Questions from Homework

Ex 10.4

4 c) $36, 18, 9, \ldots, \frac{9}{128}$

$a = 36$

$r = \frac{18}{36} = \frac{1}{2}$

$t_n = \frac{9}{128}$

$t_n = ar^{n-1}$

$\frac{9}{128} = (36)\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{9}{128} \div 36$

$\frac{\cancel{9}^{1}}{128} \times \frac{1}{\cancel{36}_{4}}$

$\frac{1}{512}$

$* \left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$

$* \frac{\log(1/512)}{\log(1/2)} = 9$

$9 = n-1$

$10 = n$

2 d) $2^{50}, 2^{48}, 2^{46}, \ldots$

$a = 2^{50}$

$r = \frac{2^{48}}{2^{50}} = 2^{48-50} = 2^{-2}$

$t_n = (a)(r)^{n-1}$

$t_{15} = (2^{50})(2^{-2})^{14}$

$t_{15} = (2^{50})(2^{-28})$

$t_{15} = 2^{22}$

4 b) $16, -8, 4, \ldots, \frac{1}{4}$

$n = ?$

$a = 16$

$r = \frac{-8}{16} = \frac{-1}{2}$

$t_n = \frac{1}{4}$

$t_n = ar^{n-1}$

$\frac{1}{4} = (16)\left(-\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(-\frac{1}{2}\right)^{n-1}$

Get common base.

$\frac{\log(1/64)}{\log(1/8)} = 6$

$* \left(-\frac{1}{2}\right)^6 = \left(-\frac{1}{2}\right)^{n-1}$

$6 = n-1$

$\boxed{7 = n}$

5 a) $t_3 = \frac{1}{9}$ | $t_7 = 9$

$t_n = ar^{n-1}$
$t_3 = ar^{3-1}$
$t_3 = ar^2$
$\frac{1}{9} = ar^2$

$t_n = ar^{n-1}$
$t_7 = ar^{7-1}$
$t_7 = ar^6$
$9 = ar^6$

Elimination by division

$\frac{9 = ar^6}{\frac{1}{9} = ar^2}$

$81 = r^4$

$\boxed{\pm 3 = r}$

$\boxed{x\sqrt{y}}$

$9 = ar^6$
$9 = a(\pm 3)^6$
$\frac{9}{729} = \frac{a(729)}{729}$
$\boxed{\frac{1}{81} = a}$

Find t_4:
- if $r = 3$

$t_n = ar^{n-1}$
$t_4 = \left(\frac{1}{81}\right)(3)^{4-1}$
$t_4 = \left(\frac{1}{81}\right)(3)^3$
$t_4 = \left(\frac{1}{81}\right)(27)$
$t_4 = \frac{27}{81} = \frac{1}{3}$

Find t_4:
- if $r = -3$

$t_n = ar^{n-1}$
$t_4 = \left(\frac{1}{81}\right)(-3)^{4-1}$
$t_4 = \left(\frac{1}{81}\right)(-3)^3$
$t_4 = \left(\frac{1}{81}\right)(27)$
$t_4 = \frac{-27}{81} = -\frac{1}{3}$

6 $t_{10} = 2560$ | $t_5 = 80$

$t_n = ar^{n-1}$
$t_{10} = ar^9$
$2560 = ar^9$
$ar^9 = 2560$

$t_n = ar^{n-1}$
$t_5 = ar^4$
$80 = ar^4$
$ar^4 = 80$

Elimination by division

$\frac{ar^9 = 2560}{ar^4 = 80}$

$r^5 = 32$
$r = (32)^{(1/5)} = \boxed{2}$
or
$r = \sqrt[5]{32} = 2$

$ar^4 = 80$
$a(2)^4 = 80$
$\frac{16a}{16} = \frac{80}{16}$
$\boxed{a = 5}$

$t_n = (5)(2)^{n-1}$
$t_{12} = (5)(2)^{12-1}$
$t_{12} = (5)(2)^{11}$
$t_{12} = (5)(2048)$
$t_{12} = 10240$

Arithmetic Series

Series: The sum (addition) of the terms of a sequence. The sum is usually finite: 1+2+3+4+5. However it could be infinite: 2+4+8+16+... You can find the sum of many finite series and certain types of infinite series by using formulas.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Arithmetic Series

Ex: $2+6+10+14+18+22+26+30 \ldots$

$a = 2$

$d = 6-2$

$d = 4$

(i) What is t_6?

