Final Review:

① a) $f(x) = \sqrt{x-5}$ $\qquad f(x+h) = \sqrt{x+h-5}$

$$f'(x) = \lim_{h\to 0} \frac{(\sqrt{x+h-5} - \sqrt{x-5})}{h} \frac{(\sqrt{x+h-5} + \sqrt{x-5})}{(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h\to 0} \frac{x+h-5-(x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5}}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{5}-\cancel{x}+\cancel{5}}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-5} + \sqrt{x-5})} = \frac{1}{2\sqrt{x-5}}$$

Final Review

$$= \frac{2(x+h)-2}{(x+h)+3}$$

① b) $f(x) = \boxed{\dfrac{2x-2}{x+3}}$ $\qquad f(x+h) = \boxed{\dfrac{2x+2h-2}{x+h+3}}$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

CD: $(x+3)(x+h+3)$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{2x+2h-2}{x+h+3} - \dfrac{2x-2}{x+3}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2x^2} \cancel{+2xh} \cancel{-2x} \cancel{+6x} +6h \cancel{-6} - (\cancel{2x^2} \cancel{+2xh} \cancel{+6x} \cancel{-2x} -2h \cancel{-6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

Final Review:

② a) $f(x) = 3x^2 + 5x - 2$

$f'(x) = 6x + 5$

b) $f(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$

$f'(x) = -\frac{3}{2}x^{-3/2} = \frac{-3}{2x^{3/2}}$

c) $f(x) = 2x^4 + \sqrt{x}$

$f(x) = 2x^4 + x^{1/2}$

$f'(x) = 8x^3 + \frac{1}{2}x^{-1/2}$

$f'(x) = 8x^3 + \frac{1}{2x^{1/2}}$

d) $f(x) = \sqrt[3]{x^2} = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

Final Review:

③ a) $y = (3x^2-2)(4x+5)$

$y' = 6x(4x+5) + 4(3x^2-2)$

$y' = 24x^2 + 30x + 12x^2 - 8$

$y' = 36x^2 + 30x - 8$

b) $g(x) = (x^2-5x+2)(4x+1)$

$g'(x) = (2x-5)(4x+1) + 4(x^2-5x+2)$

$g'(x) = 8x^2 + 2x - 20x - 5 + 4x^2 - 20x + 8$

$g'(x) = 12x^2 - 38x + 3$

Final Review:

④ a) $f(x) = \dfrac{2x^2+3}{3x-2}$

$$f'(x) = \frac{4x(3x-2)-3(2x^2+3)}{(3x-2)^2}$$

$$f'(x) = \frac{12x^2-8x-6x^2-9}{(3x-2)^2}$$

$$f'(x) = \frac{6x^2-8x-9}{(3x-2)^2}$$

b) $y = \dfrac{\sqrt{x}}{3+x^2}$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - 2x(\sqrt{x})}{(3+x^2)^2}$$

$$y' = \frac{\cancel{2\sqrt{x}}\,\dfrac{3+x^2}{\cancel{2\sqrt{x}}} - 2x^{3/2}\cdot 2\sqrt{x}}{2\sqrt{x}(3+x^2)^2}$$

$$y' = \frac{3+x^2-4x^2}{2\sqrt{x}(3+x^2)^2} = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$$

Final Review:

⑤ $y = (x^2 - 3)^8$ at $x = 2$

(i) Find y:

$y = (2^2 - 3)^8$

$y = (4 - 3)^8$

$y = 1$

(ii) Find y':

$y = (x^2 - 3)^8$

$y' = 8(x^2 - 3)^7 (2x)$

$y' = 16x(x^2 - 3)^7$

(iii) Find m:

$y' = 16(2)[(2)^2 - 3]^7$

$y' = 32(1)^7$

$y' = 32 \leftarrow m$

(iv) Find equation:

$y - y_1 = m(x - x_1)$

$y - 1 = 32(x - 2)$

$y - 1 = 32x - 64$

$\boxed{0 = 32x - y - 63}$

Final Review:

