Correct Homework Sheet

② $\frac{1-\boxed{\cos 2\theta}}{\sin^2\theta} = 2$

$\frac{1-(\cos^2\theta - \sin^2\theta)}{\sin^2\theta}$

$\frac{\boxed{1-\cos^2\theta}+\sin^2\theta}{\sin^2\theta}$

$\frac{\sin^2\theta+\sin^2\theta}{\sin^2\theta}$

$\frac{2\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}$

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③ $\boxed{\sin(x+y)}\boxed{\sin(x-y)} = \cos^2 y - \cos^2 x$

$(\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

$\sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$(1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)$

$\cos^2 y - \cancel{\cos^2 x\cos^2 y} - \cos^2 x + \cancel{\cos^2 x\cos^2 y}$

$\cos^2 y - \cos^2 x$

⑤ $\boxed{\tan^4\theta} = \boxed{\sec^4\theta}(1-2\cos^2\theta+\cos^4\theta)$

$\frac{\sin^4\theta}{\cos^4\theta}$

$\frac{1}{\cos^4\theta}(1-2\cos^2\theta+\cos^4\theta)$

$\frac{(\sin^2\theta)(\sin^2\theta)}{\cos^4\theta}$

$\boxed{\frac{1-2\cos^2\theta+\cos^4\theta}{\cos^4\theta}}$

$\frac{(1-\cos^2\theta)(1-\cos^2\theta)}{\cos^4\theta}$

$\boxed{\frac{1-2\cos^2\theta+\cos^4\theta}{\cos^4\theta}}$

7. $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = 2\boxed{\sec\theta}$

$$\frac{\cos^2\theta + (1+\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$2\left(\frac{1}{\cos\theta}\right)$$

$$\frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{2}{\cos\theta}$$

$$\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$$

$$\frac{2}{\cos\theta}$$

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$$\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta - \cos^4\theta} = \frac{\boxed{\csc^2\theta}}{\boxed{\cot^2\theta}}$$

$$\frac{\boxed{(\sin^2\theta + \cos^2\theta)}\cancel{(\sin^2\theta - \cos^2\theta)}}{\cos^2\theta\cancel{(\sin^2\theta - \cos^2\theta)}}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

$$\frac{1}{\sin^2\theta} \div \frac{\cos^2\theta}{\sin^2\theta}$$

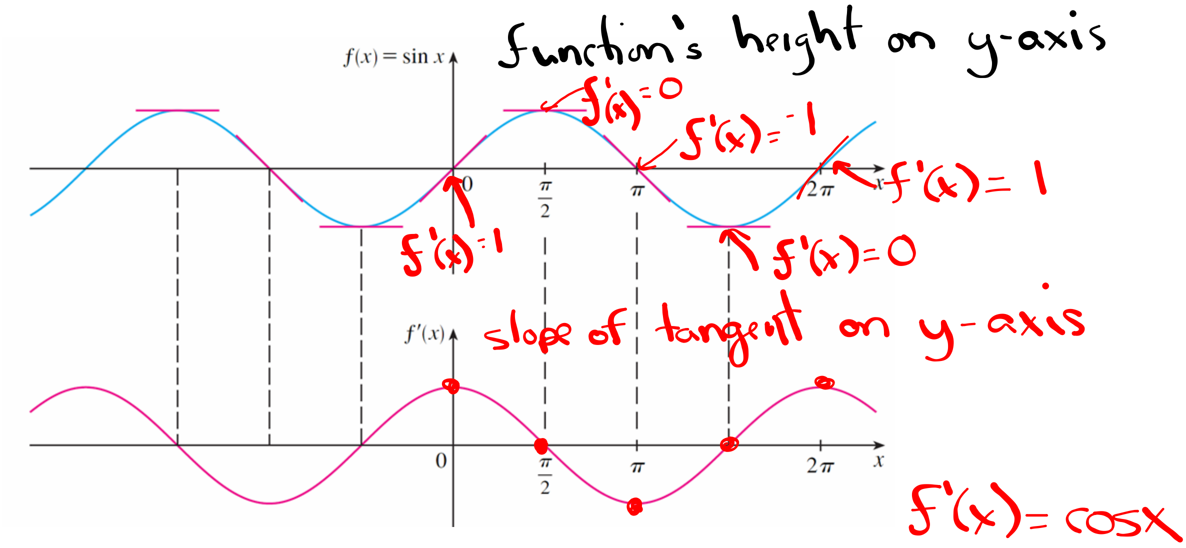
$$\frac{1}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}}{\cos^2\theta}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



$f'(x)$

x	$f'(x)$
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

$y = \sin x$

$u = x$

$du = 1$

$\frac{dy}{dx} = \cos x \cdot 1$

Given that "u" represents some differentiable function...

