

Exercise 2.4

$$\textcircled{1} \text{ f) } f(t) = (at+b)(ct^2-d)$$

$$f'(t) = \overbrace{(a)}^{\text{red}}(ct^2-d) + \overbrace{(at+b)}^{\text{red}}(\overbrace{2ct}^{\text{red}})$$

$$f'(t) = \underline{act^2} - ad + \underline{2act} + \underline{2bct}$$

$$f'(t) = 3act^2 + 2bct - ad$$

$$\textcircled{2} \text{ a) } y = (1-2x)(3x-4), x=2$$

① Find  $y'$ :

$$y' = -2(3x-4) + (1-2x)(3)$$

$$y' = -6x + 8 + 3 - 6x$$

$$y' = -12x + 11$$

② sub in  $x=2$ 

$$y' = -12x + 11$$

$$y' = -12(2) + 11$$

$$y' = -13$$

$$m = -13$$

$$\textcircled{3} y = (2-\sqrt{x})(1+\sqrt{x}+3x) \text{ @ } (1,5)$$

$$y = (2-x^{1/2})(1+x^{1/2}+3x)$$

① Find  $y'$ :

$$y' = \left(-\frac{1}{2}x^{-1/2}\right)(1+x^{1/2}+3x) + (2-x^{1/2})\left(\frac{1}{2}x^{-1/2}+3\right)$$

$$y' = \left(-\frac{1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x) + (2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$$

② sub in  $x=1$ 

$$y' = \left(-\frac{1}{2\sqrt{1}}\right)(1+\sqrt{1}+3(1)) + (2-\sqrt{1})\left(\frac{1}{2\sqrt{1}}+3\right)$$

$$y' = \left(-\frac{1}{2}\right)(5) + (1)\left(\frac{1}{2}\right)$$

$$y' = \frac{-5}{2} + \frac{1}{2} = \frac{-4}{2} = -2$$

$$m = -2$$

$$\textcircled{3} y - y_1 = m(x - x_1)$$

$$y - 5 = 1(x - 1) \dots$$

$$y - 5 = x - 1$$

$$y = x + 4$$

$$\text{or } x - y + 4 = 0$$

Exercise 2.4  
 a)  $g(v) = (v - \sqrt{v})(v^2 + \sqrt{v})$

$$g(v) = \underbrace{(v - v^{1/2})}_{f(x)} \underbrace{(v^2 + v^{1/2})}_{g(x)} \quad \begin{matrix} 1 + (-\frac{1}{2}) \\ \frac{2}{3} + (-\frac{1}{3}) \end{matrix}$$

$$g'(v) = \left(1 - \frac{1}{2}v^{-1/2}\right)(v^2 + v^{1/2}) + (v - v^{1/2})\left(2v + \frac{1}{2}v^{-1/2}\right)$$

$$g'(v) = v^2 + v^{1/2} - \frac{1}{2}v^{3/2} - \frac{1}{2}v^0 + 2v^2 + \frac{1}{2}v^{1/2} - 2v^{3/2} - \frac{1}{2}v^0$$

$$g'(v) = \frac{3v^2}{1} - \frac{5}{2}v^{3/2} + \frac{3}{2}v^{1/2} - \frac{1}{1}$$

$$g'(v) = \frac{6v^2}{2} - \frac{5v^{3/2}}{2} + \frac{3v^{1/2}}{2} - \frac{2}{2}$$

$$g'(v) = \frac{6v^2 - 5v^{3/2} + 3v^{1/2} - 2}{2}$$

Product Rule:

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

b)  $g(u) = \sqrt{u} (2 - u^2 + 5u^4)$

$$g(u) = \underbrace{u^{1/2}}_{f(x)} \underbrace{(2 - u^2 + 5u^4)}_{g(x)}$$

$$g'(u) = \frac{1}{2}u^{-1/2}(2 - u^2 + 5u^4) + u^{1/2}(-2u + 20u^3)$$

$$g'(u) = \frac{1}{2}u^{-1/2} - \frac{1}{2}u^{3/2} + \frac{5}{2}u^{7/2} - 2u^{3/2} + 20u^{7/2}$$

