

Questions From Homework

① Given:

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = -0.5 \text{ cm}^2/\text{min}$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = ?$$

$$-0.5 = 8\pi(4) \frac{dr}{dt}$$

$$r = 4 \text{ cm}$$

$$-0.5 = 32\pi \frac{dr}{dt}$$

$$\frac{-0.5}{32\pi} = \frac{dr}{dt}$$

$$\boxed{\frac{-1}{64\pi} \text{ cm/min} = \frac{dr}{dt}}$$

$$-0.004974 \text{ cm/min} = \frac{dr}{dt}$$

① Given:

$$\frac{dA}{dt} = 4 \text{ cm}^2/\text{min}$$

$$\frac{db}{dt} = 1 \text{ cm/min}$$

$$\frac{dh}{dt} = ?$$

$$h = 20 \text{ cm}$$

$$A = 80 \text{ cm}^2$$

(i) Find b

$$A = \frac{1}{2}bh$$

$$80 = \frac{1}{2}b(20)$$

$$80 = 10b$$

$$8 = b$$

(ii) Find $\frac{dh}{dt}$

$$A = \left(\frac{1}{2}b\right)h$$

$$\frac{dA}{dt} = \frac{1}{2} \frac{db}{dt} h + \frac{1}{2} b \frac{dh}{dt}$$

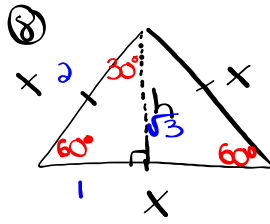
$$4 = \frac{1}{2}(1)(20) + \frac{1}{2}(8) \frac{dh}{dt}$$

$$4 = 10 + 4 \frac{dh}{dt}$$

$$-6 = 4 \frac{dh}{dt}$$

$$-1.5 \text{ cm/min} = \frac{dh}{dt}$$

Questions From Homework



Let $x =$ base

$$A = \frac{1}{2}bh$$

Given:

$$A = \frac{1}{2}xh$$

$$\frac{dx}{dt} = -20 \text{ cm/s}$$

$$A = \frac{1}{2}x\left(\frac{x\sqrt{3}}{2}\right)$$

$$\frac{dA}{dt} = ?$$

$$A = \frac{\sqrt{3}}{4}x^2$$

$$\frac{x}{h} = \frac{2}{\sqrt{3}}$$

$$2h = x\sqrt{3}$$

$$h = \frac{x\sqrt{3}}{2}$$

Find x :

$$A = \frac{\sqrt{3}}{4}x^2$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}x \frac{dx}{dt}$$

$$4 \cdot 100 = \frac{\sqrt{3}}{4}x^2 \cdot 4$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} \left(\frac{20}{\sqrt{3}}\right) (-20)$$