6) a) $f(x) = 3(2x^2-4)^4$

$f'(x) = 12(2x^2-4)^3(4x)$

$f'(x) = 48x(2x^2-4)^3$

b) $f(x) = \frac{16}{\sqrt{x-1}} = 16(x-1)^{-1/2}$

$f'(x) = -8(x-1)^{-3/2}(1)$

$f'(x) = \frac{-8}{(x-1)^{3/2}}$

Final Review:

1. a) $$y=\left[\frac{2x+1}{x-1}\right]^5$$

$$2x-2-2x-1$$

$$y'=5\left[\frac{2x+1}{x-1}\right]^4\left[\frac{2(x-1)-1(2x+1)}{(x-1)^2}\right]$$

$$y'=5\cdot\frac{(2x+1)^4}{(x-1)^4}\cdot\frac{-3}{(x-1)^2}=\frac{-15(2x+1)^4}{(x-1)^6}$$

b) $$y=(x^2-1)^3(3x-2)^2$$

$$y'=3(x^2-1)^2(2x)(3x-2)^2+(x^2-1)^3(2)(3x-2)(3)$$

$$y'=6x(x^2-1)^2(3x-2)^2+6(x^2-1)^3(3x-2)$$

$$3x^2-2x+x^2-1$$

$$y'=6(x^2-1)^2(3x-2)\left[x(3x-2)+(x^2-1)\right]$$

$$y'=6(x^2-1)^2(3x-2)(4x^2-2x-1)$$

c) $$y=\frac{(2x+1)^2}{(x^4-x+1)^2}$$

$$y'=\frac{2(2x+1)(2)(x^4-x+1)^2-(2x+1)^2(2)(x^4-x+1)(4x^3-1)}{(x^4-x+1)^4}$$

$$y'=\frac{4(2x+1)(x^4-x+1)^2-2(4x^3-1)(2x+1)^2(x^4-x+1)}{(x^4-x+1)^4}$$

$$2x^4-2x+2 \quad -(8x^4+4x^3-2x-1)$$

$$y'=\frac{2(2x+1)\cancel{(x^4-x+1)}\left[2(x^4-x+1)-(4x^3-1)(2x+1)\right]}{(x^4-x+1)^{\cancel{4}\,3}}$$

$$y'=\frac{2(2x+1)(-6x^4-4x^3+3)}{(x^4-x+1)^3}$$

Final Review:

8 a) $f(x) = \sin^3 x + \cos^3 x$

$f(x) = (\sin x)^3 + (\cos x)^3$

$f'(x) = 3(\sin x)^2(\cos x) + 3(\cos x)^2(-\sin x)$

$f'(x) = 3\sin^2 x\cos x - 3\sin x\cos^2 x$

$f'(x) = 3\sin x\cos x(\sin x - \cos x)$

b) $y = 3\sec(2x^2+1)$

$u = 2x^2+1$
$du = 4x$

$y' = 3\sec(2x^2+1)\tan(2x^2+1)\cdot 4x$

$y' = 12x\sec(2x^2+1)\tan(2x^2+1)$

Do Ex. 2.9 on page 112 of Text
Questions 1, 4, 6, 9, 11, 12, 14

a, d

Page 103

③ Find $\left.\frac{dy}{dx}\right]_{x=4}$ if $y=u^2-2u^5$ and $u=x-\sqrt{x}$

① Find u

$u=x-\sqrt{x}$

$u=(4)-\sqrt{4}$

$\boxed{u=2}$

② Find $\frac{dy}{du}$

$y=u^2-2u^5$

$\frac{dy}{du}=2u-10u^4$

③ Find $\frac{du}{dx}$

$u=x-x^{1/2}$

$\frac{du}{dx}=1-\frac{1}{2}x^{-1/2}$

$$\left.\frac{dy}{dx}\right]_{x=4}=(2u-10u^4)\left(1-\frac{1}{2\sqrt{x}}\right)$$

$$=(2(2)-10(2)^4)\left(1-\frac{1}{2\sqrt{4}}\right)$$

$$=(4-160)\left(1-\frac{1}{4}\right)$$

$$=(-156)\left(\frac{3}{4}\right)$$

$$=\frac{-468}{4}$$

$$=-117$$

⑨ If $F(x)=f(g(x))$, where $\boxed{g(2)=4}$, $\boxed{g'(2)=3}$, $\boxed{f'(4)=5}$ find $F'(2)$

