

Final Review:

$$\textcircled{1} \text{ a) } f(x) = \sqrt{x-5} \qquad f(x+h) = \sqrt{x+h-5}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h-5} - \sqrt{x-5}) (\sqrt{x+h-5} + \sqrt{x-5})}{h (\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x+h-5 - (x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{x+h-5} - \cancel{x-5}}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-5} + \sqrt{x-5})} = \frac{1}{2\sqrt{x-5}}$$

Final Review

$$= \frac{2(x+h)-2}{(x+h)+3}$$

$$\textcircled{1} \text{ b) } f(x) = \frac{2x-2}{x+3}$$

$$f(x+h) = \frac{2x+2h-2}{x+h+3}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{2x+2h-2}{x+h+3} - \frac{2x-2}{x+3}}{h}$$

CD: $(x+3)(x+h+3)$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + \cancel{6h} - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - \cancel{2h} - \cancel{6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{8h}}{\cancel{h}(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

Final Review:

$$\textcircled{a) } f(x) = 3x^2 + 5x - 2$$

$$f'(x) = 6x + 5$$

$$\textcircled{b) } f(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$$

$$f'(x) = \frac{-3}{2} x^{-3/2} = \frac{-3}{2x^{3/2}}$$

$$\textcircled{c) } f(x) = 2x^4 + \sqrt{x}$$

$$f(x) = 2x^4 + x^{1/2}$$

$$f'(x) = 8x^3 + \frac{1}{2} x^{-1/2}$$

$$f'(x) = 8x^3 + \frac{1}{2x^{1/2}}$$

$$\textcircled{d) } f(x) = \sqrt[3]{x^2} = x^{2/3}$$

$$f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$$

Final Review:

$$\textcircled{2} \text{ a) } y = (3x^2 - 2)(4x + 5)$$

$$y' = 6x(4x + 5) + 4(3x^2 - 2)$$

$$y' = 24x^2 + 30x + 12x^2 - 8$$

$$y' = 36x^2 + 30x - 8$$

$$\text{b) } g(x) = (x^2 - 5x + 2)(4x + 1)$$

$$g'(x) = (2x - 5)(4x + 1) + 4(x^2 - 5x + 2)$$

$$g'(x) = 8x^2 + 2x - 20x - 5 + 4x^2 - 20x + 8$$

$$g'(x) = 12x^2 - 38x + 3$$

Final Review:

$$\textcircled{4} \text{ a) } f(x) = \frac{2x^2+3}{3x-2}$$

$$f'(x) = \frac{4x(3x-2) - 3(2x^2+3)}{(3x-2)^2}$$

$$f'(x) = \frac{12x^2 - 8x - 6x^2 - 9}{(3x-2)^2}$$

$$f'(x) = \frac{6x^2 - 8x - 9}{(3x-2)^2}$$

$$\text{b) } y = \frac{\sqrt{x}}{3+x^2}$$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - 2x(\sqrt{x})}{(3+x^2)^2}$$

$$y' = \frac{\frac{\cancel{2\sqrt{x}}}{\cancel{2\sqrt{x}}} \frac{3+x^2}{\cancel{2\sqrt{x}}} - 2x^{3/2} \cdot \cancel{2\sqrt{x}}}{\cancel{2\sqrt{x}}(3+x^2)^2}$$

$$y' = \frac{3+x^2-4x^2}{2\sqrt{x}(3+x^2)^2} = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$$

Final Review:

$$\textcircled{5} \quad y = (x^2 - 3)^8 \quad \text{at } x = \underline{2}$$

(i) Find y :

$$y = (2^2 - 3)^8$$

$$y = (4 - 3)^8$$

$$y = \underline{1}$$

(ii) Find y' :

$$y = (x^2 - 3)^8$$

$$y' = 8(x^2 - 3)^7 (2x)$$

$$y' = 16x(x^2 - 3)^7$$

(iii) Find m :

$$y' = 16(2)(2^2 - 3)^7$$

$$y' = 32(1)^7$$

$$y' = \underline{32} \leftarrow m$$

(iv) Find equation:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 32(x - 2)$$

$$y - 1 = 32x - 64$$

$$\boxed{0 = 32x - y - 63}$$

Final Review:

$$\textcircled{6} \text{ a) } f(x) = 3(2x^2 - 4)^4$$
$$f'(x) = 12(2x^2 - 4)^3(4x)$$
$$f'(x) = 48x(2x^2 - 4)^3$$

$$\text{b) } f(x) = \frac{16}{\sqrt{x-1}} = 16(x-1)^{-1/2}$$
$$f'(x) = -8(x-1)^{-3/2}(1)$$
$$f'(x) = \frac{-8}{(x-1)^{3/2}}$$