$$\frac{400}{\sqrt{3}} = \frac{\sqrt{3}x^2}{\sqrt{3}}$$

$$\frac{dA}{dt} = \frac{-20(3)^{\frac{3}{2}}}{(3)^{\frac{1}{2}}}$$

$$\frac{400}{\sqrt{3}} = x^2$$

$$\frac{dA}{dt} = -20(3)^{\frac{1}{2} - \frac{1}{4}}$$

$$\pm \sqrt{\frac{400}{\sqrt{3}}} = x$$

$$\frac{dA}{dt} = -20(3)^{\frac{1}{4}}$$

$$\frac{20}{\sqrt{3}} = x$$

$$15.2 \text{ cm} \approx x$$

$$\frac{dA}{dt} = -20\sqrt[4]{3} \text{ cm/s}$$

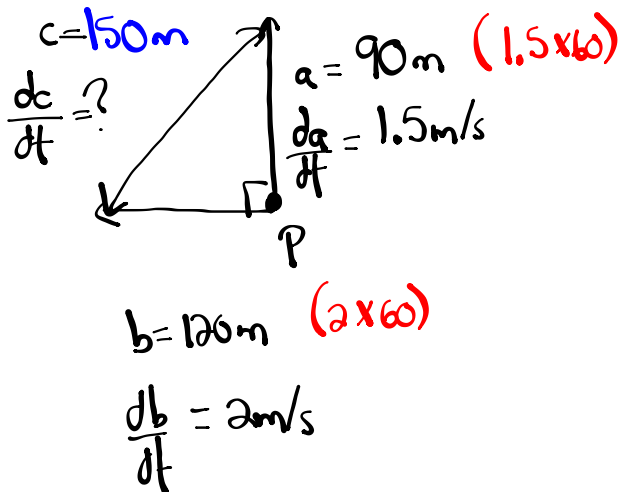
$$\frac{dA}{dt} \approx -26.3 \text{ cm/s}$$

Related Rates

A man starts jogging north at a rate of 1.5 m/s and a woman starts at the same point P , at the same time jogging west at a rate of 2 m/s. At what rate is the distance between the man and the woman increasing one minute later?

60 s.c.

(Hint: draw a diagram)



(i) Find c

$$a^2 + b^2 = c^2$$

$$(90)^2 + (120)^2 = c^2$$

$$8100 + 14400 = c^2$$

$$22500 = c^2$$

$$\underline{\underline{150m = c}}$$

(ii) Find $\frac{dc}{dt}$

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

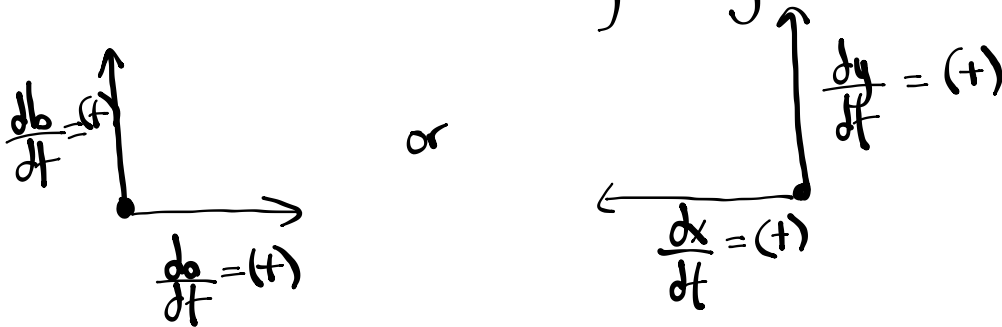
$$2(90)(1.5) + 2(120)(2) = 2(150) \frac{dc}{dt}$$