$F'(x)=f'(g(x))\cdot g'(x)$

$F'(2)=f'(g(2))\cdot g'(2)$

$F'(2)=\boxed{f'(4)}\cdot\boxed{g'(2)}$

$F'(2)=5\cdot 3=\underline{15}$

Page 112

④ e) $y = \sqrt{x}(5-\sqrt{x}) = 5\sqrt{x} - x = 5x^{1/2} - x$

$y' = \frac{5}{2}x^{-1/2} - 1$

$y' = \frac{5}{2\sqrt{x}} - 1 = \frac{5-2\sqrt{x}}{2\sqrt{x}}$

Using Product:

$y = \sqrt{x}(5-\sqrt{x})$

$y' = \frac{1}{2}x^{-1/2}(5-\sqrt{x}) + x^{1/2}\left(-\frac{1}{2}x^{-1/2}\right)$

$y' = \frac{1}{2\sqrt{x}}(5-\sqrt{x}) + \sqrt{x}\left(-\frac{1}{2\sqrt{x}}\right)$

$y' = \frac{5}{2\sqrt{x}} - \frac{\sqrt{x}}{2\sqrt{x}} - \frac{\sqrt{x}}{2\sqrt{x}}$

$y' = \frac{5}{2\sqrt{x}} - \frac{2\sqrt{x}}{2\sqrt{x}}$

$y' = \frac{5}{2\sqrt{x}} - 1$

f) $y = \frac{x^2 - 2x}{\sqrt{x}}$

$y' = \frac{x^{1/2}(2x-2) - \frac{1}{2}x^{-1/2}(x^2-2x)}{x}$

$y' = \frac{2x^{3/2} \cdot 2x^{1/2} - 2x^{1/2} \cdot 2x^{1/2} - \frac{1}{\cancel{2\sqrt{x}}}(x^2-2x) \cdot \cancel{2x^{1/2}}}{x \cdot 2x^{1/2}}$

$y' = \frac{4x^2 - 4x - x^2 + 2x}{2x^{3/2}} = \frac{3x^2 - 2x}{2x^{3/2}}$

$= \frac{x(3x-2)}{2x^{3/2}}$

$= \frac{x^{-1/2}(3x-2)}{2}$

$= \boxed{\frac{3x-2}{2\sqrt{x}}}$

Page 112

⑥ if $y = u^2 - u^3 + 2u^4$ and $u = \frac{x}{2x-1}$ Find $\left.\frac{dy}{dx}\right|_{x=1}$

(i) $\frac{dy}{du} = 2u - 3u^2 + 8u^3$

(ii) $\frac{du}{dx} = \frac{1(2x-1) - 2x}{(2x-1)^2}$ $\quad 2\cancel{x} - 1 - \cancel{2x}$

$\frac{du}{dx} = \frac{-1}{(2x-1)^2}$

(iii) Find u

$u = \frac{x}{2x-1}$

$u = \frac{(1)}{2(1)-1}$

$u = 1$

(iv) $\left.\frac{dy}{dx}\right|_{x=1} = \left[\frac{dy}{du}\right] \cdot \left[\frac{du}{dx}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[2u - 3u^2 + 8u^3\right]\left[\frac{-1}{(2x-1)^2}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[2(1) - 3(1)^2 + 8(1)^3\right]\left[\frac{-1}{(2(1)-1)^2}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[2 - 3 + 8\right]\left[-1\right] = -7$

Page 112

9) d, $y = x\sqrt{x^2+5}$, $(-2,-6)$ $x_1 = -2$ $y_1 = -6$

① Find y'

$$y' = 1\sqrt{x^2+5} + x\left[\frac{1}{\cancel{2}}(x^2+5)^{-1/2} \cdot \cancel{2}x\right]$$

$$y' = \sqrt{x^2+5} + \frac{x^2}{\sqrt{x^2+5}}$$

② Find m or $y'(-2)$

$$y'(-2) = \sqrt{(-2)^2+5} + \frac{(-2)^2}{\sqrt{(-2)^2+5}}$$

$$y' = 3 + \frac{4}{3} = \boxed{\frac{13}{3}}$$

③ Find equation:

$$y+6 = \frac{13}{3}(x+2)$$

$$y+6 = \frac{13x}{3} + \frac{26}{3} - 6$$

$$\boxed{y = \frac{13x}{3} + \frac{8}{3}}$$

$$3y = 13x + 8$$

$$\boxed{0 = 13x - 3y + 8}$$

Page 112

(12) Find the points on the curve $y = \frac{1}{2x-1}$ where the tangent line is perpendicular to $x - 2y = 1$

