② f) $f(t) = (at+b)(ct^2-d)$

$f'(t) = (a)(ct^2-d) + (at+b)(2ct)$

$f'(t) = \underline{act^2} - ad + \underline{2act^2} + 2bct$

$f'(t) = 3act^2 + 2bct - ad$

③a) $y = (1-2x)(3x-4)$, $x = 2$

① Find y':

$y' = -2(3x-4) + (1-2x)(3)$

$y' = -6x + 8 + 3 - 6x$

$y' = -12x + 11$

② sub in $x = 2$

$y' = -12x + 11$

$y' = -12(2) + 11$

$y' = -13$

$m = -13$

⑤ $y = (2-\sqrt{x})(1+\sqrt{x}+3x)$ @ $(1,5)$

$y = (2 - x^{1/2})(1 + x^{1/2} + 3x)$

① find y':

$$y' = \left(-\frac{1}{2}x^{-1/2}\right)(1+x^{1/2}+3x) + (2-x^{1/2})\left(\frac{1}{2}x^{-1/2}+3\right)$$

$$y' = \left(\frac{-1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x) + (2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$$

② sub in $x = 1$

$$y' = \left(\frac{-1}{2\sqrt{1}}\right)(1+\sqrt{1}+3(1)) + (2-\sqrt{1})\left(\frac{1}{2\sqrt{1}}+3\right)$$

$$y' = \left(-\frac{1}{2}\right)(5) + (1)\left(\frac{7}{2}\right)$$

$$y' = \frac{-5}{2} + \frac{7}{2} = \frac{2}{2} = 1$$

$m = 1$

③ $y - y_1 = m(x - x_1)$

$y - 5 = 1(x - 1)$

$y - 5 = x - 1$

$y = x + 4$

or $x - y + 4 = 0$

② h) $g(v) = (v - \sqrt{v})(v^2 + \sqrt{v})$

$$g(v) = \underbrace{(v - v^{1/2})}_{f(x)}\underbrace{(v^2 + v^{1/2})}_{g(x)}$$

$1 + (-\frac{1}{2})$

$\frac{2}{2} + (-\frac{1}{2})$

$-\frac{1}{2} + \frac{2}{1}$

$-\frac{1}{2} + \frac{4}{2}$

$$g'(v) = \left(1 - \frac{1}{2}v^{-1/2}\right)(v^2 + v^{1/2}) + (v - v^{1/2})\left(2v + \frac{1}{2}v^{-1/2}\right)$$

$$g'(v) = v^2 + v^{1/2} - \frac{1}{2}v^{3/2} - \frac{1}{2}\cancel{v^0} + 2v^2 + \frac{1}{2}v^{1/2} - 2v^{3/2} - \frac{1}{2}\cancel{v^0}$$

$$g'(v) = \frac{3v^2}{1} - \frac{5}{2}v^{3/2} + \frac{3}{2}v^{1/2} - \frac{1}{1}$$

$$g'(v) = \frac{6v^2}{2} - \frac{5v^{3/2}}{2} + \frac{3v^{1/2}}{2} - \frac{2}{2}$$

$$g'(v) = \frac{6v^2 - 5v^{3/2} + 3v^{1/2} - 2}{2}$$

Product Rule:

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

② g) $g(u) = \sqrt{u}\,(2 - u^2 + 5u^4)$

$$g(u) = \underbrace{u^{1/2}}_{f(x)}\underbrace{(2 - u^2 + 5u^4)}_{g(x)}$$

$$g'(u) = \frac{1}{2}u^{-1/2}(2 - u^2 + 5u^4) + u^{1/2}(-2u + 20u^3)$$

$$g'(u) = u^{-1/2} - \frac{1}{2}u^{3/2} + \frac{5}{2}u^{7/2} - 2u^{3/2} + 20u^{7/2}$$

