Questions from homework

(9) $1+4+9+16 = \sum_{n=1}^{4} n^2$ general term

$a=1$

$d=?$

$r=?$

(7) $2+4+6+8+10+12 = \sum_{n=1}^{6} 2n$ general term

$a=2$

$d=2$

$t_n = 2+(n-1)(2)$

$t_n = 2+2n-2$

$t_n = 2n$

Limit (of a sequence - t_n)

A finite number L that the value of t_n gets closer and closer to, or "approaches," as n becomes very large, or "approaches infinity." The value of t_n can be made as close as you like to L by using a sufficiently large value for n.

general term

The notation for a limit is $\boxed{\lim_{n\to\infty} t_n = L}$

$$\lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$$

$$t_1 = \left(\frac{1}{2}\right)^1 = \frac{1}{2} = 0.5$$

$$t_2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$$

$$t_3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125$$

$$t_{10} = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024} = 0.000977$$

Converging on $\underline{\underline{0}}$

Converging Sequence (has a limit)

A seqence in which the terms approach a limit

For example, $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \ldots$ converges to $\underline{\underline{1}}$

0.25, 0.4, 0.5, 0.57, ...

The above sequence was generated using the following general term.	$t_n = \frac{n}{n+3}$

What happens if *"n"* is a very large number?

$$t_{10} = \frac{10}{13} = 0.7692$$

$$t_{100} = \frac{100}{103} = 0.9709$$

$$t_{1000} = \frac{1000}{1003} = 0.997$$

Converging on 1.

Diverging Sequence

does not have a limit

A seqence in which the terms do not approach a limit

For example, *1, 2, 3, 4,...* diverges. (no limit exists)

The above sequence was generated using the following general term.	$t_n = n$

What happens if *"n"* is a very large number?

$t_{10} = 10$

$t_{100} = 100$

$t_{1000} = 1000$

There ain't no stoppin' it

There is no limit

Diverging

Decide whether each sequence *converges* or *diverges* then state the limit.

geometric sequence

2, 4, 8, 16, 32,... *diverges* $\lim_{n\to\infty} 2^n = \text{DNE}$

$t_n = (2)(2)^{n-1}$

$t_n = (2)^n$

geometric sequence

3, 1.5, 0.75, 0.375,... *converges* $\lim_{n\to\infty} (3)\left(\frac{1}{2}\right)^{n-1} = 0$

$t_n = (3)\left(\frac{1}{2}\right)^{n-1}$

Infinite Sequences

Suppose we have a sequence defined by $t_n = \dfrac{n}{2n+1}, n \in N$

Generate the first 4 terms of the sequence

$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \frac{7}{15}, \frac{8}{17}, \ldots$

You may notice that as "n" increases "t_n" approaches ½ — converging

Symbolically this is written $\displaystyle \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$

and is read "The limit as n approaches infinity of n over $(2n + 1)$ is ½."

Algebraically we solve by dividing the numerator and the denominator by the highest power of n. (degree)

$$\lim_{n\to\infty} \frac{n}{2n+1}$$

$$= \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{2n}{n} + \frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{1}{2 + \frac{1}{n}}$$

approaches 0

$$= \frac{1}{2+0}$$

$$= \frac{1}{2}$$

if the degree of the numerator and denominator are the same, then your limit will be the quotient of the leading coefficients.

$$\lim_{n\to\infty} \frac{1n^2-2}{4+3n^2} = \frac{1}{3}$$

$$\lim_{n\to\infty} \frac{\frac{n^2}{n^2}-\frac{2}{n^2}}{\frac{4}{n^2}+\frac{3n^2}{n^2}} = \lim_{n\to\infty} \frac{1-2/n^2}{\frac{4}{n^2}+3} = \frac{1}{3}$$

if the degree of the numerator is larger than the degree of the denominator, then your limit will not exist.

$$\lim_{n\to\infty} \frac{n^2+2n}{n-3} = \text{DNE}$$

$$\lim_{n\to\infty} \frac{\frac{n^2}{n^2}+\frac{2n}{n^2}}{\frac{n}{n^2}-\frac{3}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{2}{n}}{\frac{1}{n}-\frac{3}{n^2}} = \frac{1}{0}$$

if the degree of the denominator is larger than the degree of the numerator, then your limit will always equal 0.

$$\lim_{n\to\infty} \frac{1}{3n^5-2} = 0$$

$$\lim_{n\to\infty} \frac{\frac{1}{n^5}}{\frac{3n^5}{n^5}-\frac{2}{n^5}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n^5}}{3-2/n^5}$$

$$= \frac{0}{3}$$

$$= 0$$

Find the limit if it exists

$$t_n = n + 5$$

$$= \lim_{n\to\infty} \frac{n+5}{1} \leftarrow n^0$$

$$= \lim_{n\to\infty} \frac{\frac{n}{n} + \frac{5}{n}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{1 + 5/n}{1/n}$$

$$= \frac{1}{0}$$

$$= \text{DNE}$$

$$t_n = \frac{3n+1}{4n-2}$$

$$= \lim_{n\to\infty} \frac{\frac{3n}{n} + \frac{1}{n}}{\frac{4n}{n} - \frac{2}{n}}$$

$$= \lim_{n\to\infty} \frac{3 + 1/n}{4 - 2/n}$$

$$= \frac{3}{4}$$

Homework

#1 b)
#2
#3
#4