(i) Find perpendicular slope:

$x - 2y = 1$

$-2y = -x + 1$

$y = \frac{-x}{-2} + \frac{1}{-2}$

$y = \frac{1}{2}x - \frac{1}{2}$

$m = \frac{1}{2}$

$\boxed{m\perp = -2}$

(ii) Find derivative:

$y = \frac{1}{2x-1} = (2x-1)^{-1}$

$y' = -1(2x-1)^{-2}(2)$

$\boxed{y' = \frac{-2}{(2x-1)^2}}$

(iii) Solve for x

$\frac{-2}{1} = \frac{-2}{(2x-1)^2}$

$-2(2x-1)^2 = -2$

$(2x-1)^2 = 1$

$4x^2 - 4x + 1 = 1$

$4x^2 - 4x = 0$

$4x(x-1) = 0$

$4x = 0 \quad | \quad x - 1 = 0$

$x = 0 \quad | \quad x = 1$

(iv) Solve for y:

if $x = 0$

$y = \frac{1}{2x-1}$

$y = \frac{1}{-1} = -1$

$\boxed{(0,-1)}$

if $x = 1$

$y = \frac{1}{2x-1}$

$y = \frac{1}{1} = 1$

$\boxed{(1,1)}$

Page 103

③ Find $\frac{dy}{dx}\Big]_{x=4}$ if $y = u^2 - 2u^5$ and $u = x - \sqrt{x}$

① Find u

$u = x - \sqrt{x}$

$u = (4) - \sqrt{4}$

$\boxed{u = 2}$

② Find $\frac{dy}{du}$

$y = u^2 - 2u^5$

$\frac{dy}{du} = 2u - 10u^4$

③ Find $\frac{du}{dx}$

$u = x - x^{1/2}$

$\frac{du}{dx} = 1 - \frac{1}{2}x^{-1/2}$

$$\frac{dy}{dx}\Big]_{x=4} = (2u - 10u^4)\left(1 - \frac{1}{2\sqrt{x}}\right)$$

$$= (2(2) - 10(2)^4)\left(1 - \frac{1}{2\sqrt{4}}\right)$$

$$= (4 - 160)\left(1 - \frac{1}{4}\right)$$

$$= (-156)\left(\frac{3}{4}\right)$$

$$= \frac{-468}{4}$$

$$= -117$$

⑨ If $F(x) = f(g(x))$, where $\boxed{g(2) = 4}$, $\boxed{g'(2) = 3}$, $\boxed{f'(4) = 5}$ find $F'(2)$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(2) = f'(g(2)) \cdot g'(2)$

$F'(2) = \boxed{f'(4)} \cdot \boxed{g'(2)}$

$F'(2) = 5 \cdot 3 = 15$

If $f(3) = -2$, $f'(3) = 3$, $g(3) = 1$, $g'(3) = 7$

and $f'(1) = 4$

Find:

(i) $(f \cdot g)(3) = f'(3)g(3) + f(3) \cdot g'(3)$

$= (3)(1) + (-2)(7)$

$= 3 - 14$

$= -11$

(ii) $\left(\dfrac{f}{g}\right)'(3) = \dfrac{f'(3)g(3) - g'(3)f(3)}{[g(3)]^2}$

$= \dfrac{(3)(1) - (7)(-2)}{(1)^2}$

$= \dfrac{3 + 14}{1}$

$= 17$

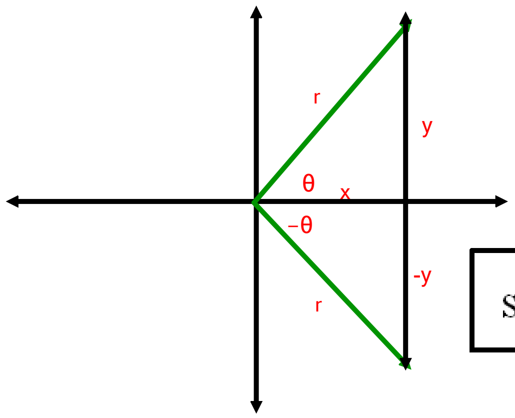
(iii) $(f \circ g)'(3) = f'(g(3)) \cdot g'(3)$