$$g'(u) = \frac{45}{2}u^{7/2} - \frac{5}{2}u^{3/2} + u^{-1/2}$$

$$g'(u) = \frac{45u^{7/2}}{2} - \frac{5u^{3/2}}{2} + \frac{1}{u^{1/2}}$$

$$g(u) = u^{1/2}(2 - u^2 + 5u^4)$$

$$g(u) = 2u^{1/2} - u^{5/2} + 5u^{9/2}$$

$$g'(u) = u^{-1/2} - \frac{5}{2}u^{3/2} + \frac{45}{2}u^{7/2}$$

$$g'(u) = \frac{45}{2}u^{7/2} - \frac{5}{2}u^{3/2} + \frac{1}{u^{1/2}}$$

⑤ $y=(2-\sqrt{x})(1+\sqrt{x}+3x)$ @ $(1,5)$

x_1 y_1

① Find derivative

$y=\underbrace{(2-x^{1/2})}_{f(x)}\underbrace{(1+x^{1/2}+3x)}_{g(x)}$

$y'=\underbrace{\left(-\frac{1}{2}x^{-1/2}\right)}_{f'(x)}\underbrace{(1+\sqrt{x}+3x)}_{g(x)}+\underbrace{(2-\sqrt{x})}_{f(x)}\underbrace{\left(\frac{1}{2}x^{-1/2}+3\right)}_{g'(x)}$

$y'=\left(\frac{-1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x)+(2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$

② Solve for slope of tangent (m)

$m=y'(1)=\left(\frac{-1}{2}\right)(5)+(1)\left(\frac{1}{2}+3\right)$

$m=y'(1)=-\frac{5}{2}+1\left(\frac{1}{2}+\frac{6}{2}\right)$

$m=y'(1)=-\frac{5}{2}+\frac{7}{2}=\frac{2}{2}=\underline{\underline{1}}$

$m=1$

③ Find the equation

$y-y_1=m(x-x_1)$

$y-5=1(x-1)$

$y-5=x-1$

$y=x+4$ ✓

$\boxed{0=x-y+4}$ ✓

multiply (Product

⑥ If:

$f(2)=3$

$f'(2)=5$

$g(2)=-1$

$g'(2)=-4$

Find $(fg)'(2)$

$$(fg)'(x)=f'(x)g(x)+f(x)g'(x)$$

$$(fg)'(2)=f'(2)g(2)+f(2)g'(2)$$

$$(fg)'(2)=(5)(-1)+(3)(-4)$$

$$(fg)'(2)=-5-12$$

$$(fg)'(2)=-17$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

f(x) = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is:*** ***the derivative of the first function times the second function, plus the first function times the derivative of the second function***

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

$$(fg)'x = f'(x)g(x) + f(x)g'(x)$$

Differentiate the following function and simplify your answer:

$$h(t) = \underbrace{(t^3 - 5t)}_{f(x)}\underbrace{(6\sqrt{t} - t^{-5})}_{g(x)}$$

$$h'(t) = (3t^2 - 5)(6t^{1/2} - t^{-5}) + (t^3 - 5t)(3t^{-1/2} + 5t^{-6})$$

$$h'(t) = 18t^{5/2} - 3t^{-3} - 30t^{1/2} + 5t^{-5} + 3t^{5/2} + 5t^{-3} - 15t^{1/2} - 25t^{-5}$$

$$h'(t) = 21t^{5/2} - 45t^{1/2} + 2t^{-3} - 20t^{-5}$$

$$\boxed{h'(t) = 21t^{5/2} - 45t^{1/2} + \frac{2}{t^3} - \frac{20}{t^5}}$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{\text{(First)}}{\text{(Second)}}$$

In words, ***the Quotient Rule*** says that the ***derivative of a quotient is:***
the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Examples:

$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Differentiate the following functions and simplify your answers:

$$F(x) = \frac{x^2 + 2x - 3}{x^3 + 1} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$F'(x) = \frac{(2x+2)(x^3+1) - 3x^2(x^2+2x-3)}{(x^3+1)^2}$$

$$F'(x) = \frac{2x^4 + 2x + 2x^3 + 2 - 3x^4 - 6x^3 + 9x^2}{(x^3+1)^2}$$

$$F'(x) = \frac{-x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3+1)^2}$$

$$F(x) = \frac{\sqrt{x}}{1 + 2x} \quad \begin{matrix} f(x) \\ g(x) \end{matrix} \quad = \frac{x^{1/2}}{1+2x}$$

$$F'(x) = \frac{\frac{1}{2}x^{-1/2}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\frac{1}{2\sqrt{x}}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = 2\sqrt{x}\,\frac{\frac{1+2x}{\cancel{2\sqrt{x}}} - 2\sqrt{x}\cdot 2\sqrt{x}}{2\sqrt{x}(1+2x)^2}$$