Final Review:

$$\textcircled{1} \text{ a) } y = \left[\frac{2x+1}{x-1} \right]^5$$

$$y' = 5 \left[\frac{2x+1}{x-1} \right]^4 \left[\frac{2x-2 - 1(2x+1)}{(x-1)^2} \right]$$

$$y' = 5 \cdot \frac{(2x+1)^4}{(x-1)^4} \cdot \frac{-3}{(x-1)^2} = \frac{-15(2x+1)^4}{(x-1)^6}$$

$$\text{b) } y = (x^2-1)^3(3x-2)^2$$

$$y' = 3(x^2-1)^2(2x)(3x-2)^2 + (x^2-1)^3(2)(3x-2)(3)$$

$$y' = 6x(x^2-1)^2(3x-2)^2 + 6(x^2-1)^3(3x-2)$$

$$y' = 6(x^2-1)^2(3x-2) \left[x(3x-2) + (x^2-1) \right]$$

$$y' = 6(x^2-1)^2(3x-2)(4x^2-2x-1)$$

$$\text{c) } y = \frac{(2x+1)^2}{(x^4-x+1)^2}$$

$$y' = \frac{2(2x+1)(2)(x^4-x+1)^2 - (2x+1)^2(2)(x^4-x+1)(4x^3-1)}{(x^4-x+1)^4}$$

$$y' = \frac{4(2x+1)(x^4-x+1)^2 - 2(4x^3-1)(2x+1)^2(x^4-x+1)}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)(x^4-x+1) \left[2x^4-2x+2 - (8x^3+4x^2-2x-1) \right]}{(x^4-x+1)^3}$$

$$y' = \frac{2(2x+1)(-6x^3-4x^2+3)}{(x^4-x+1)^3}$$

Final Review:

$$\textcircled{8} \text{ a) } f(x) = \sin^3 x + \cos^3 x$$

$$f(x) = (\sin x)^3 + (\cos x)^3$$

$$f'(x) = 3(\sin x)^2(\cos x) + 3(\cos x)^2(-\sin x)$$

$$f'(x) = 3\sin^2 x \cos x - 3\sin x \cos^2 x$$

$$f'(x) = 3\sin x \cos x (\sin x - \cos x)$$

$$\text{b) } y = 3\sec(2x^2 + 1)$$

$$u = 2x^2 + 1$$

$$du = 4x$$

$$y' = 3\sec(2x^2 + 1)\tan(2x^2 + 1) \cdot 4x$$

$$y' = 12x \sec(2x^2 + 1)\tan(2x^2 + 1)$$

Do Ex. 2.9 on page 112 of Text
Questions 1, 4, 6, 9, 11, 12, 14

↑
a, d

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③ Find $\frac{dy}{dx}$ at $x=4$ if $y = u^2 - 2u^5$ and $u = x - \sqrt{x}$

① Find u $u = x - \sqrt{x}$ $u = (4) - \sqrt{4}$ $u = 2$	② Find $\frac{dy}{du}$ $y = u^2 - 2u^5$ $\frac{dy}{du} = 2u - 10u^4$	③ Find $\frac{du}{dx}$ $u = x - x^{1/2}$ $\frac{du}{dx} = 1 - \frac{1}{2}x^{-1/2}$
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$$\begin{aligned} \frac{dy}{dx} \Big|_{x=4} &= (2u - 10u^4) \left(1 - \frac{1}{2\sqrt{x}}\right) \\ &= (2(2) - 10(2)^4) \left(1 - \frac{1}{2\sqrt{4}}\right) \\ &= (4 - 160) \left(1 - \frac{1}{4}\right) \\ &= (-156) \left(\frac{3}{4}\right) \\ &= \frac{-468}{4} \\ &= -117 \end{aligned}$$

⑨ If $F(x) = f(g(x))$, where $\begin{matrix} g(2) = 4 \\ g'(2) = 3 \\ f'(4) = 5 \end{matrix}$ find $F'(2)$

$$\begin{aligned} F'(x) &= f'(g(x)) \cdot g'(x) \\ F'(2) &= f'(g(2)) \cdot g'(2) \\ F'(2) &= f'(4) \cdot g'(2) \\ F'(2) &= 5 \cdot 3 = \underline{\underline{15}} \end{aligned}$$