$\frac{dy}{dx} = \cos x$

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Let's Practice...

Differentiate the following:

$y = \sin 3x$

$u = 3x$

$du = 3$

$y' = \cos u \cdot du$

$y' = \cos(3x) \cdot 3$

$\boxed{y' = 3\cos(3x)}$

$y = \sin(x+2)$

$u = x + 2$

$du = 1$

$\boxed{y' = \cos(x+2)}$

$y = \sin(kx+d)$

$u = kx + d$

$du = k$

$\boxed{\frac{dy}{dx} = k\cos(kx+d)}$

Ex #2.

Differentiate:

a) $y = \sin(x^3)$

$u = x^3$

$du = 3x^2$

$y' = \cos(x^3) \cdot 3x^2$

$\boxed{y' = 3x^2 \cos(x^3)}$

$u = x$

$du = 1$

$y = (\sin x)^3$

b) $y = \sin^3 x$

$\frac{dy}{dx} = 3(\sin x)^2 (\cos x \cdot 1)$

$\boxed{\frac{dy}{dx} = 3\sin^2 x \cos x}$

c) $y = \sin^3(x^2 - 1)$

$y = [\sin(x^2-1)]^3$

$u = x^2 - 1$

$du = 2x$

$y' = 3[\sin(x^2-1)]^2 \cdot \cos(x^2-1) \cdot 2x$

$\boxed{y' = 6x \sin^2(x^2-1)\cos(x^2-1)}$

Ex #3.

Differentiate:

$$y = (x^2)(\cos x)$$

Product Rule

$u = x$
$du = 1$

$$\frac{dy}{dx} = 2x\cos x + x^2(-\sin x)(1)$$

$$\frac{dy}{dx} = 2x\cos x - x^2\sin x$$

$$\boxed{\frac{dy}{dx} = x(2\cos x - x\sin x)}$$

Homework

Do Questions 1 and 3 from Exercise 7.2 Page 313

Worksheet on derivatives of trigonometric functions

① a) $y = \sin(\cos x)$ $u = \cos x$

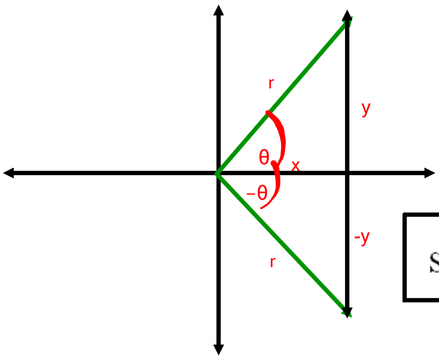
$du = -\sin x$

$$\frac{dy}{dx} = \cos u \cdot du$$

$$\frac{dy}{dx} = \cos(\cos x) \cdot -\sin x$$

$$\frac{dy}{dx} = -\sin x \cos(\cos x)$$

Negative Angles



$\sin\theta = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$

$\sin(-\theta) = -\frac{y}{r}$ $\cos(-\theta) = \frac{x}{r}$

$\boxed{\sin(-\theta) = -\sin\theta}$ $\boxed{\cos(-\theta) = \cos\theta}$

Exercise 7.2

① a) $y = \cos(-4x)$ $u = -4x$, $du = -4$

$y' = -\sin u \cdot du$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$\boxed{y' = -4\sin(4x)}$

