# **Understanding Logarithms**

#### Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of  $y = \log_c x$ , c > 0,  $c \ne 1$
- determining the characteristics of the graph of  $y = \log_c x$ , c > 0,  $c \ne 1$
- · explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- · evaluating logarithms using a variety of methods

Exide Glog<sub>3</sub>x - Glog<sub>3</sub>x<sup>4</sup> + log<sub>3</sub>x
$$log_3x^5 - log_3(x^4)^3 + log_3x$$

$$log_3x^5 - log_3x^3 + log_3x$$

$$log_3(\frac{x^5 \cdot x}{x^3})$$

$$log_3(\frac{x^6}{x^3})$$

$$log_3x^4$$

$$4 log_3x$$

# **General Properties of Logarithms:**

If c > 0 and  $c \neq 1$ , then...

- (i)  $\log_{\mathbf{c}} 1 = 0$
- (ii)  $\log_{\mathbf{c}} \mathbf{c}^{x} = x$
- (iii)  $c^{\log_c x} = x$

Did You Know?

The input value for a logarithm is called an argument. For example, in the expression log<sub>6</sub> 1, the argument is 1.

(i) 
$$\log_5 1 = 0$$
 (ii)  $\log_5 3^3 = 3$  (iii)  $\gamma^{\log_5 49} = 49$ 

### **Product Law of Logarithms**

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c MN = \log_c M + \log_c N$$

Proof

Let  $\log_c M = x$  and  $\log_c N = y$ , where M, N, and c are positive real numbers with  $c \neq 1$ .

Write the equations in exponential form as  $M = c^x$  and  $N = c^y$ :

$$\begin{split} MN &= (c^x)(c^y) \\ MN &= c^{x+y} \\ \log_c MN &= x+y \\ \log_c MN &= \log_c M + \log_c N \end{split} \qquad \text{Apply the product law of powers.}$$

#### **Quotient Law of Logarithms**

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

Proof

Let  $\log_c M = x$  and  $\log_c N = y$ , where M, N, and c are positive real numbers with  $c \neq 1$ .

Write the equations in exponential form as  $M = c^x$  and  $N = c^y$ :

$$\frac{M}{N} = \frac{c^x}{c^y}$$

$$\frac{M}{N} = c^{x-y}$$

Apply the quotient law of powers.

$$\log_c \frac{M}{N} = x - y$$

Write in logarithmic form.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

Substitute for x and y.

#### **Power Law of Logarithms**

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^p = P \log_c M$$

How could you prove the quotient law using the product law and the power law?

Proof

Let  $\log_c M = x$ , where M and c are positive real numbers with  $c \neq 1$ .

Write the equation in exponential form as  $M = c^x$ . Let P be a real number.

$$\begin{aligned} M &= c^x \\ M^p &= (c^x)^p \\ M^p &= c^{xp} \end{aligned} & \text{Simplify the exponents.} \\ \log_c M^p &= xP & \text{Write in logarithmic form.} \\ \log_c M^p &= (\log_c M)P & \text{Substitute for } x. \\ \log_c M^p &= P\log_c M \end{aligned}$$

The laws of logarithms can be applied to logarithmic functions, expressions, and equations.

# Product Law of Logarithms

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c \underline{MN} = \log_c M + \log_c N$$

 $= \log 100$   $= \log 100$   $= \log 100$ 

# **Quotient Law of Logarithms**

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

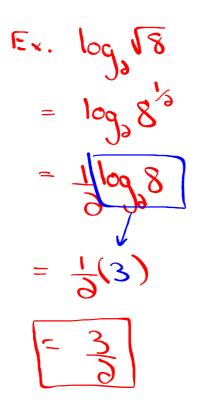
 $Ex: log_6 36 - log_6 4 = log_6 (\frac{36}{4})$   $= log_6 9$ 

## **Power Law of Logarithms**

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^p = P \log_c M$$

How could you prove the quotient law using the product law and the power law?



