Warm Up

If the length of a square is growing at a rate of 2cm/sec, what would the original length have to be if the *area* of the square is increasing at a rate of $12cm^2$/sec?

Questions From Homework

④ Given:

$\frac{dA}{dt} = 150 cm^2/s$

$\frac{dr}{dt} = ?$

$A = 1256 cm^2$

① Find r:

$A = 4\pi r^2$

$\frac{1256}{4\pi} = \frac{4\pi r^2}{4\pi}$

$100 = r^2$

$\pm 10 = r$

$r = 10 cm$

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$150 = 8\pi(10)\frac{dr}{dt}$

$150 = 80\pi \frac{dr}{dt}$

$\frac{150}{80\pi} = \frac{dr}{dt}$

$\frac{15}{8\pi} cm/s = \frac{dr}{dt}$

$0.5968 cm/s \approx \frac{dr}{dt}$

⑭ Given:

$\frac{dr}{dt} = 10 cm/s$

$\frac{dA}{dt} = ?$

$t = 4 sec$

① Find r

$r = 10 cm/s \times 4 s$

$r = 40 cm$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

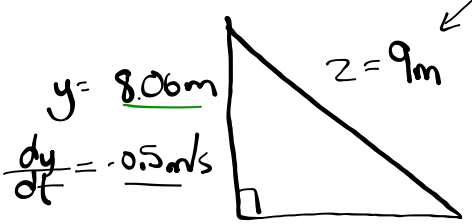
$\frac{dA}{dt} = 2\pi(40)(10)$

$\frac{dA}{dt} = 800\pi cm^2/s = 2513.27$

Related Rates
(Lampposts and Ladders)

A ladder 9m long is set against a wall and begins to slide down. The top of the ladder slides down at a rate of 0.5m/s. How quickly is the bottom sliding away from the wall when it is 4m from the wall to begin with?

(Hint: draw a diagram)



constant ← Ladder is not changing

(i) Find y:

$y^2 = z^2 - x^2$

$y^2 = 81 - 16$

$y^2 = 65$

$y = \pm\sqrt{65}$

$y = \sqrt{65}$

$y \approx 8.06$ m

(ii) Find $\frac{dx}{dt}$:

$x^2 + y^2 = z^2$ ← constant

$x^2 + y^2 = 81$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(4)\frac{dx}{dt} + 2(8.06)(-0.5) = 0$

$8\frac{dx}{dt} - 8.06 = 0$

$8\frac{dx}{dt} = 8.06$

$\frac{dx}{dt} = 1.0075$ m/s

Without calculator

$2(4)\frac{dx}{dt} + 2(\sqrt{65})(-0.5) = 0$

$8\frac{dx}{dt} - \sqrt{65} = 0$

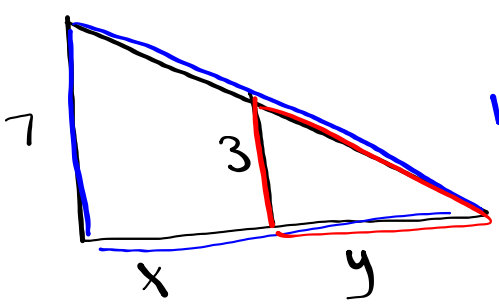
$8\frac{dx}{dt} = \sqrt{65}$

$\frac{dx}{dt} = \frac{\sqrt{65}}{8}$ m/s

Bigfoot is 3m tall and walks curiously towards a lamppost that is 7m tall. If he walks at a rate of 2m/s, at what rate is the length of his shadow changing?

draw a diagram
Let x = distance between Bigfoot and lamppost
Let y = length of shadow

use similar triangles



$\frac{dx}{dt} = -2\,m/s$ $\frac{dy}{dt} = ?$

towards

base / height: $\frac{x+y}{7} = \frac{y}{3}$ (base / height)

$3(x+y) = 7y$

$3x+3y = 7y$

$3x = 4y$

$3\frac{dx}{dt} = 4\frac{dy}{dt}$

$3(-2) = 4\frac{dy}{dt}$

$-6 = 4\frac{dy}{dt}$

$-\frac{6}{4} = \frac{dy}{dt}$

$-\frac{3}{2}\,m/s = \frac{dy}{dt}$

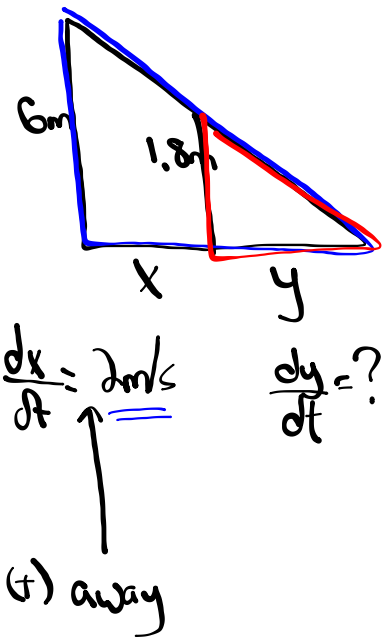
$\boxed{-1.5\,m/s = \frac{dy}{dt}}$

A man is 1.8m tall and walks away from a 6m lamppost at a rate of 2m/s. How fast is his shadow changing when he is 5m from the post?

ignore

draw a diagram
Let x = distance between man and lamppost
Let y = length of shadow

Use similar triangles



$$\frac{x+y}{6} = \frac{y}{1.8}$$

$$1.8(x+y) = 6y$$

$$1.8x + 1.8y = 6y$$

$$1.8x = 4.2y$$

$$1.8\frac{dx}{dt} = 4.2\frac{dy}{dt}$$

$$1.8(2) = 4.2\frac{dy}{dt}$$

$$3.6 = 4.2\frac{dy}{dt}$$

$$\frac{3.6}{4.2} = \frac{dy}{dt}$$

$$\frac{6}{7}\ m/s = \frac{dy}{dt}$$

$$\frac{dy}{dt} \approx 0.86 m/s$$

Homework