

Questions from Homework

Page 105

$$\textcircled{6} \quad s = t^3 - 3t^2 - 5t, \quad t \geq 0$$

$$s' = 3t^2 - 6t - 5 \quad (\text{velocity})$$

$$\textcircled{4} \quad 3t^2 - 6t - 5$$

$$0 = 3t^2 - 6t - 9 \quad \text{factor}$$

$$0 = 3(t^2 - 2t - 3)$$

$$0 = 3(t+1)(t-3)$$

$$t+1=0 \quad | \quad t-3=0$$

$$t=-1 \quad | \quad \boxed{t=3}$$

$$\textcircled{7} \quad s = t^2 - 4t + 4, \quad t \geq 0$$

$$s' = 2t - 4 \quad (\text{velocity})$$

$$\text{a) } \begin{array}{l} s'(1) = 2(1) - 4 \\ s'(1) = -2 \text{ m/s} \end{array} \quad \left| \quad \begin{array}{l} s'(3) = 2(3) - 4 \\ s'(3) = 2 \text{ m/s} \end{array} \right.$$

$$\text{b) Let } v \text{ or } s' = 0 \quad \text{c) Let } v \text{ or } s' > 0$$

$$s' = 2t - 4$$

$$0 = 2t - 4$$

$$0 = 2(t - 2)$$

$$t - 2 = 0$$

$$\boxed{t=2}$$

$$2t - 4 > 0$$

$$2t > 4$$

$$t > 2$$

$$\textcircled{8} \quad s = t^3 - 15t^2 + 63t$$

$$v = 3t^2 - 30t + 63$$

$$\text{a) Let } v = 0$$

$$0 = 3t^2 - 30t + 63$$

$$0 = 3(t^2 - 10t + 21)$$

$$0 = 3(t-7)(t-3)$$

$$t-7=0 \quad | \quad t-3=0$$

$$t=7s \quad | \quad t=3s$$

$$\text{b) } v > 0$$

$$3t^2 - 30t + 63 > 0$$

$$3(t-7)(t-3) > 0$$

$$\textcircled{1} \quad t-7 > 0 \quad | \quad t-3 > 0$$

$$\boxed{t > 7} \quad | \quad t > 3$$

$$\textcircled{2} \quad t-7 < 0 \quad | \quad t-3 < 0$$

$$t < 7 \quad | \quad \boxed{t < 3}$$

Questions from Homework

Page 105

Velocity

④ $h = 24.5t - 4.9t^2$

$v = 24.5 - 9.8t$

b) Let $v = 0$

$0 = 24.5 - 9.8t$

$9.8t = 24.5$

$t = 2.5s$

c) Find $h(2.5)$

$h = 24.5(2.5) - 4.9(2.5)^2$

$h = 30.6m$

d) Let $h = 0$

$0 = 24.5t - 4.9t^2$

$0 = t(24.5 - 4.9t)$

$$t = 0 \quad \left| \quad \begin{array}{l} 24.5 - 4.9t = 0 \\ 24.5 = 4.9t \\ \boxed{5 = t} \end{array} \right.$$

Page 109

③ d) $s = \sqrt{t^2 + t} = (t^2 + t)^{1/2}$

$v = \frac{1}{2}(t^2 + t)^{-1/2}(2t + 1)$

$v = \frac{2t + 1}{2(t^2 + t)^{1/2}}$

$a = \frac{2(t^2 + t)^{-1/2}(2) - (2t + 1)(1)(t^2 + t)^{-3/2}(2t + 1)}{[2(t^2 + t)^{1/2}]^2}$

$a = \frac{4(t^2 + t)^{-1/2} - (2t + 1)^2(t^2 + t)^{-3/2}}{4(t^2 + t)}$

$a = \frac{(t^2 + t)^{-1/2} [4(t^2 + t) - (2t + 1)^2]}{4(t^2 + t)}$

$a = \frac{4t^2 + 4t - 4t^2 - 4t - 1}{4(t^2 + t)^{3/2}}$

$a = \frac{-1}{4(t^2 + t)^{3/2}}$

Questions from Homework

Page 129

$$\textcircled{4} \text{ d), } s = \frac{5t}{1+t}$$

$$s' = \frac{5(1+t) - 5t(1)}{(1+t)^2} = \frac{5+5t-5t}{(1+t)^2} = \frac{5}{(1+t)^2} \text{ (velocity)}$$

$$s'' = \frac{(0)(1+t)^2 - 5(2)(1+t)(1)}{(1+t)^4} = \frac{-10(1+t)}{(1+t)^{4-1}} = \frac{-10}{(1+t)^3}$$

$$s''(4) = \frac{-10}{(1+4)^3} = \frac{-10}{125} = -\frac{2}{25} \text{ m/s}^2$$

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time*.

Differentiate with respect to time

If $xy^2 = 12$ and $\frac{dy}{dt} = 6$, Find $\frac{dx}{dt}$ when $y = 2$

① Differentiate w/ respect to time:

$$xy^2 = 12$$

$$\frac{dx}{dt}y^2 + 2xy\frac{dy}{dt} = 0$$

② Find x:

$$xy^2 = 12$$

$$x(2)^2 = 12$$

$$4x = 12$$

$$x = 3$$

③ Find $\frac{dx}{dt}$

$$\frac{dx}{dt}y^2 + 2xy\frac{dy}{dt} = 0$$

$$\frac{dx}{dt}(2)^2 + 2(3)(2)(6) = 0$$

$$4\frac{dx}{dt} + 72 = 0$$

$$4\frac{dx}{dt} = -72$$

$$\frac{dx}{dt} = -18$$

If $x^3 + y^3 = 9$ and $\frac{dx}{dt} = 4$, Find $\frac{dy}{dt}$ when $x = 2$

① Differentiate w/ respect to time:

$$x^3 + y^3 = 9$$

$$3x^2 \frac{dx}{dt} + 3y^2 \frac{dy}{dt} = 0$$

② Find y:

$$x^3 + y^3 = 9$$

$$(2)^3 + y^3 = 9$$

$$8 + y^3 = 9$$

$$y^3 = 1$$

$$\underline{y = 1}$$

③ Find $\frac{dy}{dt}$

$$3x^2 \frac{dx}{dt} + 3y^2 \frac{dy}{dt} = 0$$

$$3(2)^2(4) + 3(1)^2 \frac{dy}{dt} = 0$$

$$48 + 3 \frac{dy}{dt} = 0$$

$$3 \frac{dy}{dt} = -48$$

$$\frac{dy}{dt} = -16$$