

Questions from Homework

$$\textcircled{a} \quad y = \sqrt{x^2 + 2x}$$

$$y = (x^2 + 2x)^{1/2}$$

$$y' = \frac{1}{2} (x^2 + 2x)^{-1/2} (2x+2)$$



$$y' = \frac{(x+1)}{f(x)} (x^2 + 2x)^{-1/2}$$

$$y'' = (x^2 + 2x)^{-3/2} - \frac{1}{2} (x+1) (x^2 + 2x)^{-3/2}$$

$$y'' = (x^2 + 2x)^{-3/2} - (x+1)^2 (x^2 + 2x)^{-3/2}$$

$$y'' = (x^2 + 2x)^{-3/2} \left[(x^2 + 2x) - (x+1)^2 \right]$$

$$y'' = \frac{x^2 + 2x - x^2 - 2x - 1}{(x^2 + 2x)^{3/2}}$$

$$y'' = \frac{-1}{\sqrt{(x^2 + 2x)^3}}$$

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$$\textcircled{8} \text{ b) } s = t^3 - 15t^2 + 63t$$

$$v = 3t^2 - 30t + 63$$

$$3t^2 - 30t + 63 > 0$$

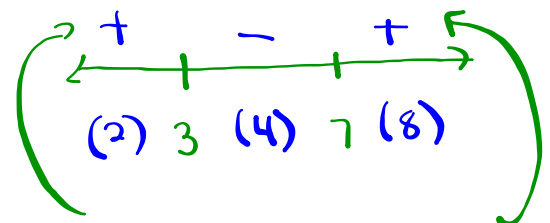
$$3(t^2 - 10t + 21) = 0$$

$$3(t-3)(t-7) = 0$$

$$t-3=0 \quad | \quad t-7=0$$

$$t=3 \text{ sec} \quad | \quad t=7 \text{ sec}$$

where does this
function have
positive y-values



$$t < 3 \quad \text{and} \quad t > 7$$

$$\text{or } [0, 3) \quad \text{and} \quad (7, \infty)$$

don't use $-\infty$ because
we are talking time.

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time*.

Related Rates

1. Draw a diagram
2. List what is given in differentiation notation $\frac{da}{dt}$, $\frac{dv}{dt}$, etc.
3. List what is to be found in differentiation notation.
4. Find an appropriate equation that relates the variables in steps 2 and 3.
5. Differentiate with respect to time.
6. Substitute the values given and solve for the unknown.

Areas and Volumes

The length of a square is 4m and is increasing at a rate of 1.25m/min. How fast is the *area* of the square increasing?

Hint!

write down what is given

find an equation that relates the two quantities

Given:

$$l = 4\text{m}$$

$$\frac{dl}{dt} = 1.25\text{m/min}$$

$$\frac{dA}{dt} = ?$$

$$A = l^2$$

$$\frac{dA}{dt} = 2l \frac{dl}{dt}$$

$$\frac{dA}{dt} = 2(4)(1.25)$$

$$\frac{dA}{dt} = 10\text{m}^2/\text{min}$$

Suppose you tossed a stone into a lake. A circular ripple starts and moves outward with its radius increasing at a rate of 5cm/sec. How fast is the *area* of the circle increasing after 3 seconds? (*Hint: what would the radius be at 3 seconds?*)

Given:

$$\frac{dr}{dt} = 5 \text{ cm/sec}$$

$$\frac{dA}{dt} = ?$$

$$\text{when } t = 3 \text{ sec}$$

$$r = 5 \text{ cm/s} \times 3 \text{ s}$$

$$r = 15 \text{ cm}$$

Area of a circle

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi (15)(5)$$

$$\frac{dA}{dt} = 150\pi \text{ cm}^2/\text{sec}$$

$$\frac{dA}{dt} = 471.24 \text{ cm}^2/\text{sec}$$

Volumes/Surface Areas of Spheres

A *spherical* snowball is melting in such a way that its volume is *decreasing* at a rate of $1 \text{ cm}^3/\text{min}$. At what rate is the radius of the snowball decreasing if the original radius is 5 cm ?

Hint!

write down what is given

find an equation that relates the two quantities

Given:

$$\frac{dV}{dt} = -1 \text{ cm}^3/\text{min}$$

$$\frac{dr}{dt} = ?$$

$$r = 5 \text{ cm}$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-1 = 4\pi (5)^2 \frac{dr}{dt}$$

$$-1 = 100\pi \frac{dr}{dt}$$

$$\frac{-1}{100\pi} \text{ cm/min} = \frac{dr}{dt}$$

$$-0.00318 \text{ cm/min} = \frac{dr}{dt}$$

A beach ball is being inflated so that its surface area is *increasing* at a rate of $100 \text{ cm}^2/\text{sec}$. Find the rate at which the radius is increasing if the original radius is 2 cm?

Soln.

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 100 \text{ cm}^2/\text{sec}$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = ?$$

$$100 = 8\pi (2) \frac{dr}{dt}$$

$$r = 2 \text{ cm}$$

$$100 = 16\pi \frac{dr}{dt}$$

$$\frac{100}{16\pi} \text{ cm/sec} = \frac{dr}{dt}$$

$$\frac{25}{4\pi} \text{ cm/sec} = \frac{dr}{dt} = 1.99 \text{ cm/sec}$$

Homework