

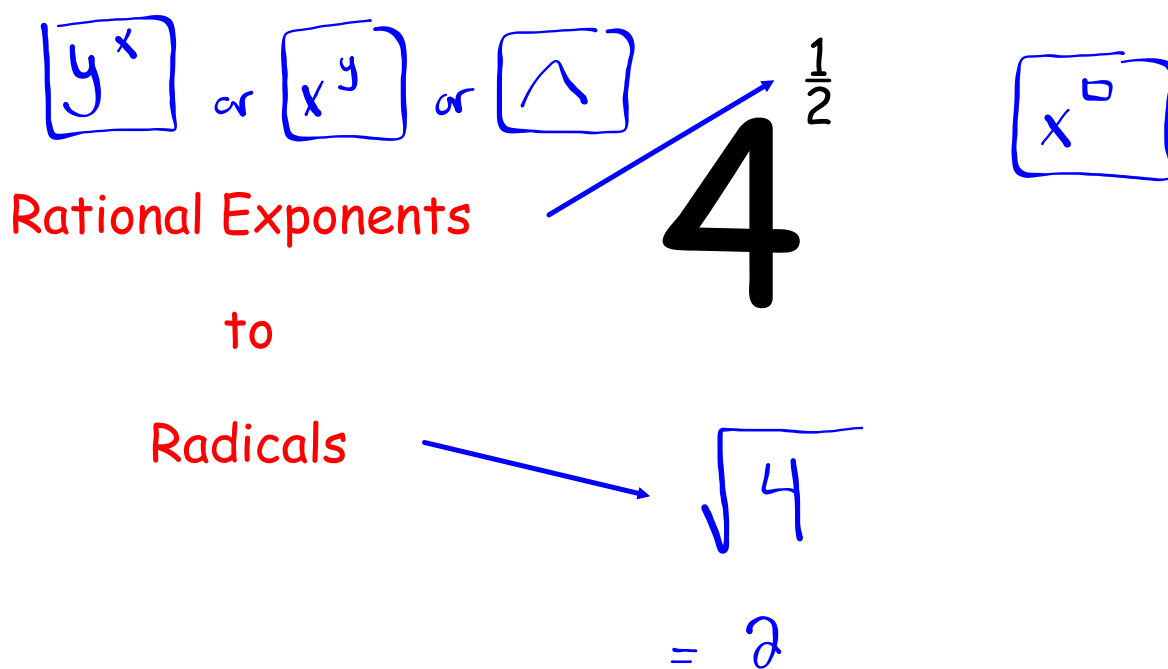
Rational Exponents

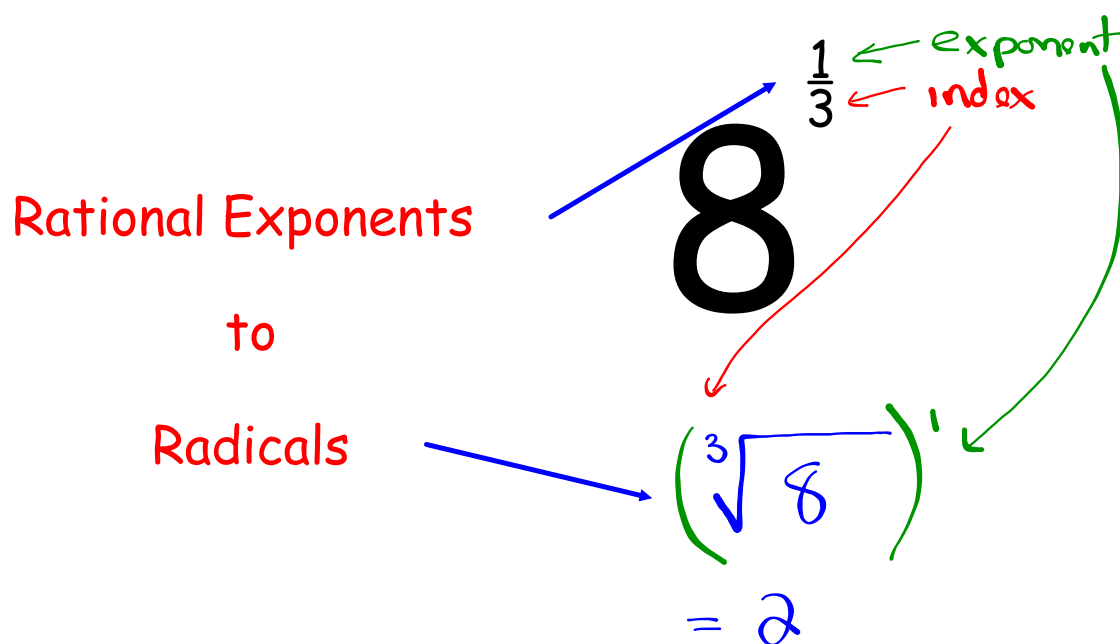
$$4^{\frac{1}{2}}$$

Rational Exponent?

$$4^{0.5}$$

Rational Numbers: Any number that can be written in fraction form is a rational number. This includes integers, terminating decimals, and repeating decimals as well as fractions. ... So, any terminating decimal is a rational number.