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$$\textcircled{4} \text{ c) } y = \sqrt{x}(5 - \sqrt{x}) = 5\sqrt{x} - x = 5x^{1/2} - x$$

$$y' = \frac{5}{2}x^{-1/2} - 1$$

$$y' = \frac{5}{2\sqrt{x}} - 1 = \frac{5 - 2\sqrt{x}}{2\sqrt{x}}$$

Using Product:

$$y = \sqrt{x}(5 - \sqrt{x})$$

$$y' = \frac{1}{2}x^{-1/2}(5 - \sqrt{x}) + x^{1/2}\left(-\frac{1}{2}x^{-1/2}\right)$$

$$y' = \frac{1}{2\sqrt{x}}(5 - \sqrt{x}) + \sqrt{x}\left(-\frac{1}{2\sqrt{x}}\right)$$

$$y' = \frac{5}{2\sqrt{x}} - \frac{\sqrt{x}}{2\sqrt{x}} - \frac{\sqrt{x}}{2\sqrt{x}}$$

$$y' = \frac{5}{2\sqrt{x}} - \frac{2\sqrt{x}}{2\sqrt{x}}$$

$$y' = \frac{5}{2\sqrt{x}} - 1$$

$$\textcircled{5} \text{ f) } y = \frac{x^2 - 2x}{\sqrt{x}}$$

$$y' = \frac{x^2(x^2 - 2x) - \frac{1}{2}x^2(x^2 - 2x)}{x^2}$$

$$y' = \frac{2x^{3/2} - 2x^{1/2} - \frac{1}{2}(x^2 - 2x) \cdot 2x^{1/2}}{x \cdot 2x^{1/2}}$$

$$y' = \frac{4x^2 - 4x - x^2 + 2x}{2x^{3/2}} = \frac{3x^2 - 2x}{2x^{3/2}}$$

$$= \frac{x(3x - 2)}{2x^{3/2}}$$

$$= \frac{x^{-1/2}(3x - 2)}{2}$$

$$= \boxed{\frac{3x - 2}{2\sqrt{x}}}$$

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① d) $y = x\sqrt{x^2+5}$, $(-2, -6)$ $x_1 = -2$
 $y_1 = -6$

① Find y'

$$y' = \sqrt{x^2+5} + x \left[\frac{1}{2} (x^2+5)^{-1/2} \cdot 2x \right]$$

$$y' = \sqrt{x^2+5} + \frac{x^2}{\sqrt{x^2+5}}$$

② Find m or $y'(-2)$

$$y'(-2) = \sqrt{(-2)^2+5} + \frac{(-2)^2}{\sqrt{(-2)^2+5}}$$

$$y' = 3 + \frac{4}{3} = \boxed{\frac{13}{3}}$$

③ Find equation:

$$y + 6 = \frac{13}{3}(x + 2)$$

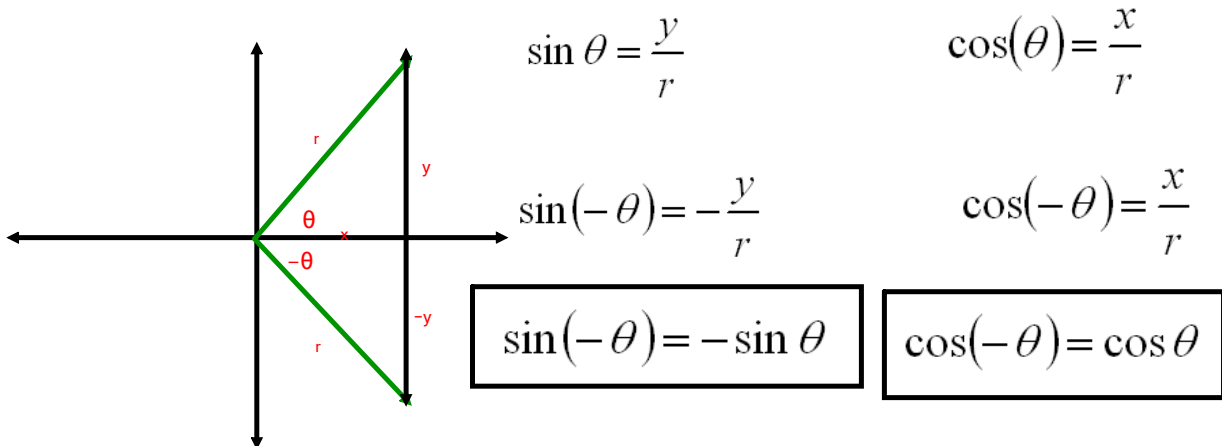
$$y + 6 = \frac{13x}{3} + \frac{26}{3} - 6$$

$$y = \frac{13x}{3} + \frac{8}{3}$$

$$3y = 13x + 8$$

$$0 = 13x - 3y + 8$$

Negative Angles



Ex: 7.2

① a) $y = \cos(-4x)$
 $y' = -\sin(-4x) \cdot -4$
 $y' = 4\sin(-4x)$
 $y' = -4\sin(4x)$

