Correct Homework Sheet

② $\frac{1-\boxed{\cos 2\theta}}{\sin^2\theta} = 2$

$\frac{1-(\cos^2\theta - \sin^2\theta)}{\sin^2\theta}$

$\frac{\boxed{1-\cos^2\theta}+\sin^2\theta}{\sin^2\theta}$

$\frac{\sin^2\theta+\sin^2\theta}{\sin^2\theta}$

$\frac{2\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}$

$2$

③ $\boxed{\sin(x+y)}\boxed{\sin(x-y)} = \cos^2 y - \cos^2 x$

$(\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

$\sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$(1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)$

$\cos^2 y - \cancel{\cos^2 x\cos^2 y} - \cos^2 x + \cancel{\cos^2 x\cos^2 y}$

$\cos^2 y - \cos^2 x$

⑤ $\boxed{\tan^4\theta} = \boxed{\sec^4\theta}(1-2\cos^2\theta+\cos^4\theta)$

$\frac{\sin^4\theta}{\cos^4\theta}$

$\frac{1}{\cos^4\theta}(1-2\cos^2\theta+\cos^4\theta)$

$\frac{(\sin^2\theta)(\sin^2\theta)}{\cos^4\theta}$

$\boxed{\frac{1-2\cos^2\theta+\cos^4\theta}{\cos^4\theta}}$

$\frac{(1-\cos^2\theta)(1-\cos^2\theta)}{\cos^4\theta}$

$\boxed{\frac{1-2\cos^2\theta+\cos^4\theta}{\cos^4\theta}}$

7) $$\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\boxed{\sec\theta}$$

$$\frac{\cos^2\theta + (1+\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$\frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$$

$$\frac{2}{\cos\theta}$$

$$2\left(\frac{1}{\cos\theta}\right)$$

$$\frac{2}{\cos\theta}$$

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$$\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta - \cos^4\theta} = \frac{\boxed{\csc^2\theta}}{\boxed{\cot^2\theta}}$$

$$\frac{\boxed{(\sin^2\theta + \cos^2\theta)}\cancel{(\sin^2\theta - \cos^2\theta)}}{\cos^2\theta\cancel{(\sin^2\theta - \cos^2\theta)}}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

$$\frac{1}{\sin^2\theta} \div \frac{\cos^2\theta}{\sin^2\theta}$$

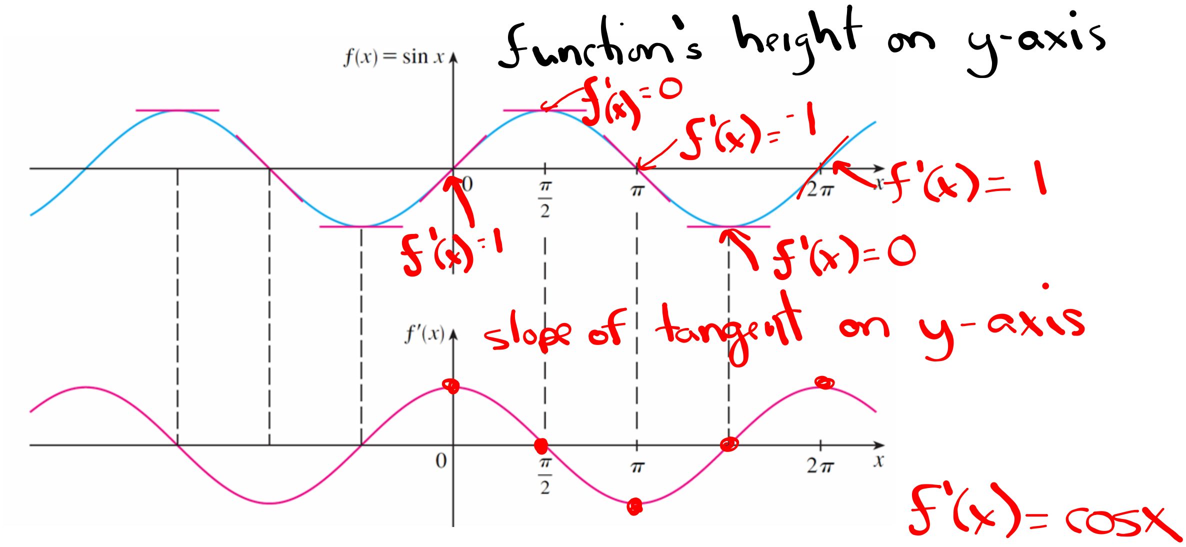
$$\frac{1}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}}{\cos^2\theta}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

