Warm-Up

horizontal (change sign)

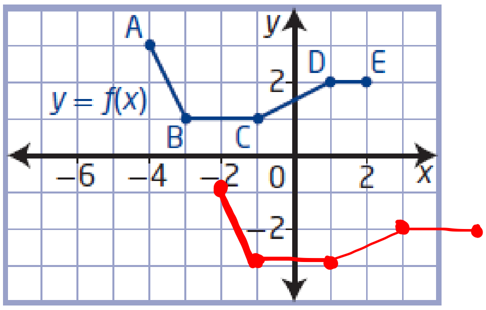
8. Copy and complete the table. $y = f(x-h)+k$

vertical

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + \underline{5}$	$(x, y) \rightarrow (x, y + 5)$	$k=5$ (Up)
horizontal	$y = f(x + \underline{7})$	$(x, y) \rightarrow (x - 7, y)$	$h=-7$ (Left)
horizontal	$y = f(x - \underline{3})$	$(x,y) \rightarrow (x+3, y)$	$h=3$ (Right)
vertical	$y = f(x) - \underline{6}$	$(x,y) \rightarrow (x, y-6)$	$k=-6$ (Down)
horizontal and vertical	$y = f(x+4)-9$ $y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$	$h=-4$ Left $k=-9$ Down
horizontal and vertical	$y=f(x-4)-6$	$(x, y) \rightarrow (x + 4, y - 6)$	$h=4$ Right $k=-6$ Down
h + v	$y=f(x+2)+3$	$(x, y) \rightarrow (x - 2, y + 3)$	$h=-2$ $k=3$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+k)$	

Questions from Homework

4.



b) $y = f(x-2)-4$

$h = 2 \quad k = -4$

$(x, y) \rightarrow (x+2, y-4)$

$A(-4,3) \rightarrow (-2,-1)$

$B(-3,1) \quad (-1,-3)$

$C(-1,1) \quad (1,-3)$

$D(1,2) \quad (3,-2)$

$E(2,2) \quad (4,-2)$

Transformations:

New Functions From Old Functions

- Translations
- Stretches
- Reflections

Reflections and Stretches

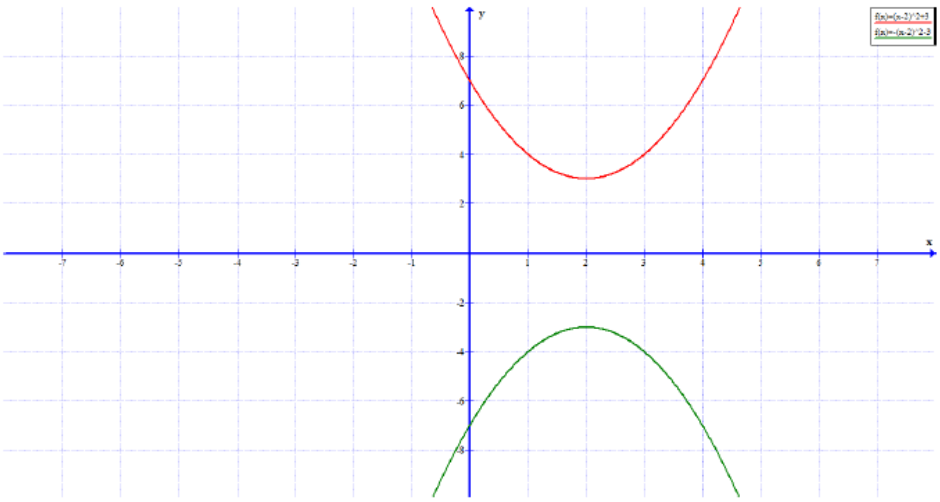
Focus on...

- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

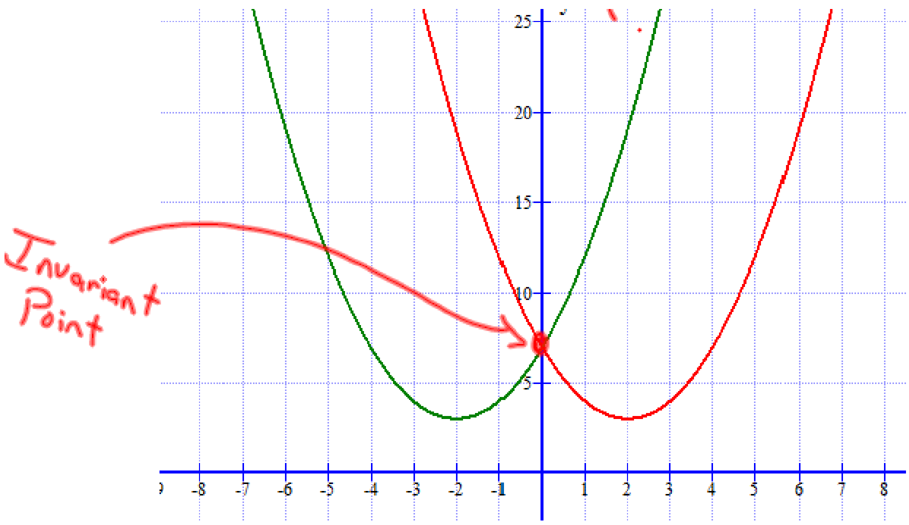
Vertical reflection $(x, y) \rightarrow (x, -y)$

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.



Horizontal Reflection $(x, y) \rightarrow (-x, y)$

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.



invariant point

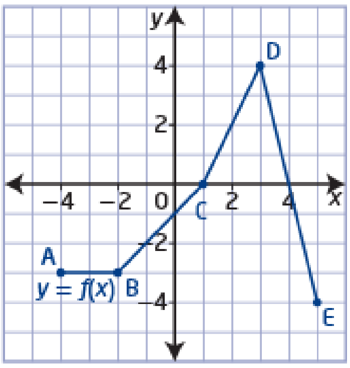
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

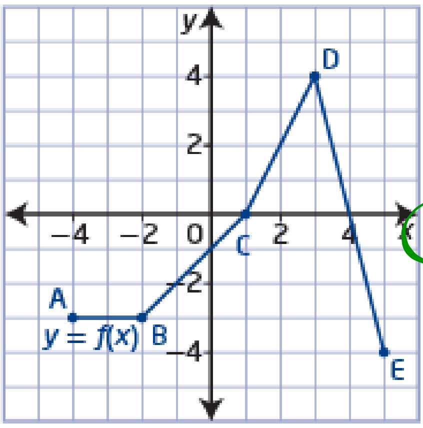
b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



Remember...

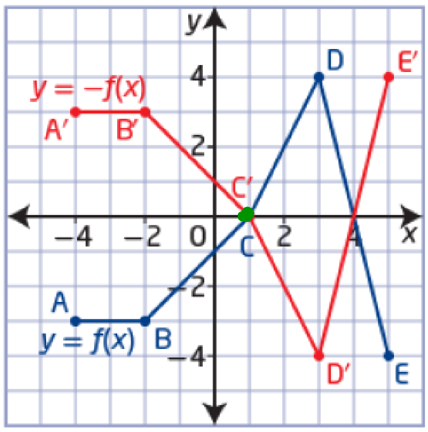
- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below (Vertical Reflection)



$(x, y) \rightarrow (x, -y)$

(-4,-3)	(-4,3)
(-2,-3)	(-2,3)
(1,0)	(1,0)
(3,4)	(3,-4)
(5,-4)	(5,4)



Invariant Point

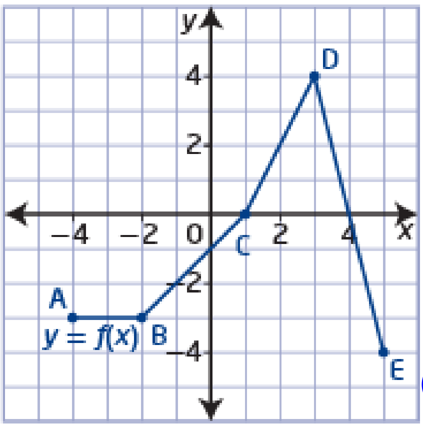
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

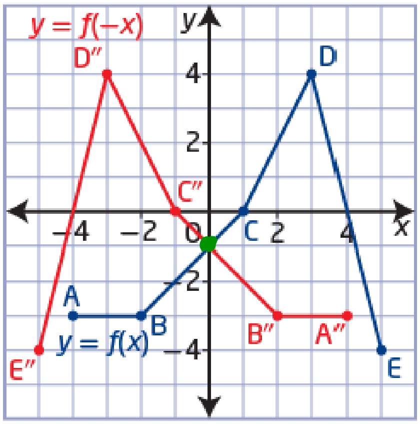
- Sketch $y = f(-x)$ on the axis below

Horizontal reflection

$(x, y) \rightarrow (-x, y)$



$(-4, -3)$	$(4, -3)$
$(-2, -3)$	$(2, -3)$
$(1, 0)$	$(-1, 0)$
$(3, 4)$	$(-3, 4)$
$(5, -4)$	$(-5, -4)$