Ex: 7.3

① d) $y = -\frac{1}{4}\csc(-8x)$
 $y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$
 $y' = -2\csc(-8x)\cot(-8x)$
 $y' = -2(-\csc 8x)(-\cot 8x)$
 $y' = -2\csc(8x)\cot(8x)$

Ex 7.2

$$\textcircled{1} \text{ a) } y = \sin(\cos x)$$

$$y' = \cos(\cos x) (-\sin x)$$

$$y' = -\sin x \cos(\cos x)$$

$$\text{p) } y = \cos^3(\sin x) = [\cos(\sin x)]^3$$

$$y' = 3[\cos(\sin x)]^2 (-\sin(\sin x)) \cos x$$

$$y' = -3 \cos^2(\sin x) \sin(\sin x) \cos x$$

$$\text{q) } y = x \cos \frac{1}{x} = (x)(\cos x^{-1})$$

$$y' = \cos x^{-1} + x (-\sin x^{-1}) (-x^{-2})$$

$$y' = \cos\left(\frac{1}{x}\right) + \frac{1}{x} \sin\left(\frac{1}{x}\right)$$

$$\text{s) } y = \frac{1 + \sin x}{1 - \sin x}$$

$$y' = \frac{\cos x (1 - \sin x) - (1 + \sin x)(-\cos x \cdot 1)}{(1 - \sin x)^2}$$

$$y' = \frac{\cos x - \cos x \sin x + 2 \cos x (1 + \sin x)}{(1 - \sin x)^2}$$

$$y' = \frac{\cos x - \cos x \sin x + 2 \cos x + 2 \sin x \cos x}{(1 - \sin x)^2}$$

Ex 7.3

$$\textcircled{1} \text{ a) } y = 3 \tan 2x$$

$$y' = 3 \sec^2(2x) \cdot 2$$

$$y' = 6 \sec^2(2x)$$

$$\text{g) } y = \sec^3 \sqrt{x} = \sec(x^{1/3})$$

$$y' = \sec(x^{1/3}) \tan(x^{1/3}) \cdot \frac{1}{3} x^{-2/3}$$

$$y' = \sec(x^{1/3}) \tan(x^{1/3}) \cdot \frac{1}{3x^{2/3}}$$

$$y' = \frac{\sec^3 \sqrt{x} \tan^3 \sqrt{x}}{3 \sqrt{x^2}}$$

7.3

$$\textcircled{1} \text{ d) } y = -\frac{1}{4} \csc(-8x)$$

$$y' = -\frac{1}{4} (-\csc(-8x) \cot(-8x)) \cdot -8$$

$$y' = -2 \csc(-8x) \cot(-8x)$$

$$y' = -2 \csc(8x) \cot(8x)$$

* Negative Angle
Identity

$$\text{m) } y = 2x(\sqrt{x} - \cot x)$$

$$y = 2x^{3/2} - 2x \cot x$$

$$y' = 3x^{1/2} - [2 \cot x + 2x(-\csc^2 x \cdot 1)]$$

$$y' = 3\sqrt{x} - 2 \cot x + 2x \csc^2 x$$

$$\text{Ex: } y = \sin(x^2) \quad u = x^2$$

$$y' = \cos(x^2) \cdot 2x$$

$$y' = 2x \cos(x^2)$$

$$\text{① n) } y = \sin(\tan x) \quad u = \tan x$$

$$y' = \cos(\tan x) \cdot \sec^2 x$$

$$y' = \sec^2 x [\cos(\tan x)]$$

$$\text{K) } y = \frac{1}{\sqrt{(\sec 2x - 1)^3}} = \frac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$$

$$y' = -\frac{3}{2} (\sec 2x - 1)^{-5/2} \cdot \sec 2x \tan 2x \cdot (2)$$

$$y' = \frac{-3 \sec(2x) \tan(2x)}{(\sec 2x - 1)^{5/2}}$$

$$y' = \frac{-3 \sec(2x) \tan(2x)}{\sqrt{(\sec 2x - 1)^5}}$$

$$\textcircled{1} \text{ a) } y = \tan^2(\cos x) = [\tan(\cos x)]^2$$

$$y' = 2[\tan(\cos x)] \cdot \sec^2(\cos x) \cdot (-\sin x)(1)$$

$$y' = -2\sin x [\tan(\cos x)] [\sec^2(\cos x)]$$

$$\text{b) } y = \frac{x^2 \tan x}{\sec x} = x^2 \left(\frac{\sin x}{\cos x} \right) \cdot \left(\frac{\cos x}{1} \right) = x^2 \sin x$$

