Questions from Homework

2 c) $t_n = \frac{1}{3n}$

$t_1 = \frac{1}{3}$ $t_2 = \frac{1}{6}$ $t_3 = \frac{1}{9}$ $t_4 = \frac{1}{12}$

$0.\overline{3}$, $0.1\overline{6}$, $0.\overline{1}$, $0.08\overline{3}$

$d = t_2 - t_1$
$= \frac{1}{6} - \frac{1}{3}$
$= \frac{3}{18} - \frac{6}{18}$
$= \frac{-3}{18} = \cancel{-\frac{1}{6}}$

$d = t_3 - t_2$
$= \frac{1}{9} - \frac{1}{6}$
$= \frac{2}{18} - \frac{3}{18}$
$= \cancel{-\frac{1}{18}}$

3 d) $5\sqrt{2}, 4\sqrt{2}, 3\sqrt{2}, \ldots 2\sqrt{2}, 1\sqrt{2}$

Given:

$a = 5\sqrt{2}$

$d = 4\sqrt{2} - 5\sqrt{2}$
$= -1\sqrt{2}$
$= -\sqrt{2}$

general term ($n = n$)

$t_n = a + (n-1)d$
$t_n = 5\sqrt{2} + (n-1)(-\sqrt{2})$
$t_n = 5\sqrt{2} - n\sqrt{2} + \sqrt{2}$
$t_n = 6\sqrt{2} - n\sqrt{2}$
$t_n = \sqrt{2}(6 - n)$

9 f) $5a - 3b, 4a - 2b, 3a - b, \ldots, -5a + 7b$

Given

$n = ?$

$a = 5a - 3b$

$d = 4a - 2b - (5a - 3b)$
$= 4a - 2b - 5a + 3b$
$= -a + b$

$t_n = -5a + 7b$

$t_n = a + (n-1)d$
$-5a + 7b = 5a - 3b + (n-1)(-a+b)$
$-10a + 10b = (n-1)(-a+b)$
$10(\cancel{-a+b}) = (n-1)(\cancel{-a+b})$
$10 = n - 1$
$\boxed{11 = n}$

Questions from Homework

$\boxed{10, 8, 6, 4, 2, 0, -2, -4, -6, -8, -10, -12}$

⑥ b) $t_9 = -6$

$t_n = a + (n-1)d$

$t_9 = a + 8d$

$-6 = a + 8d$

$t_{12} = -12$

$t_n = a + (n-1)d$

$t_{12} = a + 11d$

$-12 = a + 11d$

$$
\begin{aligned}
-12 &= a + 11d \\
(-)\ -6 &= a + 8d \\
\hline
\frac{-6}{3} &= \frac{3d}{3}
\end{aligned}
$$

$\boxed{-2 = d}$

$-6 = a + 8d$

$-6 = a + 8(-2)$

$-6 = a - 16$

$\boxed{10 = a}$

general term:

$t_n = a + (n-1)d$

$t_n = 10 + (n-1)(-2)$

$t_n = 10 - 2n + 2$

$t_n = -2n + 12$

Geometric Sequences

Ex: 2, 4, 8, 16, 32

2 2 2 2

Sequences of numbers that follow a pattern of multiplying a fixed number from one term to the next are called geometric sequences.

- To find the next term, multiply the previous term by a common ratio.
- In the sequence 2, 4, 8, 16, 32 we are multiplying by 2.
- This common ratio is called **"r" ($r = t_2/t_1$).**
- The first term is still called **"a"** or **"t_1".**
- The second term is called **"t_2".**
- The last term or an indicated term is called **"t_n".** general term
- The position of a term or the number of terms is called **"n".**

Geometric Sequences

Remember $r = t_2/t_1$

Find "r" and the next term!