Rational Exponents
to
Radicals

$81^{\frac{1}{4}}$ (index)

$\sqrt[4]{81}$

$= 3$

The diagram illustrates the relationship between rational exponents and radicals. It shows the number 81 with a rational exponent of 1/4. A red arrow points from the exponent 1/4 to the radical symbol in the expression below, indicating that the denominator of the exponent becomes the index of the radical. A blue arrow points from the text 'Rational Exponents to Radicals' to the radical expression. The final result is shown as = 3.

What if...

Rational Exponents

to

Radicals

125 ^{$\frac{2}{3}$}

(exponent)
(index)

$$= \left(\sqrt[3]{125} \right)^2$$

$$= (5)^2$$

$$= 25$$

Express the radical as a power.

$$\left(\sqrt[3]{15}\right)^1 = 15^{\frac{1}{3}}$$

$$\left(\sqrt[2]{25}\right)^7 = 25^{\frac{7}{2}}$$

$$\left(\sqrt[5]{9}\right)^2 = 9^{\frac{2}{5}}$$

Let's Take a Closer Look!!

Fill in the chart. (You can use your calculator!!)

x	$x^{\frac{1}{2}} = \sqrt{x}$	x	$x^{\frac{1}{3}} = \sqrt[3]{x}$
1	$1^{\frac{1}{2}} = \sqrt{1} = 1$	1	$1^{\frac{1}{3}} = \sqrt[3]{1} = 1$
4	$4^{\frac{1}{2}} = \sqrt{4} = 2$	8	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
9	$9^{\frac{1}{2}} = \sqrt{9} = 3$	27	$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
16	$16^{\frac{1}{2}} = \sqrt{16} = 4$	64	$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$
25	$25^{\frac{1}{2}} = \sqrt{25} = 5$	125	$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

perfect cubes

What do you notice?

$$\begin{aligned} \sqrt[3]{27} &= \sqrt[3]{\underline{3 \cdot 3 \cdot 3}} \\ &= 3 \end{aligned}$$

Our Conclusion

- Raising a number to an exponent of $1/2$ is equivalent to taking the **square root**!
- Raising a number to an exponent of $1/3$ is equivalent to taking the **cube root**!

$$X^{1/n} = \sqrt[n]{X}$$

$$X^{2/3} = \left(\sqrt[3]{X}\right)^2$$

Practice Questions

Calculate each of the following without using a calculator:

$$27^{1/3}$$

$$= \sqrt[3]{27}$$

$$= \textcircled{3}$$

$$100^{1/2}$$

$$= \sqrt{100}$$

$$= \textcircled{10}$$

$$16^{1/4}$$

$$= \sqrt[4]{16}$$

$$= \textcircled{2}$$



Calculate each of the following without using a calculator:

$$\begin{aligned} & \underline{\underline{36^{0.5}}} \\ &= 36^{\frac{1}{2}} \\ &= \sqrt{36} \\ &= \textcircled{6} \end{aligned}$$

$$\begin{aligned} & \underline{\underline{32^{0.2}}} \\ &= 32^{\frac{2}{10}} \\ &= 32^{\frac{1}{5}} \\ &= \sqrt[5]{32} \\ &= \textcircled{2} \end{aligned}$$

$$\begin{aligned} & \underline{\underline{625^{0.25}}} \\ &= 625^{\frac{25}{100}} \\ &= 625^{\frac{1}{4}} \\ &= \sqrt[4]{625} \\ &= \textcircled{5} \end{aligned}$$

Calculate each of the following
without using a calculator:

$$\begin{aligned} & \underline{4}^{\underline{3/2}} \\ & = (\sqrt{4})^3 \\ & = (2)^3 \\ & = 8 \end{aligned}$$



$$\begin{aligned} & \underline{27}^{\underline{2/3}} \\ & = (\sqrt[3]{27})^2 \\ & = (3)^2 \\ & = 9 \end{aligned}$$

Therefore:

$$\mathbf{x^{m/n} = \left(\sqrt[n]{x} \right)^m}$$

Write as a power:

$$\left(\sqrt[4]{625}\right)^9$$

$$= 625^{9/4}$$



**Calculate the following
without using a calculator:**

$$128^{1/7}$$
$$= \sqrt[7]{128}$$
$$= 2$$



$$343^{2/3}$$
$$= (\sqrt[3]{343})^2$$
$$= (7)^2$$
$$= 49$$

**Calculate the following
without using a calculator:**

$$289^{3/2}$$

$$= (\sqrt{289})^3$$

$$= (17)^3$$

$$= 4913$$



$$625^{3/4}$$

$$= (\sqrt[4]{625})^3$$

$$= (5)^3$$

$$= 125$$

Stop....



Check out page 227.

Questions:

5, 6,

$$2.\underline{5}$$

7a,b, f

$$= 2\frac{1}{2} \quad \text{mixed number}$$

8,

10a,c,f,

$$= \frac{5}{2} \quad \text{improper fraction}$$

11, 15