Homework

Page 28 #1, 3, 4

$*f(-4) = 2(-4)+1$
$= -8+1$
$= -7$

Vertical

Horizontal

$f(x) = 2x+1$

x	y
* -4	-7
-2	-3
0	1
2	5
4	9

$g(x) = -f(x)$

x	y
-4	7
-2	3
0	-1
2	-5
4	-9

$h(x) = f(-x)$

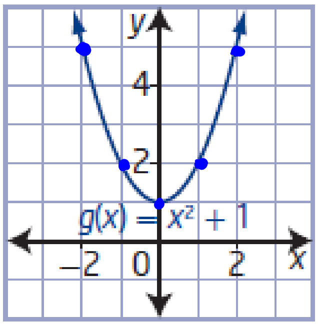
x	y
4	-7
2	-3
0	1
-2	5
-4	9

Questions from Homework

3. Consider each graph of a function.

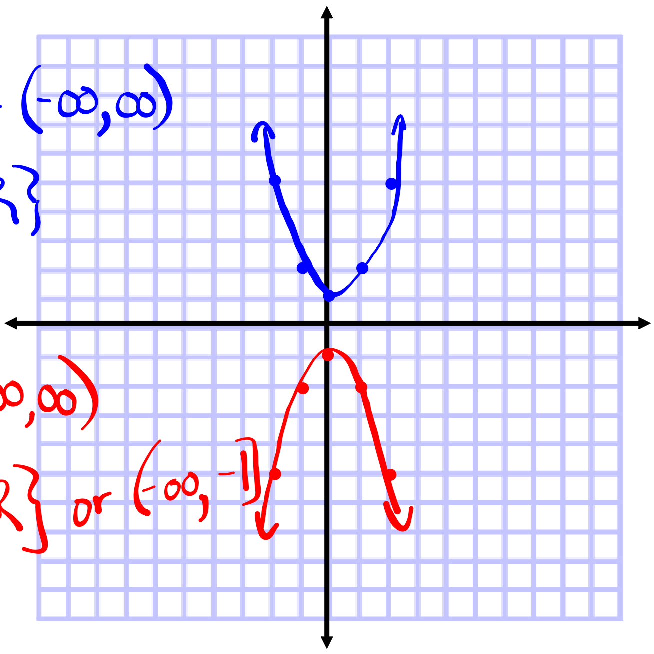
- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes. (Vertical)
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.

b)



$(x, y) \rightarrow (x, -y)$

(-2,5)	(-2,-5)
(-1,2)	(-1,-2)
(0,1)	(0,-1)
(1,2)	(1,-2)
(2,5)	(2,-5)



D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \leq -1, y \in \mathbb{R}\}$ or $(-\infty, -1]$

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

stretch

- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between Ex: 0.5, $\frac{1}{4}$ 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

* If you can't see a value in place of "a" or "b" then we let them equal 1

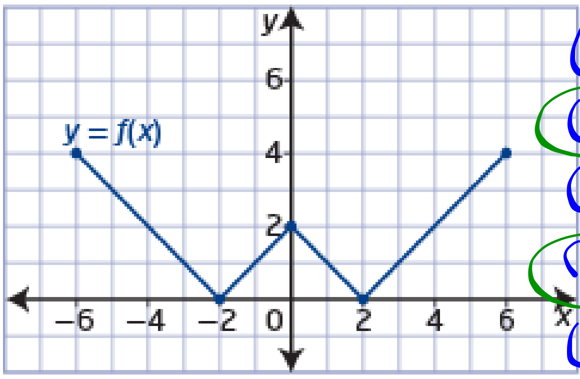
Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

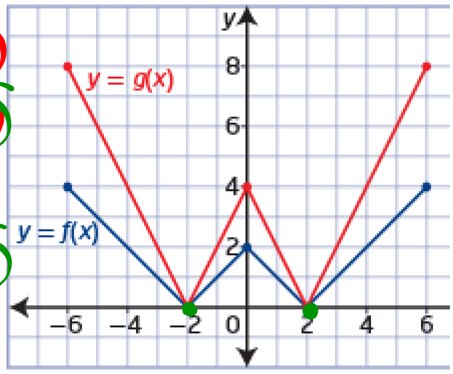
(negative)

$a = 2 \rightarrow$ Vertical Stretch by a factor of 2

a) $g(x) = 2f(x)$ $(x, y) \rightarrow (x, 2y)$



$f(x)$	$g(x)$
(-6,4)	(-6,8)
(-2,0)	(-2,0)
(0,2)	(0,4)
(2,0)	(2,0)
(6,4)	(6,8)



The invariant points are $(-2, 0)$ and $(2, 0)$.