$y = \cos(-4x)$

$y = \cos(4x)$ $u = 4x$, $du = 4$

$y' = -\sin(4x) \cdot 4$

$\boxed{y' = -4\sin(4x)}$

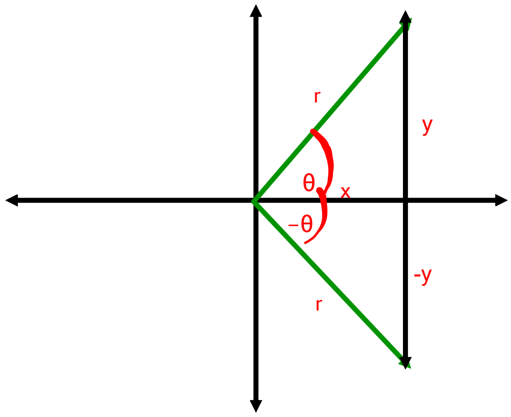
Example:

$y = \sin(-3x)$ $u = -3x$, $du = -3$

$y' = \cos(-3x) \cdot -3$

$y' = -3\cos(-3x)$

$y' = -3\cos(3x)$



$\csc\theta = \frac{r}{y}$ $\cot\theta = \frac{x}{y}$

$\csc(-\theta) = -\frac{r}{y}$ $\cot(-\theta) = -\frac{x}{y}$

$\csc(-\theta) = -\csc\theta$ $\cot(-\theta) = -\cot\theta$

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$u = -8x$
$du = -8$

① a) $y = -\frac{1}{4}\csc(-8x)$

$$\frac{dy}{dx} = -\frac{1}{4}\left(-\csc(-8x)\cot(-8x)\right)\cdot -8$$

$$\frac{dy}{dx} = -2\csc(-8x)\cot(-8x)$$

$$\boxed{\frac{dy}{dx} = -2\csc(8x)\cot(8x)}$$

Questions from Homework

① i) $y = 3\sin^4(2-x)^{-1}$

$y = 3\left[\sin(2-x)^{-1}\right]^4$

$u = (2-x)^{-1}$

$du = -(2-x)^{-2}(-1)$

$du = (2-x)^{-2}$

$\frac{dy}{dx} = 12\left[\sin(2-x)^{-1}\right]^3 \left(\cos(2-x)^{-1}\right) \cdot (2-x)^{-2}$

$\frac{dy}{dx} = \frac{12\sin^3(2-x)^{-1}\cos(2-x)^{-1}}{(2-x)^2}$

Questions from Homework

① m) $y = (1+\cos^2 x)^6$ $\quad (\cos x)^2$

$\frac{dy}{dx} = 6(1+\cos^2 x)^5 (2(\cos x)(-\sin x))$

$\frac{dy}{dx} = -6(1+\cos^2 x)^5 (\underline{2\sin x \cos x})$ Double Angle Identity

$\frac{dy}{dx} = -6(1+\cos^2 x)^5 (\underline{\sin 2x})$

n) $y = \sin\left(\frac{1}{x}\right) = \sin(x^{-1})$

$y' = \cos(x^{-1}) \cdot (-x^{-2})$

$y' = \cos\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2}$

$y' = -\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

Questions from Homework

① u) $y = \cos^2\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)$ $u = \dfrac{1-\sqrt{x}}{1+\sqrt{x}}$ du: quotient rule

$$y = \left[\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right]^2$$

$$\frac{dy}{dx} = 2\left[\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right]\left(-\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right)\left[\frac{-\frac{1}{2\sqrt{x}}(1+\sqrt{x}) - \frac{1}{2\sqrt{x}}(1-\sqrt{x})}{(1+\sqrt{x})^2}\right]$$

$$\frac{dy}{dx} = -2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\cdot\left[\frac{-\frac{(1+\sqrt{x})}{2\sqrt{x}} - \frac{(1-\sqrt{x})}{2\sqrt{x}}}{(1+\sqrt{x})^2}\right]$$

$$\frac{dy}{dx} = -2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left[\frac{\frac{-1-\cancel{\sqrt{x}}-1+\cancel{\sqrt{x}}}{2\sqrt{x}}}{(1+\sqrt{x})^2}\right]$$

$$\frac{dy}{dx} = -2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left[\frac{\frac{-\cancel{2}}{\cancel{2}\sqrt{x}}}{(1+\sqrt{x})^2}\right]$$

$$\frac{dy}{dx} = -2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left[\frac{\frac{-1}{\cancel{\sqrt{x}}}\cancel{\sqrt{x}}}{(1+\sqrt{x})^2\sqrt{x}}\right]$$

$$\frac{dy}{dx} = \underbrace{2\cos\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)}_{\text{Double Angle}}\left[\frac{1}{\sqrt{x}(1+\sqrt{x})^2}\right]$$

$$\boxed{\frac{dy}{dx} = \sin 2\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left[\frac{1}{\sqrt{x}(1+\sqrt{x})^2}\right]}$$