Express as a single logarithm:

$$\log_{3}(x)^{3} - \log_{3}(x)^{3} - \log_{3}(2)^{4}$$

$$\log_3\left(\frac{\lambda_3}{\lambda_3}\right)$$

8

#### Questions from Homework

$$\frac{\log u = m}{\log u} \quad (\log solu)$$

h) 
$$\log_4 8 = 1.5$$
  $\longrightarrow \text{Let } x = \log_4 8$ 

$$\log_4 8 = 1.5$$

$$\log_4 8 = 1.5$$

$$4^x = 8$$

$$8^x = 8^3$$

$$3x = 3$$

$$x = 3$$

$$x = 3$$

$$\frac{\text{P}}{\text{S}} = \frac{1}{\text{S}} = \frac{4}{\text{exp}}$$

$$\frac{1}{\text{S}} = \frac{1}{\text{exp}}$$

$$\frac{3}{\text{exp}} = \frac{1}{\text{exp}}$$

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$$\bigoplus_{\text{log}} e) \quad \partial^{1-x} = 3 \quad (\exp_{\text{form}})$$

$$X = 1 - \log_3 3$$

#### **Questions from Homework**

Exercise 2

(3) b) 
$$\log_3 30 = 5$$
 g)  $\log_3 (60) = -3$   $\log_3 (60) = -3$   $\log_3 (60) = -3$   $\log_3 (60) = -3$ 

j) 
$$log_{9}l_{3} = 0.35$$

$$\frac{log(l_{3})}{log_{9}} = 0.35 \text{ or } \frac{l}{q}$$

h) 
$$10^{5} = 3$$
 (Exp. form)

Base ans

 $\log_{10}(3) = 5^{x}$  (Exp form)

 $cns$ 
 $\log_{10}(3) = 5^{x}$ 

1

#### Example 1

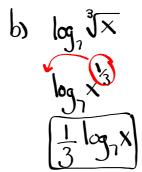
#### Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of x, y, and z.

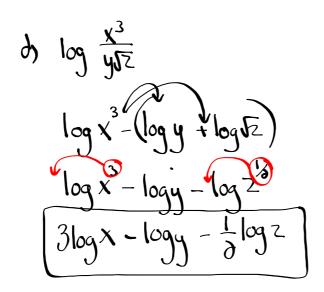
- a)  $\log_5 \frac{xy}{z}$
- **b)**  $\log_7 \sqrt[3]{X}$
- c)  $\log_6 \frac{1}{X^2}$
- **d)**  $\log \frac{X^3}{V\sqrt{Z}}$

a) 
$$\log_5 \frac{xy}{z}$$

$$\log_5 x + \log_5 y - \log_5 z$$



c) 
$$\log_6 \frac{1}{x^3}$$
 $\log_6 1 - \log_6 x^3$ 
 $0 - 2\log_6 x$ 
 $-2\log_6 x$ 



#### Example 2

#### Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

- a)  $\log_6 8 + \log_6 9 \log_6 2$
- **b)**  $\log_7 7\sqrt{7}$
- c)  $2 \log_2 12 \left(\log_2 6 + \frac{1}{3} \log_2 27\right)$

a) 
$$\log_{6} 8 + \log_{9} 9 - \log_{6} 3$$

$$\log_{6} \left(\frac{8 \cdot 9}{3}\right)$$

$$\log_{6} \left(\frac{8 \cdot 9}{3}\right)$$

$$\log_{7} 7 + \log_{7} 7$$

# Example 3

## Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) 
$$\log_7 x^2 + \log_7 x - \frac{5 \log_7 x}{2}$$

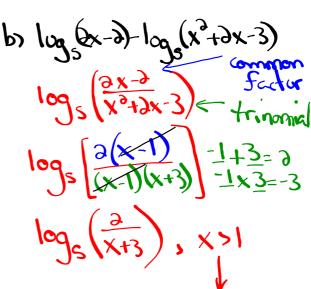
**b)** 
$$\log_5 (2x-2) - \log_5 (x^2 + 2x - 3)$$

a) 
$$\log_{1}x^{3} + \log_{1}x - 5\log_{1}x$$

$$\log \left(\frac{x_3 \cdot x}{x_2 \cdot x}\right)$$

$$\log_{1}\left(\frac{x^{3}}{x^{5/3}}\right)^{3}$$

$$\frac{1}{2}\log_2 x$$



For the original expression to be defined, both logarithmic terms must be defined.