$$g'(u) = \frac{45}{2}u^{7/2} - \frac{5}{2}u^{3/2} + u^{-1/2}$$

$$g'(u) = \frac{45u^{7/2}}{2} - \frac{5u^{3/2}}{2} + \frac{1}{u^{1/2}}$$

$$g(u) = u^{1/2} (2 - u^2 + 5u^4)$$

$$g(u) = 2u^{1/2} - u^{5/2} + 5u^{9/2}$$

$$g'(u) = u^{-1/2} - \frac{5}{2}u^{3/2} + \frac{45}{2}u^{7/2}$$

$$g'(u) = \frac{45u^{7/2}}{2} - \frac{5}{2}u^{3/2} + \frac{1}{u^{1/2}}$$

## Exercise 2.4

$$\textcircled{1} y = (2 - \sqrt{x})(1 + \sqrt{x} + 3x) \text{ @ } (1, 5)$$

① Find derivative

$$y = \underbrace{(2 - x^{1/2})}_{f(x)} \underbrace{(1 + x^{1/2} + 3x)}_{g(x)}$$

$$y' = \underbrace{\left(-\frac{1}{2}x^{-1/2}\right)}_{f'(x)} \underbrace{(1 + \sqrt{x} + 3x)}_{g(x)} + \underbrace{(2 - \sqrt{x})}_{f(x)} \underbrace{\left(\frac{1}{2}x^{-1/2} + 3\right)}_{g'(x)}$$

$$y' = \left(-\frac{1}{2\sqrt{x}}\right)(1 + \sqrt{x} + 3x) + (2 - \sqrt{x})\left(\frac{1}{2\sqrt{x}} + 3\right)$$

② Solve for slope of tangent (m)

$$m = y'(1) = \left(-\frac{1}{2}\right)(5) + (1)\left(\frac{1}{2} + 3\right)$$

$$m = y'(1) = -\frac{5}{2} + 1\left(\frac{1}{2} + \frac{6}{2}\right)$$

$$m = y'(1) = -\frac{5}{2} + \frac{7}{2} = \frac{2}{2} = \underline{\underline{1}} \quad \leftarrow m=1$$

③ Find the equation

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 1(x - 1)$$

$$y - 5 = x - 1$$

$$y = x + 4 \quad \checkmark$$

$$\boxed{0 = x - y + 4} \quad \checkmark$$

## Exercise 2.4

⑥ If: Find  $(fg)'(a)$  *multiply* **Product Rule**

$$f(a) = \underline{\underline{3}}$$

$$f'(a) = \underline{\underline{5}}$$

$$g(a) = \underline{\underline{-1}}$$

$$g'(a) = \underline{\underline{-4}}$$

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$(fg)'(a) = \underline{\underline{f'(a)}} \underline{\underline{g(a)}} + \underline{\underline{f(a)}} \underline{\underline{g'(a)}}$$

$$(fg)'(a) = (5)(-1) + (3)(-4)$$

$$(fg)'(a) = -5 - 12$$

$$(fg)'(a) = -17$$


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## Exercise 2.5

④ If: Find  $\left(\frac{f}{g}\right)'(a)$  *Division* **(Quotient Rule)**

$$f(a) = \underline{\underline{3}}$$

$$f'(a) = \underline{\underline{5}}$$

$$g(a) = \underline{\underline{-1}}$$

$$g'(a) = \underline{\underline{-4}}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\left(\frac{f}{g}\right)'(a) = \frac{\underline{\underline{f'(a)}} \underline{\underline{g(a)}} - \underline{\underline{f(a)}} \underline{\underline{g'(a)}}}{[\underline{\underline{g(a)}}]^2}$$

$$\left(\frac{f}{g}\right)'(a) = \frac{(5)(-1) - (3)(-4)}{(-1)^2}$$

$$\left(\frac{f}{g}\right)'(a) = \frac{-5 + 12}{1}$$

$$\boxed{\left(\frac{f}{g}\right)'(a) = 7}$$

## Differentiation Rules

### Product Rule:

**The Product Rule** If  $f$  and  $g$  are both differentiable, then

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$$f(x) = (\text{First}) \times (\text{Second})$$

In words, *the Product Rule* says that the *derivative of a product of two functions is: the derivative of the first function times the second function, plus the first function times the derivative of the second function*

*Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.*

$$(fg)'_x = f'(x)g(x) + f(x)g'(x)$$

Differentiate the following function and simplify your answer:

$$h(t) = \overset{f(x)}{(t^3 - 5t)} \overset{g(x)}{(6\sqrt{t} - t^{-5})}$$

$$h'(t) = (3t^2 - 5)(6t^{1/2} - t^{-5}) + (t^3 - 5t)(3t^{-1/2} + 5t^{-6})$$

$$h'(t) = \underline{18t^{5/2}} - \underline{3t^{-3}} - \underline{30t^{1/2}} + \underline{5t^{-5}} + \underline{3t^{5/2}} + \underline{5t^{-3}} - \underline{15t^{1/2}} - \underline{25t^{-5}}$$

$$h'(t) = \underline{21t^{5/2}} - \underline{45t^{1/2}} + \underline{2t^{-3}} - \underline{20t^{-5}}$$

$$h'(t) = 21t^{5/2} - 45t^{1/2} + \frac{2}{t^3} - \frac{20}{t^5}$$

## Quotient Rule:

**The Quotient Rule** If  $f$  and  $g$  are differentiable, then

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{\text{(First)}}{\text{(Second)}}$$

In words, *the Quotient Rule* says that the *derivative of a quotient is: the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.*

$$\left( \frac{f}{g} \right)' (x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Examples:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Differentiate the following functions and simplify your answers:

$$F(x) = \frac{x^2 + 2x - 3}{x^3 + 1} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$F'(x) = \frac{(2x+2)(x^3+1) - 3x^2(x^2+2x-3)}{(x^3+1)^2}$$

$$F'(x) = \frac{2x^4 + 2x + 2x^3 + 2 - 3x^4 - 6x^3 + 9x^2}{(x^3+1)^2}$$

$$F'(x) = \frac{-x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3+1)^2}$$

$$F(x) = \frac{\sqrt{x}}{1+2x} \quad \begin{matrix} f(x) \\ g(x) \end{matrix} = \frac{x^{1/2}}{1+2x}$$

$$F'(x) = \frac{\frac{1}{2}x^{-1/2}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\frac{1}{2\sqrt{x}}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\frac{1+2x}{2\sqrt{x}} - 2\sqrt{x} \cdot 2\sqrt{x}}{2\sqrt{x}(1+2x)^2} \quad \text{CD: } 2\sqrt{x}$$

$$F'(x) = \frac{1+2x-4x}{2\sqrt{x}(1+2x)^2} = \frac{1-2x}{2\sqrt{x}(1+2x)^2}$$

Differentiate the following functions, do not simplify your answers:

$$f(x) = \frac{8 - 9x^7}{3x - 7}$$

$$f'(x) = \frac{-63x^6(3x-7) - 3(8-9x^7)}{(3x-7)^2}$$

$$f(x) = \frac{x^3 - 7x^2 + 2}{x^8 - 4x^5}$$

$$f'(x) = \frac{(3x^2 - 14x)(x^8 - 4x^5) - (x^3 - 7x^2 + 2)(8x^7 - 20x^4)}{(x^8 - 4x^5)^2}$$

## Exercise 2.5

①  $f(x) = \frac{x - \frac{1}{x}}{x(x+1)}$  ← Complex Fraction  
CO:  $x$

$$f(x) = \frac{x-1}{x^2+x}$$

$$f'(x) = \frac{(1)(x+x) - (x-1)(2x+1)}{(x^2+x)^2}$$

$$f'(x) = \frac{x^2+x - (2x^2+x-2x-1)}{(x^2+x)^2}$$

$$f'(x) = \frac{x^2+x - 2x^2 - x + 2x + 1}{(x^2+x)^2}$$

$$f'(x) = \frac{-x^2 + 2x + 1}{(x^2+x)^2}$$

$$f'(x) = -\frac{x^2 - 2x - 1}{(x^2+x)^2}$$

## Exercise 2.5

$$\textcircled{2} \text{ a) } f(x) = \frac{2+x}{1-2x}$$

linear ↙ ↘  $\{x | x \in \mathbb{R}\}$

linear ↙ ↘  $\{x | x \in \mathbb{R}\}$

Non permissible values: (cannot divide by 0)

$$1 - 2x \neq 0$$

$$\frac{1}{2} \neq \frac{2x}{2}$$

$$\frac{1}{2} \neq x$$

$$\text{Domain: } \{x | x \neq \frac{1}{2}, x \in \mathbb{R}\}$$

Set Notation

$$\text{Domain: } (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$$

Interval Notation

## Exercise 2.5

$$\textcircled{3} \text{ c) } y = \frac{1}{x^2+1}, \quad (-2, \frac{1}{5}) \quad \begin{array}{l} x_1 = -2 \\ y_1 = \frac{1}{5} \end{array}$$