Questions from Homework

$u = y$
$du = \frac{dy}{dx}$

$u = 2x$
$du = 2$

② a) $\sin y = \cos(2x)$

$\cos y \cdot \frac{dy}{dx} = -\sin(2x) \cdot 2$

$\cos y \frac{dy}{dx} = -2\sin(2x)$

$\frac{dy}{dx} = \frac{-2\sin 2x}{\cos y}$

Questions from Homework

③ b) $y = \frac{\sin x}{\cos x}$ @ $\left(\frac{\pi}{4}, 1\right)$ $x_1 = \frac{\pi}{4}$ $y_1 = 1$

(i) Find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{1}{\cos^2 x}$$

(ii) Find m:

$$\frac{dy}{dx} = \frac{1}{\cos^2(\pi/4)}$$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\frac{dy}{dx} = \frac{1}{1/2}$$

$$\frac{dy}{dx} = \underline{\underline{2}}$$

$m = 2$

(iii) Find equation:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2\left(x - \frac{\pi}{4}\right)$$

$$y - 1 = 2x - \frac{\pi}{2} \rightarrow \boxed{y = 2x - \frac{\pi}{2} + 1}$$

$$2y - 2 = 4x - \pi$$

$$\boxed{0 = 4x - 2y + 2 - \pi}$$

or

$$y = \frac{\sin x}{\cos x}$$

$$y = \tan x$$

$$\frac{dy}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = \sec^2(\pi/4)$$

$$\frac{dy}{dx} = (\sqrt{2})^2$$

$$\frac{dy}{dx} = 2$$

Let's Practice...

Differentiate the following:

$$f(x) = \frac{1}{1+\tan x}$$

$$f(x) = (1+\tan x)^{-1}$$

$$f'(x) = -1(1+\tan x)^{-2}(\sec^2 x)$$

$$\boxed{f'(x) = \frac{-\sec^2 x}{(1+\tan x)^2}}$$

Differentiate:

$$f(x) = 2\csc^3(3x^2)$$

$-\csc u \cot u \cdot du$

$-\csc(3x^2)\cot(3x^2) \cdot 6x$

$f(x) = 2[\csc(3x^2)]^3$

$u = 3x^2$

$du = 6x$

$f'(x) = 6[\csc(3x^2)]^2(-\csc(3x^2)\cot(3x^2) \cdot 6x)$

$f'(x) = -36x \csc^2(3x^2)\csc(3x^2)\cot(3x^2)$

$$\boxed{f'(x) = -36x \csc^3(3x^2)\cot(3x^2)}$$

Homework

Worksheet on derivatives of trigonometric functions

Page 314 #3 c,e Ex (7.2)

Page 319 #1 Ex (7.3)

③ a) Find the equation of the tangent line to the given curve at the given point.

a) $y = 2\sin x$ at $(\pi/6, 1)$ $x_1 = \frac{\pi}{6}$ $y_1 = 1$

(i) Find the derivative: (ii) Find m: (iii) Find equation:

$y = 2\sin x$

$y' = 2\cos x$

$y' = 2\cos x$

$y' = 2\cos(\pi/6)$

$y' = 2\left(\frac{\sqrt{3}}{2}\right)$

$y' = \sqrt{3}$

$m = \sqrt{3}$

$y - y_1 = m(x - x_1)$

$y - 1 = \sqrt{3}\left(x - \frac{\pi}{6}\right)$

$y - 1 = x\sqrt{3} - \frac{\pi\sqrt{3}}{6}$

$6y - 6 = 6x\sqrt{3} - \pi\sqrt{3}$

$\boxed{0 = 6x\sqrt{3} - 6y + 6 - \pi\sqrt{3}}$

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③ c) $y = \frac{1}{\cos x} - 2\cos x$ @ $\left(\frac{\pi}{3}, 1\right)$