CD: $2\sqrt{x}$

$$F'(x) = \frac{1+2x-4x}{2\sqrt{x}(1+2x)^2} = \frac{1-2x}{2\sqrt{x}(1+2x)^2}$$

Differentiate the following functions, do not simplify your answers:

$$f(x)=\frac{8-9x^7}{3x-7}$$

$$f'(x)=\frac{-63x^6(3x-7)-3(8-9x^7)}{(3x-7)^2}$$

$$f(x)=\frac{x^3-7x^2+2}{x^8-4x^5}$$

$$f'(x)=\frac{(3x^2-14x)(x^8-4x^5)-(x^3-7x^2+2)(8x^7-20x^4)}{(x^8-4x^5)^2}$$

Homework

Exercise 2.5

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

① d)

$$g(x) = \frac{x^3 - 1}{x^2 + x + 1} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$g'(x) = \frac{3x^2(x^2+x+1) - (x^3-1)(2x+1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{3x^4+3x^3+3x^2-(2x^4+x^3-2x-1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{(x^2+x+1)^2}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{(x^2+x+1)(x^2+x+1)}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{x^4+x^3+x^2+x^3+x^2+x+x^2+x+1}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{x^4+2x^3+3x^2+2x+1}$$

$$\boxed{g'(x) = 1}$$

Exercise 2.5

① l) $f(x) = \dfrac{x \cdot 1 - \frac{1}{\cancel{x}} \cdot \cancel{x}}{(x+1)x}$

$f(x) = \dfrac{x-1}{x^2+x}$

$f(x) = \dfrac{1 - \frac{1}{x}}{x+1}$

$f(x) = \dfrac{\frac{x}{x} - \frac{1}{x}}{x+1}$

$f(x) = \dfrac{\frac{x-1}{x}}{x+1}$

$f(x) = \dfrac{x-1}{x} \cdot \dfrac{1}{x+1}$

$f(x) = \dfrac{x-1}{x^2+x}$

Exercise 2.5

3 a) $y = \dfrac{x}{x-2}$ $(4, 2)$

① $y' = \dfrac{1(x-2) - x}{(x-2)^2} = \dfrac{x-2-x}{(x-2)^2} = \dfrac{-2}{(x-2)^2}$

② $y' = \dfrac{-2}{(x-2)^2} = \dfrac{-2}{(4-2)^2} = \dfrac{-2}{(2)^2} = \dfrac{-2}{4} = -\dfrac{1}{2}$

③ $y - y_1 = m(x - x_1)$

$y - 2 = -\dfrac{1}{2}(x-4)$

$y - 2 = -\dfrac{1}{2}x + 2$

$y = -\dfrac{1}{2}x + 4$

Exercise 2.5

③ Find an equation of the tangent line to the curve at the given point.

c) $y = \frac{1}{x^2+1}$, $(-2, \frac{1}{5})$

$x_1 = -2$
$y_1 = \frac{1}{5}$

(i) Find the derivative.

$y = \frac{1}{x^2+1}$ f(x) g(x) ← division

$y' = \frac{\cancel{(0)(x^2+1)} - (1)(2x)}{(x^2+1)^2}$

$y' = \frac{-2x}{(x^2+1)^2}$

(ii) Sub in x-value to find m.

$y'(-2) = \frac{-2(-2)}{((-2)^2+1)^2} = \boxed{\frac{4}{25}}$ m

(iii) Find the equation of the tangent line

$y - y_1 = m(x - x_1)$

$y - \frac{1}{5} = \frac{4}{25}(x + 2)$

$y - \frac{1}{5} = \frac{4}{25}x + \frac{8}{25}$

$y = \frac{4}{25}x + \frac{8}{25} + \frac{1}{5}$

$y = \frac{4}{25}x + \frac{8}{25} + \frac{5}{25}$

$\boxed{y = \frac{4}{25}x + \frac{13}{25}}$ ✓

$25y = 4x + 13$

$\boxed{0 = 4x - 25y + 13}$ ✓

Exercise 2.5

③ d) $y = \dfrac{x^3 - 1}{1 + 2x^2}$, $(1, 0)$

x_1, y_1

(i) Find derivative

$$y' = \frac{3x^2(1+2x^2) - 4x(x^3-1)}{(1+2x^2)^2}$$

$$y' = \frac{3x^2 + 6x^4 - 4x^4 + 4x}{(1+2x^2)^2}$$

$$y' = \frac{2x^4 + 3x^2 + 4x}{(1+2x^2)^2}$$

(ii) Find m (sub in x=1)