$$y' = x^2 (\cos x)(1) + 2x (\sin x)$$

$$y' = x^2 \cos x + 2x \sin x$$

$$y' = x [x \cos x + 2 \sin x]$$

$$h) y = (x^2)(\csc x)$$

$$y' = (x^2)(-\csc x \cot x)(1) + 2x \csc x$$

$$y' = -x^2 \csc x \cot x + 2x \csc x$$

$$y' = x \csc x (-x \cot x + 2)$$

$$y' = x \csc x (2 - x \cot x)$$

$$u = (1-2x)^2$$

$$du = 2(1-2x)(-2)$$

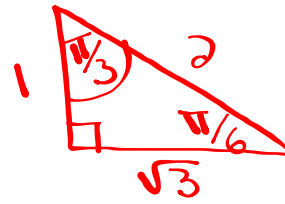
$$i) y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$$

$$y' = 3[\cot(1-2x)^2]^2 [-\csc^2(1-2x)^2] [2(1-2x)(-2)]$$

$$y' = 3 \cot^2(1-2x)^2 [\csc^2(1-2x)^2] [-4(1-2x)]$$

$$y' = 12(1-2x) \cot^2(1-2x)^2 \csc^2(1-2x)^2$$

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③ c) Find equation of tangent \rightarrow

$$y = \frac{1}{\cos x} - 2\cos x \quad \text{at} \quad \left(\frac{\pi}{3}, 1\right) \quad \begin{array}{l} x_1 = \frac{\pi}{3} \\ y_1 = 1 \end{array}$$

① find y'

$$y = (\cos x)^{-1} - 2\cos x$$

$$y' = -1(\cos x)^{-2}(-\sin x) - 2(-\sin x)$$

$$y' = \frac{\sin x}{\cos^2 x} + 2\sin x$$

② Find m or $y'(\frac{\pi}{3})$

$$y'(\frac{\pi}{3}) = \frac{\sin(\frac{\pi}{3})}{\cos^2(\frac{\pi}{3})} + 2\sin(\frac{\pi}{3})$$

$$y'(\frac{\pi}{3}) = \frac{(\frac{\sqrt{3}}{2})}{(\frac{1}{2})^2} + 2(\frac{\sqrt{3}}{2})$$

$$y'(\frac{\pi}{3}) = \frac{\sqrt{3}}{1} \cdot \frac{4}{1} + \sqrt{3}$$

$$y'(\frac{\pi}{3}) = 2\sqrt{3} + \sqrt{3} = \boxed{3\sqrt{3}}$$

③ Find equation:

$$y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$y - 1 = 3x\sqrt{3} - \pi\sqrt{3}$$

$$0 = 3x\sqrt{3} - y - \pi\sqrt{3} + 1$$

⑥ Find the points on the curve $y = \frac{x}{x-1}$ where the tangent line is parallel to the line $x+4y=1$

$$\textcircled{1} \quad x+4y=1$$

$$4y = -x+1$$

$$y = \left(-\frac{1}{4}\right)x + \frac{1}{4}$$

$$m = -\frac{1}{4}$$

$$\textcircled{2} \quad y = \frac{x}{x-1}$$

$$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$$

$$y' = \frac{-1}{(x-1)^2}$$

$$\textcircled{3} \quad \frac{-1}{(x-1)^2} = -\frac{1}{4}$$

$$-(x-1)^2 = -4$$

$$(x-1)^2 = 4$$

$$x^2 - 2x + 1 = 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\begin{array}{l|l} x-3=0 & x+1=0 \\ x=3 & x=-1 \end{array}$$

$$\textcircled{4} \quad y = \frac{x}{x-1}$$

$$y = \frac{3}{3-1}$$

$$y = \frac{3}{2}$$

$$\boxed{(3, \frac{3}{2})}$$

$$y = \frac{-1}{-1-1}$$

$$y = \frac{1}{2}$$

$$\boxed{(-1, \frac{1}{2})}$$

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

$$\textcircled{1} 6x - y = 4$$

$$6x - 4 = y$$

$$y = 6x - 4$$

$$m = 6$$

$$\textcircled{2} y = x\sqrt{x} = x(x)^{1/2} = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$$

$$\textcircled{3} \frac{3\sqrt{x}}{2} = \frac{6}{1}$$

$$3\sqrt{x} = 12$$

$$\sqrt{x} = 4$$

$$x = 16$$

$$\textcircled{4} y = x\sqrt{x}$$

$$y = (16)\sqrt{16}$$

$$y = 64$$

$$(16, 64)$$