(i) Find  $\frac{dy}{dx}$ 

$$y = \frac{1}{x^2+1}$$

$$\frac{dy}{dx} = \frac{-(0)(x^2+1) - (1)(2x)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-2x}{(x^2+1)^2}$$

(ii) Find slope

$$\frac{dy}{dx} = \frac{-2x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-2(-2)}{((-2)^2+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{4}{25}} \quad \text{slope "m"}$$

(iii) Find equation:

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{5} = \frac{4}{25}(x - (-2))$$

$$y - \frac{1}{5} = \frac{4}{25}(x+2)$$

$$y - \frac{1}{5} = \frac{4x}{25} + \frac{8}{25}$$

$$y = \frac{4x}{25} + \frac{8}{25} + \frac{1}{5}$$

$$y = \frac{4x}{25} + \frac{8}{25} + \frac{5}{25}$$

$$\boxed{y = \frac{4x}{25} + \frac{13}{25}} \quad \checkmark$$

$$0 = \frac{4x}{25} - y + \frac{13}{25}$$

$$\boxed{0 = 4x - 25y + 13}$$

# Homework

## Exercise 2.5

① d)

$$g(x) = \frac{x^3 - 1}{x^2 + x + 1} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$g'(x) = \frac{3x^2(x^2 + x + 1) - (x^3 - 1)(2x + 1)}{(x^2 + x + 1)^2}$$

$$g'(x) = \frac{3x^4 + 3x^3 + 3x^2 - (2x^4 + x^3 - 2x - 1)}{(x^2 + x + 1)^2}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{(x^2 + x + 1)^2}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{(x^2 + x + 1)(x^2 + x + 1)}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{x^4 + x^3 + x^2 + x^3 + x^2 + x + x^2 + x + 1}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{x^4 + 2x^3 + 3x^2 + 2x + 1}$$

$$g'(x) = 1$$

Exercise 2.5

$$\textcircled{1} \text{ b) } f(x) = \frac{x \cdot 1 - \frac{1}{x} \cdot x}{(x+1)x}$$

$$f(x) = \frac{x-1}{x^2+x}$$

$$f(x) = \frac{1 - \frac{1}{x}}{x+1}$$

$$f(x) = \frac{\frac{x}{x} - \frac{1}{x}}{x+1}$$

$$f(x) = \frac{\frac{x-1}{x}}{x+1}$$

$$f(x) = \frac{x-1}{x} \cdot \frac{1}{x+1}$$

$$f(x) = \frac{x-1}{x^2+x}$$

## Exercise 2.5

$$\textcircled{3} \text{ a) } y = \frac{x}{x-2} \quad (4, 2)$$

$$\textcircled{1} \quad y' = \frac{1(x-2) - x}{(x-2)^2} = \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

$$\textcircled{2} \quad y' = \frac{-2}{(x-2)^2} = \frac{-2}{(4-2)^2} = \frac{-2}{(2)^2} = \frac{-2}{4} = \left( \frac{-1}{2} \right)$$

$$\textcircled{3} \quad y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-1}{2}(x - 4)$$

$$y - 2 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 4$$

## Exercise 2.5

③ Find an equation of the tangent line to the curve at the given point.

c)  $y = \frac{1}{x^2+1}$ ,  $(-2, \frac{1}{5})$        $x_1 = -2$   
 $y_1 = \frac{1}{5}$

(i) Find the derivative:

$$y = \frac{1}{x^2+1} \quad \leftarrow \begin{array}{l} \text{division} \\ f(x) \\ g(x) \end{array}$$

$$y' = \frac{(0)(x^2+1) - (1)(2x)}{(x^2+1)^2}$$

$$y' = \frac{-2x}{(x^2+1)^2}$$

(ii) Sub in x-value to find m:

$$y'(-2) = \frac{-2(-2)}{((-2)^2+1)^2} = \frac{4}{25} m$$

(iii) Find the equation of the tangent line

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{5} = \frac{4}{25}(x + 2)$$

$$y - \frac{1}{5} = \frac{4}{25}x + \frac{8}{25}$$

$$y = \frac{4}{25}x + \frac{8}{25} + \frac{1}{5}$$

$$y = \frac{4}{25}x + \frac{8}{25} + \frac{5}{25}$$

$$y = \frac{4}{25}x + \frac{13}{25} \quad \checkmark$$

$$25y = 4x + 13$$

$$0 = 4x - 25y + 13 \quad \checkmark$$

Exercise 2.5

$$\textcircled{3} \text{ d), } y = \frac{x^3 - 1}{1 + 2x^2}, \quad (1, 0)$$

$x_1, y_1$

(i) Find derivative

$$y' = \frac{3x^2(1+2x^2) - 4x(x^3-1)}{(1+2x^2)^2}$$

$$y' = \frac{3x^2 + 6x^4 - 4x^4 + 4x}{(1+2x^2)^2}$$

$$y' = \frac{2x^4 + 3x^2 + 4x}{(1+2x^2)^2}$$

(ii) Find m (sub in  $x=1$ )

$$m = y'(1) = \frac{2(1)^4 + 3(1)^2 + 4(1)}{[1+2(1)^2]^2}$$

$$m = y'(1) = \frac{2+3+4}{9} = \frac{9}{9} = 1$$

↑  
m

$$\textcircled{iii} \quad y - y_1 = m(x - x_1)$$

$$y - 0 = 1(x - 1)$$

$$\boxed{y = x - 1}$$

$$\boxed{x - y - 1 = 0}$$

## Exercise 2.5

#6 slope of the tangent equals 0 (horizontal)

① Find derivative:

$$y = \frac{x^2}{2x+5}$$

$f(x)$  above  $x^2$ ,  $g(x)$  below  $2x+5$

$$y' = \frac{2x(2x+5) - x^2(2)}{(2x+5)^2}$$

$$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$$

$$y' = \frac{2x^2 + 10x}{(2x+5)^2}$$

② Solve for x:

$$y' = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$0 = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$0 = 2x^2 + 10x$$

$$0 = 2x(x+5)$$

$$\begin{array}{l|l} 2x=0 & x+5=0 \\ x=0 & x=-5 \end{array}$$

③ Solve for y:

if  $x=0$

$$y = \frac{x^2}{2x+5}$$

$$y = \frac{(0)^2}{2(0)+5} = \frac{0}{5} = 0$$

$$(0, 0)$$

if  $x=-5$

$$y = \frac{x^2}{2x+5}$$

$$y = \frac{(-5)^2}{2(-5)+5} = \frac{25}{-5} = -5$$

$$(-5, -5)$$

Review to Date:

$$\textcircled{1} \text{ a) } f(x) = \underline{2x^3 + 3x}$$

$$\textcircled{1} f(x+h) = \underline{2(x+h)^3 + 3(x+h)}$$

$$f(x+h) = \underline{2(x^3 + 3x^2h + 3xh^2 + h^3) + 3(x+h)}$$

$$f(x+h) = \underline{2x^3 + 6x^2h + 6xh^2 + 2h^3 + 3x + 3h}$$

$$\textcircled{2} f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 + \cancel{3x} + 3h - (\cancel{2x^3} + \cancel{3x})}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}(6x^2 + 6x\underline{h} + 2\underline{h^2} + 3)}{\cancel{h}} = 6x^2 + 3$$

Review to Date:

$$\textcircled{a} \text{ e) } f(x) = 6x\sqrt{x^3} - 2(\sqrt[3]{x})$$

$$6 \cdot \frac{5}{2} = \frac{30}{2} = 15$$

$$f(x) = 6x(x^{3/2}) - 2x^{1/3}$$

$$-2 \cdot \frac{1}{3} = -\frac{2}{3}$$

$$f(x) = 6x^{5/2} - 2x^{1/3}$$

$$f'(x) = 15x^{3/2} - \frac{2}{3}x^{-2/3}$$

$$f'(x) = 15x^{3/2} - \frac{2}{3x^{2/3}}$$

$$\textcircled{c} \text{ b) } f(x) = \frac{x+3}{4-x^2} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$f'(x) = \frac{1(4-x^2) + 2x(x+3)}{(4-x^2)^2}$$

$$f'(x) = \frac{4-x^2 + 2x^2 + 6x}{(4-x^2)^2} = \frac{x^2 + 6x + 4}{(4-x^2)^2}$$

Review to Date:

③  $y = x^3 + 3x$      $x = \underline{1}$     Equation of tangent

(i) Find  $y$       (ii) Find the derivative      (iii) Find  $m$ .