$$270 + 480 = 300 \frac{dc}{dt}$$

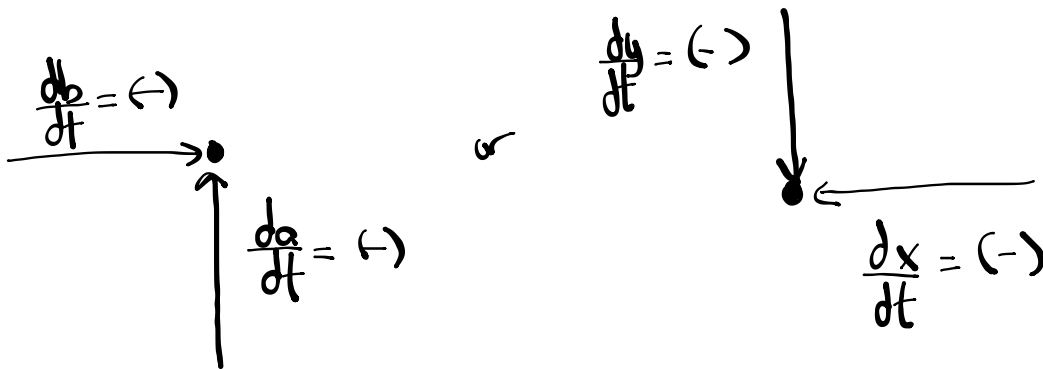
$$750 = 300 \frac{dc}{dt}$$

$2.5 \text{ m/s} = \frac{dc}{dt}$

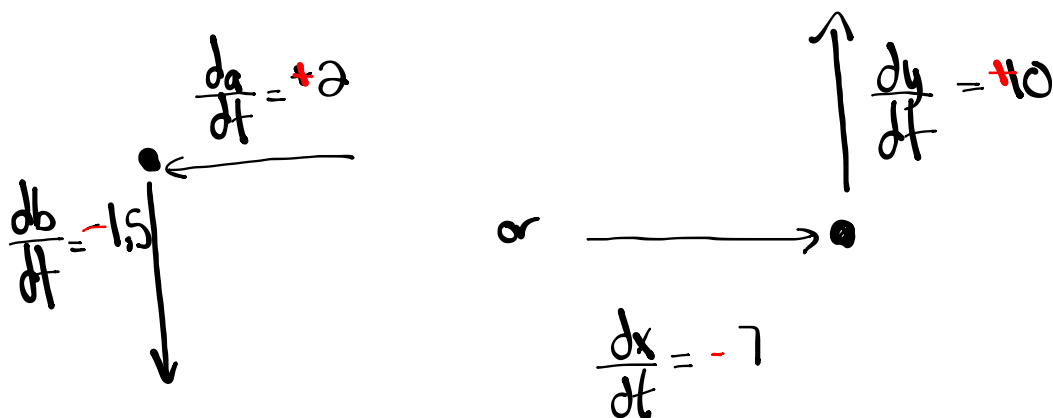
If both are travelling away from a common point



If both are travelling towards a common point

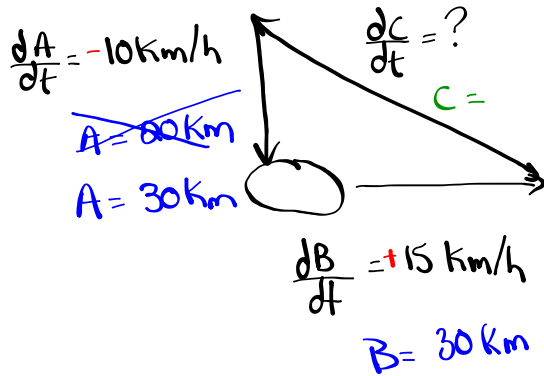


If one is travelling toward and the other is travelling away. The larger is positive and the smaller rate is negative



Ship A is 50 km north of Hawaii and sailing towards it (south) at 10 km/h. At the same time Ship B leaves Hawaii and sails east at 15 km/h. How fast is the distance between the ships changing 2 hours later?

(Hint: draw a diagram)



① Find c :

$$a^2 + b^2 = c^2$$

$$(30)^2 + (30)^2 = c^2$$

$$900 + 900 = c^2$$

$$1800 = c^2$$

$$\pm \sqrt{1800} = c$$

$$\sqrt{1800} = c$$

$$\sqrt{900 \cdot 2} = c$$

$$\underline{\underline{30\sqrt{2} = c}}$$

(ii) Find $\frac{dc}{dt}$:

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(30)(-10) + 2(30)(15) = 2(30\sqrt{2}) \frac{dc}{dt}$$

$$-600 + 900 = 60\sqrt{2} \frac{dc}{dt}$$

$$300 = 60\sqrt{2} \frac{dc}{dt}$$

$$\frac{300}{60\sqrt{2}} = \frac{dc}{dt}$$

$$\boxed{\frac{5}{\sqrt{2}} \text{ km/h} = \frac{dc}{dt}}$$

$$3.54 \text{ km/h} \approx \frac{dc}{dt}$$

Jack is headed south at 60 km/h towards JMH and Jill is headed west towards the school at 50 km/h. At what rate is the distance between them closing when Jack is 2 km and Jill is 3 km from the school?

(Hint: draw a diagram)

Homework