 3 - 6x^{-1}$

$y' = 1 - 12x^{-3}$

② e) $f(t) = (6 + t^{-2})(8t^{10} - 5t^3)$

$f'(t) = (-2t^{-3})(8t^{10} - 5t^3) + (6 + t^{-2})(80t^9 - 15t^2)$

$f'(t) = -16t^7 + 10t^0 + 480t^9 - 90t^2 + 80t^7 - 15t^0$

$f'(t) = 480t^9 + 64t^7 - 90t^2 - 5t^0$ $\quad t^0 = 1$

$f'(t) = 480t^9 + 64t^7 - 90t^2 - 5$

f) $f(t) = (at + b)(ct^2 - d)$

$f'(t) = a(ct^2 - d) + (at + b)(2ct)$

$f'(t) = act^2 - ad + 2act^2 + 2bct$

$f'(t) = 3act^2 + 2bct - ad$

⑤ $y = (2-\sqrt{x})(1+\sqrt{x}+3x)$ @ $(1,5)$

(I) $y' = \left(-\frac{1}{2}x^{-\frac{1}{2}}\right)(1+\sqrt{x}+3x) + (2-\sqrt{x})\left(\frac{1}{2}x^{-\frac{1}{2}}+3\right)$

$y' = \left(-\frac{1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x) + (2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$

(II) $y' = \left(-\frac{1}{2\sqrt{1}}\right)(1+\sqrt{1}+3(1)) + (2-\sqrt{1})\left(\frac{1}{2\sqrt{1}}+3\right)$

$y' = \left(\frac{-1}{2}\right)(5) + 1\left(\frac{1}{2}+\frac{3}{1}\right)$

$y' = -\frac{5}{2} + 1\left(\frac{7}{2}\right)$

$y' = -\frac{5}{2} + \frac{7}{2} = \frac{2}{2} = \underline{\underline{1}} \leftarrow m$

(III) $y - y_1 = m(x - x_1)$

$y - 5 = 1(x - 1)$

$y - 5 = x - 1$

$y = x + 4$

or $\boxed{x - y + 4 = 0}$