$$2x-2>0 \qquad x^2+2x-3>0 \qquad \text{What other methods could} \\ 2x>2 \qquad (x+3)(x-1)>0 \qquad \text{you have used to solve this} \\ x>1 \quad \text{and} \quad x<-3 \text{ or } x>1$$

The conditions x > 1 and x < -3 or x > 1 are both satisfied when x > 1.

Hence, the variable x needs to be restricted to x > 1 for the original expression to be defined and then written as a single logarithm.

Therefore,  $\log_5 (2x - 2) - \log_5 (x^2 + 2x - 3) = \log_5 \frac{2}{x + 3}$ , x > 1.

## **Key Ideas**

• Let P be any real number, and M, N, and c be positive real numbers with  $c \neq 1$ . Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_{c} MN = \log_{c} M + \log_{c} N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^p = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

Many quantities in science are measured using a logarithmic scale. Two
commonly used logarithmic scales are the decibel scale and the pH scale.

# Homework Finish Exercise 3

Exercise 2

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$$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$
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Exercise 3.

Oi) log<sub>10</sub> 
$$\frac{x^3y^4}{2^5}$$

log  $x^3$  + log  $y^4$  - log  $z^6$ 

3 log  $x$  + 4 log  $y$  - 6 log  $z$ 

(3) d) 
$$4 \log_{3} x - \frac{1}{3} \log_{3} (x^{3}+1) + \log_{3} (x-1)$$

$$\log_{3} x^{4} - \log_{3}(x^{3}+1)^{1/3} + \log_{3}(x-1)$$

$$\int_{\mathcal{S}} \int_{\mathcal{S}} \frac{1 + e^{\chi} \sqrt{1 + 1}}{\sqrt{1 + e^{\chi} \sqrt{1 + 1}}} \int_{\mathcal{S}} e^{\zeta d}$$

$$\log_{s}(x^{2}-1) - \log_{s}(x-1)$$

$$\log_{s}\left(\frac{x^{2}-1}{x-1}\right) \cdot \int_{s=0}^{\infty} \int_{s=0}^{\infty} \log_{s}(x-1) dx dx$$

$$\log_{s}\left(\frac{x^{2}-1}{x-1}\right) \cdot \int_{s=0}^{\infty} \log_{s}(x-1) dx dx$$

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$$\log_{s}\left(\frac{x^{2}-1}{x-1}\right) \cdot \int_{s=0}^{\infty} \log_{s}(x-1) dx$$

# **Questions from Homework**

Exercise 3

$$3e > \frac{1}{3} \left[ \log_5 x + \log_5 y - \log_5 z^2 \right]$$

$$\frac{1}{3} \left[ \log_5 x + \log_5 y^3 - \log_5 z^3 \right]$$

$$\frac{1}{3} \left[ \log_5 \left( \frac{xy}{z^3} \right)^{\frac{1}{3}} \right]$$

$$\log_5 \left( \frac{xy}{z^3} \right)^{\frac{1}{3}}$$

$$\log_5 \sqrt{\frac{xy^3}{z^3}}$$

$$\log_5 \sqrt{\frac{xy^3}{z^3}}$$

# Do I really understand??...

- a) Express the following as a single logarithm...  $2 \log_2 3^2 + \log_2 6 3 \log_2 3$
- b) Evaluate the following...  $\log_2(32)^{\frac{1}{3}}$
- c) Express the following as a single logarithm...  $\frac{1}{2} [(\log_5 a + 2\log_5 b) 3\log_5 c]$
- d) Express as a single logarithm in simplest form...

$$\frac{3}{4} \left[ 12 (\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7} \right]$$

Quiz!

Exercise 2

Exercise 3

Assignment -> x and y-intercepts