$$m = y'(1) = \frac{2(1)^4 + 3(1)^2 + 4(1)}{[1+2(1)^2]^2}$$

$$m = y'(1) = \frac{2+3+4}{9} = \frac{9}{9} = 1$$

m

(iii) $y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 1)$

$\boxed{y = x - 1}$

$\boxed{x - y - 1 = 0}$

Exercise 2.5

#6 slope of the tangent equals $\underline{\underline{0}}$ (horizontal)

① Find derivative:

$$y = \frac{x^2}{2x+5} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$y' = \frac{2x(2x+5) - x^2(2)}{(2x+5)^2}$$

$$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$$

$$y' = \frac{2x^2 + 10x}{(2x+5)^2}$$

② Solve for x:

$$\underline{\underline{y'}} = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$(2x+5)^2 \; \frac{0}{1} = \frac{2x^2+10x}{\cancel{(2x+5)^2}} \; \cancel{(2x+5)^2}$$

$$0 = 2x^2 + 10x$$

$$0 = 2x(x+5)$$

$2x = 0$, $x = 0$ | $x + 5 = 0$, $x = -5$

③ Solve for y:

if $x=0$

$$y = \frac{x^2}{2x+5}$$

$$y = \frac{(0)^2}{2(0)+5} = \frac{0}{5} = 0$$

$(0,0)$

if $x=-5$

$$y = \frac{x^2}{2x+5}$$

$$y = \frac{(-5)^2}{2(-5)+5} = \frac{25}{-5} = -5$$

$(-5,-5)$

Review to Date:

① a) $f(x) = 2x^3 + 3x$

① $f(x+h) = 2(x+h)^3 + 3(x+h)$

$f(x+h) = 2(x^3 + 3x^2h + 3xh^2 + h^3) + 3(x+h)$

$f(x+h) = 2x^3 + 6x^2h + 6xh^2 + 2h^3 + 3x + 3h$

② $f'(x) = \lim\limits_{h \to 0} \dfrac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 \cancel{+3x} + 3h - (\cancel{2x^3} + \cancel{3x})}{h}$

$f'(x) = \lim\limits_{h \to 0} \dfrac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$

$f'(x) = \lim\limits_{h \to 0} \dfrac{\cancel{h}(6x^2 + 6xh + 2h^2 + 3)}{\cancel{h}} = 6x^2 + 3$

Review to Date:

2 e) $f(x) = 6x\sqrt{x^3} - 2(\sqrt[3]{x})$

$f(x) = 6x^1(x^{3/2}) - 2x^{1/3}$

$f(x) = 6x^{5/2} - 2x^{1/3}$

$f'(x) = 15x^{3/2} - \frac{2}{3}x^{-2/3}$

$f'(x) = 15x^{3/2} - \frac{2}{3x^{2/3}}$

$6 \cdot \frac{5}{2} = \frac{30}{2} = 15$

$-2 \cdot \frac{1}{3} = \frac{-2}{3}$

5 b) $f(x) = \frac{x+3}{4-x^2}$ $\begin{matrix} f(x) \\ g(x) \end{matrix}$

$$f'(x) = \frac{1(4-x^2) + 2x(x+3)}{(4-x^2)^2}$$

$$f'(x) = \frac{4-x^2+2x^2+6x}{(4-x^2)^2} = \frac{x^2+6x+4}{(4-x^2)^2}$$

Review to Date:

③ $y = x^3 + 3x$ $x = \underline{1}$ Equation of tangent

(i) Find y

$y = (1)^3 + 3(1)$

$y = \underline{4}$

(ii) Find the derivative

$y = x^3 + 3x$

$y' = 3x^2 + 3$

(iii) Find m:

$y'(1) = 3(1)^2 + 3$

$y'(1) = 6$

$m = 6$

(iv) $y - y_1 = m(x - x_1)$

$y - 4 = 6(x - 1)$

$y - 4 = 6x - 6$

$+4$ $+4$

$\boxed{y = 6x - 2}$

$\boxed{0 = 6x - y - 2}$

Review to Date: $(f\cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$

④ c) $f(x) = \sqrt[3]{x^2}\left(x^4 - 5\sqrt[3]{x^5}\right)$

$f(x) = \underbrace{x^{2/3}}_{f(x)}\underbrace{\left(x^4 - 5x^{5/3}\right)}_{g(x)}$

$f'(x) = \frac{2}{3}x^{-1/3}\left(x^4 - 5x^{5/3}\right) + x^{2/3}\left(4x^3 - \frac{25}{3}x^{2/3}\right)$

$f'(x) = \boxed{\frac{2}{3}x^{11/3}}\boxed{-\frac{10}{3}x^{4/3}}\boxed{+4x^{11/3}}\boxed{-\frac{25}{3}x^{4/3}}$

$f'(x) = \frac{14}{3}x^{11/3} - \frac{35}{3}x^{4/3}$

$-\frac{1}{2}x^{-1/2}$

$3(2) = 6$
$2(3) = 6$

⑤ c) $g(x) = \frac{7\sqrt{x} - 10}{1 - \sqrt{x}} = \frac{7x^{1/2} - 10}{1 - x^{1/2}}$ $\begin{matrix} f(x) \\ g(x) \end{matrix}$

$$g'(x) = \frac{\frac{7}{2}x^{-1/2}\left(1 - x^{1/2}\right) + \frac{1}{2}x^{-1/2}\left(7x^{1/2} - 10\right)}{\left(1 - \sqrt{x}\right)^2}$$

$$g'(x) = \frac{\frac{7}{\cancel{2x^{1/2}}}\left(1 - x^{1/2}\right) + \frac{1}{\cancel{2x^{1/2}}}\left(7x^{1/2} - 10\right)\cdot\cancel{2x^{1/2}}}{2x^{1/2}\left(1 - \sqrt{x}\right)^2}$$

$$g'(x) = \frac{7\left(1 - x^{1/2}\right) + 1\left(7x^{1/2} - 10\right)}{2x^{1/2}\left(1 - \sqrt{x}\right)^2}$$

$$g'(x) = \frac{7 - \cancel{7x^{1/2}} + \cancel{7x^{1/2}} - 10}{2\sqrt{x}\left(1 - \sqrt{x}\right)^2} = \frac{-3}{2\sqrt{x}\left(1 - \sqrt{x}\right)^2}$$

Ex $f(x) = 5x^3 + \frac{3}{x^2} - 2$

$$f(x) = 5x^3 + 3x^{-2} - 2$$

$$f'(x) = 15x^2 - 6x^{-3} - 0$$

$$f'(x) = 15x^2 - \frac{6}{x^3}$$

Review to Date:

④ c) $f(x) = x^{2/3}(x^4 - 5x^{5/3})$ Power Rule

$f(x) = x^{\frac{14}{3}} - 5x^{\frac{7}{3}}$

$f'(x) = \frac{14}{3}x^{\frac{11}{3}} - \frac{35}{3}x^{\frac{4}{3}}$

④ d) $f(x) = (x^2 - 3x + 4)(2x^2 + 4x)$

$f'(x) = (2x - 3)(2x^2 + 4x) + (4x + 4)(x^2 - 3x + 4)$

$f'(x) = 4x^3 + 8x^2 - 6x^2 - 12x + 4x^3 - 12x^2 + 16x + 4x^2 - 12x + 16$

$f'(x) = 8x^3 - 6x^2 - 8x + 16$

$f'(x) = 2(4x^3 - 3x^2 - 4x + 8)$ *

$6x - y = 4$

$6x - 4 = y$

$y = 6x - 4$

$m = 6$

$y = x\sqrt{x} = x(x^{1/2}) = x^{3/2}$

$y' = \frac{3}{2}x^{1/2}$

$6 = \frac{3}{2}x^{1/2}$

$12 = 3x^{1/2}$

$4 = x^{1/2}$

$16 = x$

1. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

5. $y - y_1 = m(x - x_1)$

$(x+h)^3$

$x^3 + 3x^2h + 3xh^2 + h^3$