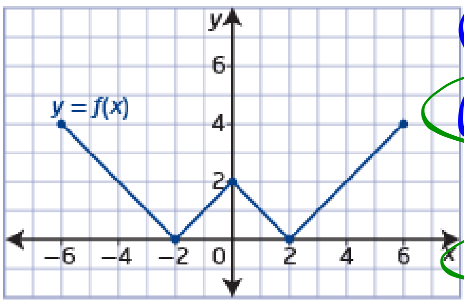
For $f(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$, ← interval notation
and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \leq y \leq 8, y \in \mathrm{R}\}$, or $[0, 8]$.

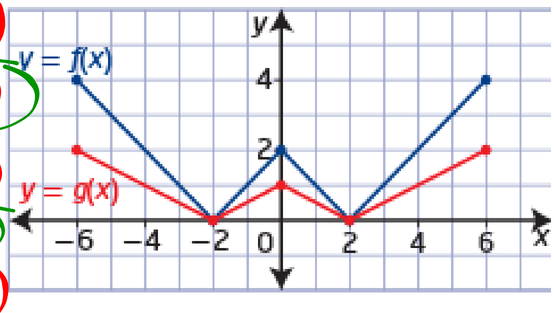
b) $g(x) = \frac{1}{2}f(x)$

$a = \frac{1}{2}$ → A vertical compression about the x-axis by a factor of $\frac{1}{2}$.

$(x, y) \rightarrow (x, \frac{1}{2}y)$



(-6,4)	(-6,2)
(-2,0)	(-2,0)
(0,2)	(0,1)
(2,0)	(2,0)
(6,4)	(6,2)



The invariant points are (−2, 0) and (2, 0).

For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \leq y \leq 4, y \in R\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in R\}$, or $[-6, 6]$,
and the range is $\leq y \leq 2, y \in R\}$, or $[0, 2]$.

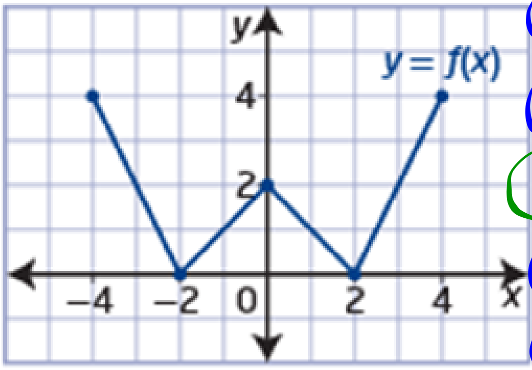
Horizontal Stretch or Compression... (Reciprocal)

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

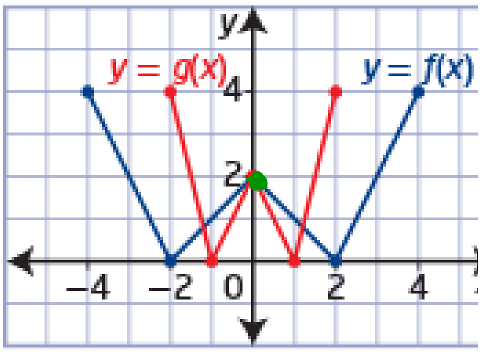
reciprocal

$b=2 \rightarrow$ Horizontal Stretch by a factor $\frac{1}{2}$

a) $g(x) = f(2x)$ $\qquad (x,y) \rightarrow (\frac{1}{2}x, y)$



(-4,4)	(-2,4)
(-2,0)	(-1,0)
(0,2)	(0,2)
(2,0)	(1,0)
(4,4)	(2,4)



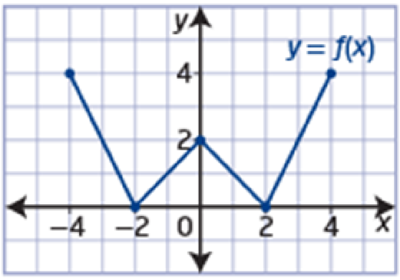
The invariant point is (0, 2).