1, 2, 4, 8,... 16

$r = \frac{t_2}{t_1} = \frac{2}{1} = 2$

16, -8, 4, -2, 1,... $-\frac{1}{2}$

$r = \frac{-8}{16} = \frac{4}{-8} = \frac{-2}{4} = -\frac{1}{2}$

0.01, 0.06, 0.36, 2.16, ... , 12.96

6 6 6

$r = 6$

Geometric Sequences

To find any given term in a geometric sequence we use the following formula:

$$\boxed{t_n = ar^{n-1}}$$

$r = \frac{t_4}{t_3} = \frac{-1}{4} \div \frac{1}{2}$

$= \frac{-1}{4} \times \frac{2}{1} = \frac{-2}{4} = \frac{-1}{2}$

Examples

Find the indicated term

1. $2, -1, \frac{1}{2}, \frac{-1}{4} \ldots$

$a = 2$

$r = \frac{-1}{2}$

$n = 9$

$t_9 = (2)\left(\frac{-1}{2}\right)^{9-1}$

$t_9 = (2)\left(\frac{-1}{2}\right)^{8}$

$t_9 = (2)\left(\frac{1}{256}\right)$

$t_9 = \frac{2}{256} = \frac{1}{128}$

look for

$\boxed{y^x}$ or $\boxed{x^y}$

or $\boxed{\wedge}$

exponent

We can also determine the number of terms in the sequence.

$$\boxed{t_n = ar^{n-1}}$$

How many terms are in the following sequences?
(Solve for "n")

9, 27, 81,... 2187

$a = 9$

$r = 3$

$t_n = 2187$

$t_n = ar^{n-1}$

$\frac{2187}{9} = \frac{(\cancel{9})(3)^{n-1}}{\cancel{9}}$

$243 = 3^{n-1}$

$\cancel{3}^5 = \cancel{3}^{n-1}$

$\frac{\log 243}{\log 3} = 5$

$5 = n-1$

$6 = n$

Solve for "n"

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \frac{1}{1024}$

$a = \frac{1}{2}$

$r = \frac{1}{4} \div \frac{1}{2}$

$= \frac{1}{4} \times \frac{2}{1}$

$= \frac{2}{4} = \frac{1}{2}$

$t_n = \frac{1}{1024}$

$t_n = ar^{n-1}$

$\frac{\frac{1}{1024}}{\frac{1}{2}} = \frac{(\cancel{\frac{1}{2}})(\frac{1}{2})^{n-1}}{\cancel{\frac{1}{2}}}$

$\frac{1}{1024} \cdot 2 = \left(\frac{1}{2}\right)^{n-1}$

$\frac{2}{1024} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$

$* \frac{\log(\frac{1}{512})}{\log(\frac{1}{2})} = 9$

$\cancel{\left(\frac{1}{2}\right)}^9 = \cancel{\left(\frac{1}{2}\right)}^{n-1}$

$9 = n-1$

$\boxed{10 = n}$

geometric $t_n = ar^{n-1}$

Find "a", "r", and "t_n" for the following sequences!

3, 12, 48, 192, 768

$t_2 = 12, \; t_5 = 768$

$t_n = ar^{n-1}$	$t_n = ar^{n-1}$
$t_2 = ar^{2-1}$	$t_5 = ar^{5-1}$
$t_2 = ar^1$	$t_5 = ar^4$
$12 = ar$	$768 = ar^4$

Elimination

$\div \; \dfrac{768 = ar^4}{12 = ar}$ $\sqrt[3]{}$

$64 = r^3$ 3 $\sqrt[x]{}$

$\boxed{4 = r}$

$12 = ar$

$\dfrac{12}{4} = \dfrac{a(4)}{4}$

$\boxed{3 = a}$

general term (n=n)

$t_n = ar^{n-1}$

$t_n = (3)(4)^{n-1}$

$(r^3)^{1/3} = (64)^{1/3}$

$r = 4$

$(r^2)^{1/2} = (64)^{1/2}$

$r = 8$

what if:

$t_n = ar^{n-1}$

$t_n = (8)(2)^{n-1}$

$t_n = (2^3)(2)^{n-1}$

$t_n = 2^{3+n-1}$

$t_n = 2^{n+2}$

Homework

#1- #6