## Derivatives of Trigonometric Functions

### The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that $x$ is measured in radians.
- The derivative graph resembles the graph of the cosine!




f'(x)

| x | f'(x) |
|---|---|
| 0 | 1 |
| π/2 | 0 |
| π | -1 |
| 3π/2 | 0 |
| 2π | 1 |

## Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[ \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[ \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[ \sin x \left( \frac{\cos h - 1}{h} \right) + \cos x \left( \frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since $x$ is constant while $h \to 0$,

    $$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$
- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

# Rules to differentiate trigonometric functions:

$y = \sin x$

$u = x$

$du = 1$

$\frac{dy}{dx} = \cos x \cdot 1$

**Given that "u" represents some differentiable function...**

$\frac{dy}{dx} = \cos x$

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

# Let's Practice...

Differentiate the following:

$y = \sin 3x$

$u = 3x$

$du = 3$

$y' = \cos u \cdot du$

$y' = \cos(3x) \cdot 3$

$\boxed{y' = 3\cos(3x)}$

$y = \sin(x+2)$

$u = x+2$

$du = 1$

$\boxed{y' = \cos(x+2)}$

$y = \sin(kx+d)$

$u = kx+d$

$du = k$

$\boxed{\frac{dy}{dx} = k\cos(kx+d)}$

# Ex #2.

Differentiate:

a) $y = \sin(x^3)$

$u = x^3$

$du = 3x^2$

$y' = \cos(x^3) \cdot 3x^2$

$\boxed{y' = 3x^2 \cos(x^3)}$

b) $y = \sin^3 x$

$u = x$

$du = 1$

$y = (\sin x)^3$

$\frac{dy}{dx} = 3(\sin x)^2 (\cos x \cdot 1)$

$\boxed{\frac{dy}{dx} = 3\sin^2 x \cos x}$

c) $y = \sin^3(x^2 - 1)$

$y = [\sin(x^2-1)]^3$

$u = x^2 - 1$

$du = 2x$

$y' = 3[\sin(x^2-1)]^2 \cdot \cos(x^2-1) \cdot 2x$

$\boxed{y' = 6x \sin^2(x^2-1)\cos(x^2-1)}$

## Ex #3.

Differentiate:

$$y = (x^2)(\cos x)$$

Product Rule

$u = x$
$du = 1$

$$\frac{dy}{dx} = 2x\cos x + x^2(-\sin x)(1)$$

$$\frac{dy}{dx} = 2x\cos x - x^2\sin x$$

$$\boxed{\frac{dy}{dx} = x(2\cos x - x\sin x)}$$

# Homework

Do Questions 1 and 3 from Exercise 7.2 Page 313

Worksheet on derivatives of trigonometric functions

① a) $y = \sin(\cos x)$ $u = \cos x$

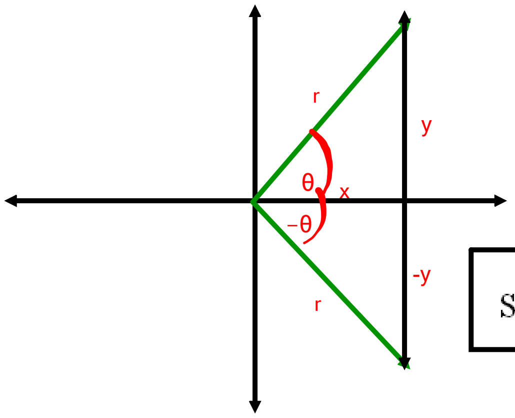
$du = -\sin x$

$\frac{dy}{dx} = \cos u \cdot du$

$\frac{dy}{dx} = \cos(\cos x) \cdot -\sin x$

$\frac{dy}{dx} = -\sin x \cos(\cos x)$

# Negative Angles




$\sin\theta = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$

$\sin(-\theta) = -\frac{y}{r}$ $\cos(-\theta) = \frac{x}{r}$

$\boxed{\sin(-\theta) = -\sin\theta}$ $\boxed{\cos(-\theta) = \cos\theta}$

Exercise 7.2

① a) $y = \cos(-4x)$ $u = -4x$, $du = -4$

$y' = -\sin u \cdot du$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$\boxed{y' = -4\sin(4x)}$

$y = \cos(-4x)$

$y = \cos(4x)$ $u = 4x$, $du = 4$

$y' = -\sin(4x) \cdot 4$

$\boxed{y' = -4\sin(4x)}$

Attachments

Derivatives Worksheet.doc