For $f(x)$, the domain is $\{x \mid -4 \le x \le 4, x \in R\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \le y \le 4, y \in R\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -2 \le x \le 2, x \in R\}$, or $[-2, 2]$, and the range is $\{y \mid 0 \le y \le 4, y \in R\}$, or $[0, 4]$.

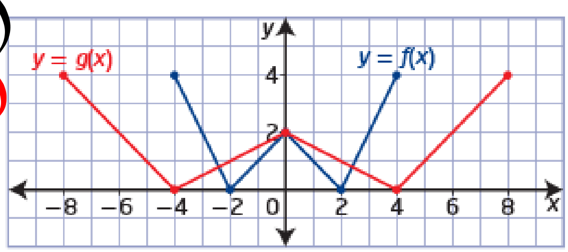
b) $g(x) = f\left(\frac{1}{2}x\right)$

$b = \frac{1}{2}$ → Horizontal stretch about the y-axis by a factor of 2



$(x, y) \to (2x, y)$

(-4,4)	(-8,4)
(-2,0)	(-4,0)
(0,2)	(0,2)
(2,0)	(4,0)
(4,4)	(8,4)



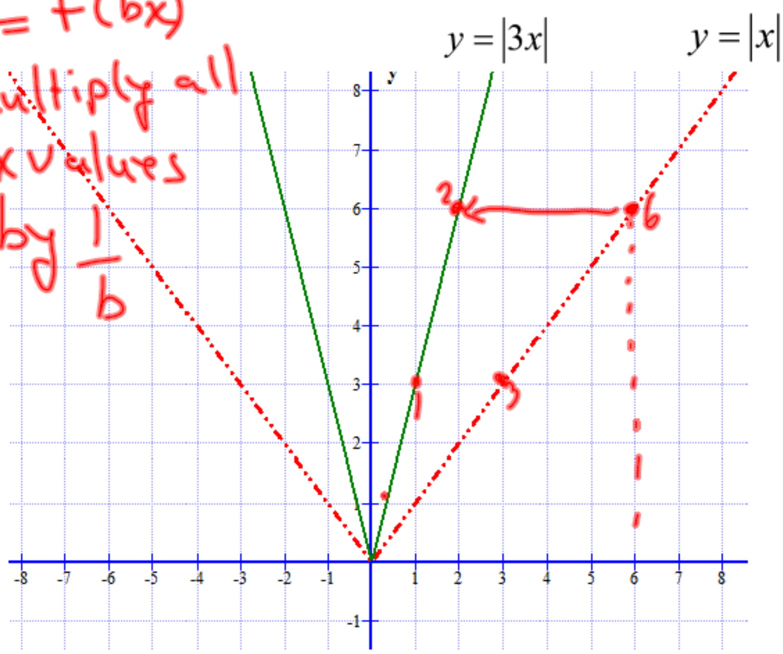
The invariant point is (0, 2).

For $f(x)$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathrm{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -8 \leq x \leq 8, x \in \mathrm{R}\}$, or $[-8, 8]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

Horizontal Stretch or Compression...

y= f(bx)
Multiply all
x values
by $\frac{1}{b}$



Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$y = -3f(-2x) + 7$

$a = -3$ → a vertical stretch about the x-axis by a factor of 3.
There is also a vertical reflection in the x-axis.

$b = -2$ → a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$.
There is a horizontal reflection in the y-axis

$h = 0$

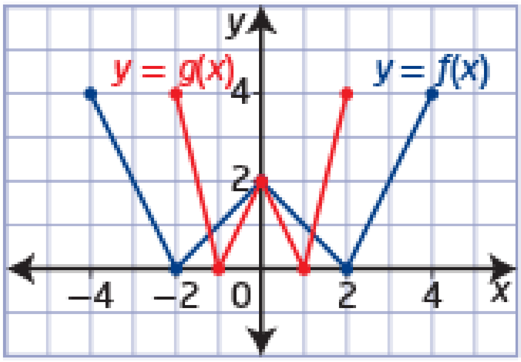
$k = 7$ → vertically translated 7 units up

$(x, y) \rightarrow \left[-\frac{1}{2}x, -3y + 7\right]$

Homework

Page 28 # 2, 5, 6, 7

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x