$y = \sec x - 2\cos x$

$y' = \sec x \tan x - 2(-\sin x)$

$y' = \sec x \tan x + 2\sin x$

$y' = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) + 2\sin\left(\frac{\pi}{3}\right)$

$y' = (2)(\sqrt{3}) + 2\left(\frac{\sqrt{3}}{2}\right)$

$y' = 2\sqrt{3} + \sqrt{3}$

$y' = 3\sqrt{3}$

$m = 3\sqrt{3}$

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③ c) $y = \frac{1}{\cos x} - 2\cos x$ @ $\left(\frac{\pi}{3}, 1\right)$

$y = (\cos x)^{-1} - 2\cos x$

$\frac{dy}{dx} = -(\cos x)^{-2}(-\sin x) - 2(-\sin x)$

$\frac{dy}{dx} = \frac{\sin x}{\cos^2 x} + 2\sin x$

$\frac{dy}{dx} = \frac{\sin(\pi/3)}{\cos^2(\pi/3)} + 2\sin\left(\frac{\pi}{3}\right)$

$\frac{dy}{dx} = \frac{\left(\sqrt{3}/2\right)}{\left(1/2\right)^2} + 2\left(\frac{\sqrt{3}}{2}\right)$

$\frac{dy}{dx} = \frac{\sqrt{3}}{2} \div \frac{1}{4} + \sqrt{3}$

$\frac{dy}{dx} = \frac{\sqrt{3}}{2} \cdot \frac{4}{1} + \sqrt{3}$

$\frac{dy}{dx} = 2\sqrt{3} + \sqrt{3}$

$\frac{dy}{dx} = 3\sqrt{3}$

$m = 3\sqrt{3}$

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3e) $y = \sin x + \cos 2x$ @ $\left(\frac{\pi}{6}, 1\right)$

$$\frac{dy}{dx} = \cos x - \sin 2x \cdot 2$$

$$\frac{dy}{dx} = \cos x - 2\sin 2x$$

$$\frac{dy}{dx} = \cos\left(\frac{\pi}{6}\right) - 2\sin\left(2 \cdot \frac{\pi}{6}\right)$$

$$\frac{dy}{dx} = \cos\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = \frac{\sqrt{3}}{2} - 2\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{dy}{dx} = \frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}$$

$$\frac{dy}{dx} = \frac{-\sqrt{3}}{2}$$

$$m = \frac{-\sqrt{3}}{2}$$

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① i) $y = \cot^3(1-2x)^2$

$y = [\cot(1-2x)^2]^3$

$u = (1-2x)^2$

$du = 2(1-2x)(-2)$
$= -4(1-2x)$

$$\frac{dy}{dx} = 3[\cot(1-2x)^2]^2(-\csc^2(1-2x)^2)(-4(1-2x))$$

$$\boxed{\frac{dy}{dx} = 12(1-2x)\cot^2(1-2x)^2\csc^2(1-2x)^2}$$

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① j) $y = \sec^2 x - \tan^2 x$

$y = \cancel{\tan^2 x} + 1 - \cancel{\tan^2 x}$

$y = 1$

$\frac{dy}{dx} = 0$

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j) $y = \sec^2 x - \tan^2 x$

$y = (\sec x)^2 - (\tan x)^2$

$\frac{dy}{dx} = 2(\sec x)(\sec x \tan x) - 2(\tan x)(\sec^2 x)$

$\frac{dy}{dx} = 2\sec^2 x \tan x - 2\sec^2 x \tan x$

$\frac{dy}{dx} = 0$

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$u = 2x$ $\quad$ $u = 3y$

$du = 2$ $\quad$ $du = 3\frac{dy}{dx}$

2 b) $\tan 2x = \cos 3y$

$$\sec^2(2x) \cdot 2 = -\sin(3y) \cdot 3\frac{dy}{dx}$$

$$2\sec^2(2x) = -3\sin(3y)\frac{dy}{dx}$$

$$\frac{2\sec^2(2x)}{-3\sin(3y)} = \frac{dy}{dx}$$

Attachments

Derivatives Worksheet.doc