$t_6 = 22$

(ii) What is S_6? (sum)

$S_6 = 2+6+10+14+18+22$

$S_6 = 72$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_6 = \frac{6}{2}[2(2)+(6-1)(4)]$

$S_6 = 3(24)$

$S_6 = 72$

Find the sum of the first 100 terms of the arithmetic series 1+4+7+10+...

a = 1
d = t_2-t_1 = 3
n = 100

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{100} = \frac{100}{2}[2(1) + (100-1)(3)]$$

$$S_{100} = 50[2 + 297]$$

$$S_{100} = 50(299)$$

$$S_{100} = 14950$$

Find the sum of the following series

$$\frac{1}{2} + 1 + \frac{3}{2} + 2, \ldots + 20$$ (40 terms)

Hint: How many terms are there?

a = ½
d = ½
$t_n = 20$
n =

(i) Find n:

$t_n = a + (n-1)d$

$20 = \frac{1}{2} + (n-1)\left(\frac{1}{2}\right)$

$20 = \cancel{\frac{1}{2}} + \frac{1n}{2} - \cancel{\frac{1}{2}}$

$2 \cdot 20 = \frac{1n}{\cancel{2}} \cdot \cancel{2}$

$\boxed{40 = n}$

(ii) Find S_n or S_{40}

$S_n = \frac{n}{2}[a + t_n]$

$S_{40} = \frac{40}{2}\left[\frac{1}{2} + 20\right]$

$S_{40} = 20\left(\frac{1}{2} + \frac{40}{2}\right)$

$S_{40} = \overset{10}{\cancel{20}}\left(\frac{41}{\cancel{2}_1}\right)$

$\boxed{S_{40} = 410}$

How many terms are in the series:
3+8+13+...+248 if its sum is 6275?

5

$a = 3$
$d = 5$
$S_n = 6275$
$t_n = 248$

$$S_n = \frac{n}{2}(a + t_n)$$

or

$$t_n = a + (n-1)d$$

$$6275 = \frac{n}{2}(3 + 248)$$

$$2 \cdot 6275 = \frac{251n}{\cancel{2}} \cdot \cancel{2}$$

$$\frac{12550}{251} = \frac{251n}{251}$$

$$50 = n$$

$$248 - 3 = 3 - 3 + (n-1)(5)$$

$$\frac{245}{5} = \frac{(n-1)(\cancel{5})}{\cancel{5}}$$

$$49 + 1 = n - 1 + 1$$

$$50 = n$$

Find the indicated sums of the following series:

S_{15} of 2+6+10....

$a=2$

$d=4$

$n=15$

$$S_{15}=\frac{15}{2}[2(2)+(15-1)(4)]$$

$$S_{15}=\frac{15}{2}[4+56]$$

$$S_{15}=\frac{15}{\cancel{2}_1}(\cancel{60}^{30})$$

$$S_{15}=450$$

S_{20} of -15-10-5+...

$a=-15$

$d=-10-(-15)$

$d=5$

$n=20$

$$S_{20}=\frac{20}{2}[2(-15)+(20-1)(5)]$$

$$S_{20}=10[-30+95]$$

$$S_{20}=10(65)$$

$$S_{20}=650$$

Homework

#1-8

7) $t_2 = -1$

$t_2 = a + (2-1)d$

$t_2 = a + d$

$-1 = a + d$

$a + d = -1$

$t_{12} = 19$

$t_1 = a + (12-1)d$

$t_1 = a + 11d$

$19 = a + 11d$

$a + 11d = 19$

$a + 11d = 19$

$(-)\ a + d = -1$

$10d = 20$

$\boxed{d = 2}$

$a + d = -1$

$a + (2) = -1$

$\boxed{a = -3}$