$$y = (1)^3 + 3(1)$$

$$y = \underline{4}$$

$$y = x^3 + 3x$$

$$y' = 3x^2 + 3$$

$$y'(1) = 3(1)^2 + 3$$

$$y'(1) = 6$$

$$m = 6$$

(iv)  $y - y_1 = m(x - x_1)$

$$y - 4 = 6(x - 1)$$

$$y - 4 = 6x - 6$$

$$\boxed{y = 6x - 2}$$

$$\boxed{0 = 6x - y - 2}$$

Review to Date:  $(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$

④ c)  $f(x) = \sqrt[3]{x^2}(x^4 - 5\sqrt[3]{x^5})$

$f(x) = x^{2/3}(x^4 - 5x^{5/3})$

$f'(x) = \frac{2}{3}x^{-1/3}(x^4 - 5x^{5/3}) + x^{2/3}(4x^3 - \frac{25}{3}x^{2/3})$

$f'(x) = \frac{2}{3}x^{11/3} - \frac{10}{3}x^{4/3} + 4x^{11/3} - \frac{25}{3}x^{4/3}$

$f'(x) = \frac{14}{3}x^{11/3} - \frac{35}{3}x^{4/3}$

$-\frac{1}{2}x^{-1/2}$   
 $3(2) = 6$   
 $2(3) = 6$

⑤ c)  $g(x) = \frac{7\sqrt{x} - 10}{1 - \sqrt{x}} = \frac{7x^{1/2} - 10}{1 - x^{1/2}}$

$g'(x) = \frac{7x^{-1/2}(1 - x^{1/2}) + \frac{1}{2}x^{-3/2}(7x^{1/2} - 10)}{(1 - \sqrt{x})^2}$

$g'(x) = \frac{7(1 - x^{1/2}) + \frac{1}{2}(7x^{1/2} - 10) \cdot \frac{1}{2x^{1/2}}}{2x^{1/2}(1 - \sqrt{x})^2}$

$g'(x) = \frac{7(1 - x^{1/2}) + \frac{1}{2}(7x^{1/2} - 10)}{2x^{1/2}(1 - \sqrt{x})^2}$

$g'(x) = \frac{7 - 7x^{1/2} + 7x^{1/2} - 10}{2\sqrt{x}(1 - \sqrt{x})^2} = \frac{-3}{2\sqrt{x}(1 - \sqrt{x})^2}$

$$\text{Ex } f(x) = 5x^3 + \frac{3}{x^2} - 2$$

$$f(x) = 5x^3 + 3x^{-2} - 2$$

$$f'(x) = 15x^2 - 6x^{-3} - 0$$

$$f'(x) = 15x^2 - \frac{6}{x^3}$$

Review to Date:

④ c)  $f(x) = x^{2/3}(x^4 - 5x^{5/3})$  Power Rule

$$f(x) = x^{14/3} - 5x^{7/3}$$

$$f'(x) = \frac{14}{3}x^{11/3} - \frac{35}{3}x^{4/3}$$

④ d)  $f(x) = (x^2 - 3x + 4)(2x^2 + 4x)$

$$f'(x) = (2x - 3)(2x^2 + 4x) + (4x + 4)(x^2 - 3x + 4)$$

$$f'(x) = \underline{4x^3} + \underline{8x^2} - \underline{6x^2} - \underline{12x} + \underline{4x^3} - \underline{12x^2} + \underline{16x} + \underline{4x^2} - \underline{12x} + \underline{16}$$

$$f'(x) = \underline{8x^3} - \underline{6x^2} - \underline{8x} + \underline{16}$$

$$f'(x) = 2(4x^3 - 3x^2 - 4x + 8) *$$

$$6x - y = 4$$

$$6x - 4 = y$$

$$y = 6x - 4$$

$$m = 6$$

$$y = x\sqrt{x} = x(x^{1/2}) = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2}$$

$$6 = \frac{3}{2}x^{1/2}$$

$$12 = 3x^{1/2}$$

$$4 = x^{1/2}$$

$$16 = x$$

$$\textcircled{1} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \left| \quad \begin{array}{l} (x+h)^3 \\ x^3 + 3x^2h + 3xh^2 + h^3 \end{array} \right.$$
$$\textcircled{5} \quad y - y_1 = m(x - x_1)$$