$= f'(1) \cdot g'(3)$

$= 4 \cdot 7$

$= 28$

Negative Angles



$$\sin\theta = \frac{y}{r} \qquad \cos(\theta) = \frac{x}{r}$$

$$\sin(-\theta) = -\frac{y}{r} \qquad \cos(-\theta) = \frac{x}{r}$$

$$\boxed{\sin(-\theta) = -\sin\theta} \qquad \boxed{\cos(-\theta) = \cos\theta}$$

Ex: 7.2

① a) $y = \cos(-4x)$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

Ex: 7.3

① d) $y = -\frac{1}{4}\csc(-8x)$

$y' = \frac{-1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$

$y' = -2\csc(-8x)\cot(-8x)$

$y' = -2(-\csc 8x)(-\cot 8x)$

$y' = -2\csc(8x)\cot(8x)$

Ex 7.2

① o) $y = \sin(\cos x)$

$y' = \cos(\cos x)(-\sin x)$

$y' = -\sin x \cos(\cos x)$

p) $y = \cos^3(\sin x) = [\cos(\sin x)]^3$

$y' = 3[\cos(\sin x)]^2(-\sin(\sin x))\cos x$

$y' = -3\cos^2(\sin x)\sin(\sin x)\cos x$

q) $y = x\cos\frac{1}{x} = (x)(\cos x^{-1})$

$y' = \cos x^{-1} + x(-\sin x^{-1})(-x^{-2})$

$y' = \cos\left(\frac{1}{x}\right) + \frac{1}{x}\sin\left(\frac{1}{x}\right)$

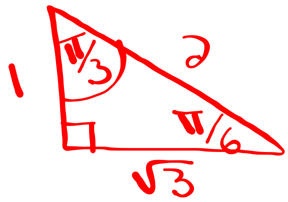
s) $y = \frac{1+\sin x}{1-\sin 2x}$

$y' = \frac{\cos x(1-\sin 2x) - (1+\sin x)(-\cos 2x \cdot 2)}{(1-\sin 2x)^2}$

$y' = \frac{\cos x - \cos x \sin 2x + 2\cos 2x(1+\sin x)}{(1-\sin 2x)^2}$

$y' = \frac{\cos x - \cos x \sin 2x + 2\cos 2x + 2\sin x\cos 2x}{(1-\sin 2x)^2}$

Page 314



③ c) Find equation of tangent →

$y = \frac{1}{\cos x} - 2\cos x$ at $(\frac{\pi}{3}, 1)$

$x_1 = \frac{\pi}{3}$

$y_1 = 1$

① find y'

$y = (\cos x)^{-1} - 2\cos x$

$y' = -1(\cos x)^{-2}(-\sin x) - 2(-\sin x)$

$y' = \frac{\sin x}{\cos^2 x} + 2\sin x$

② Find m or $y'(\frac{\pi}{3})$

$y'(\pi/3) = \frac{\sin(\pi/3)}{\cos^2(\pi/3)} + 2\sin(\pi/3)$

$y'(\pi/3) = \frac{(\sqrt{3}/2)}{(1/2)^2} + \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$

$y'(\pi/3) = \frac{\sqrt{3}}{\cancel{2}_1} \cdot \frac{\cancel{4}^2}{1} + \sqrt{3}$

$y'(\pi/3) = 2\sqrt{3} + \sqrt{3} = \boxed{3\sqrt{3}}$

③ Find equation:

$y - 1 = 3\sqrt{3}(x - \frac{\pi}{3})$

$y - 1 = 3x\sqrt{3} - \pi\sqrt{3}$

$0 = 3x\sqrt{3} - y - \pi\sqrt{3} + 1$

Ex 7.3

① a) $y = 3\tan 2x$

$y' = 3\sec^2(2x) \cdot 2$

$y' = 6\sec^2(2x)$

① d) $y = -\frac{1}{4}\csc(-8x)$

$y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$

$y' = -2\csc(-8x)\cot(-8x)$

$y' = -2\csc(8x)\cot(8x)$

*Negative Angle Identity

Ex 7.3

g) $y = \sec\sqrt[3]{x} = \sec(x^{1/3})$

$y' = \sec(x^{1/3})\tan(x^{1/3}) \cdot \frac{1}{3}x^{-2/3}$

$y' = \sec(x^{1/3})\tan(x^{1/3}) \cdot \frac{1}{3x^{2/3}}$

$y' = \frac{\sec\sqrt[3]{x}\tan\sqrt[3]{x}}{3\sqrt[3]{x^2}}$

h) $y = (x^2)(\csc x)$

$y' = (x^2)(-\csc x\cot x)(1) + 2x\csc x$

$y' = -x^2\csc x\cot x + 2x\csc x$

$y' = x\csc x(-x\cot x + 2)$

$y' = x\csc x(2 - x\cot x)$

Ex 7.3

$u = (1-2x)^2$
$du = 2(1-2x)(-2)$

i) $y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$

$y' = 3[\cot(1-2x)^2]^2[-\csc^2(1-2x)^2][2(1-2x)(-2)]$

$y' = 3\cot^2(1-2x)^2[-\csc^2(1-2x)^2][-4(1-2x)]$

$y' = 12(1-2x)\cot^2(1-2x)^2\csc^2(1-2x)^2$

m) $y = 2x(\sqrt{x} - \cot x)$

$y = 2x^{3/2} - 2x\cot x$

$y' = 3x^{1/2} - [2\cot x + 2x(-\csc^2 x \cdot 1)]$

$y' = 3\sqrt{x} - 2\cot x + 2x\csc^2 x$

Ex 7.3

① k)

$$y = \frac{1}{\sqrt{(\sec 2x - 1)^3}}$$

$$y = \frac{1}{(\sec 2x - 1)^{3/2}}$$

$$y = (\sec 2x - 1)^{-3/2}$$

$$\frac{dy}{dx} = -\frac{3}{\cancel{2}}(\sec 2x - 1)^{-5/2}(\sec 2x \tan 2x)(\cancel{2})$$

$$\frac{dy}{dx} = \frac{-3\sec 2x \tan 2x}{(\sec 2x - 1)^{5/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{-3\sec 2x \tan 2x}{\sqrt{(\sec 2x - 1)^5}}}$$

Ex 7.3

① l) $y = \dfrac{x^2 \tan x}{\sec x}$

$y = x^2\left(\dfrac{\sin x}{\cos x}\right) \div \left(\dfrac{1}{\cos x}\right)$

$y = x^2\left(\dfrac{\sin x}{\cancel{\cos x}}\right)\left(\dfrac{\cancel{\cos x}}{1}\right)$

$y = x^2 \sin x$

$\dfrac{dy}{dx} = 2x\sin x + x^2\cos x$

$$\boxed{\dfrac{dy}{dx} = x(2\sin x + x\cos x)}$$

o) $y = \tan^2(\cos x)$

$y = [\tan(\cos x)]^2$

$u = \cos x$
$du = -\sin x \cdot 1$
$= -\sin x$

$\dfrac{dy}{dx} = 2[\tan(\cos x)](\sec^2(\cos x))(-\sin x)$

$$\boxed{\dfrac{dy}{dx} = -2\sin x \tan(\cos x)\sec^2(\cos x)}$$

Ex 7.3

① p) $y = \left[\tan (x^2-x)^{-2}\right]^{-3}$

$u = (x^2-x)^{-2}$

$du = -2(x^2-x)^{-3}(2x-1)$

$\frac{dy}{dx} = -3\underbrace{\left[\tan(x^2-x)^{-2}\right]^{-4}}_{\text{denom}}\left(\sec^2(x^2-x)^{-2}\right)\left(-2\underbrace{(x^2-x)^{-3}}_{\text{denom}}(2x-1)\right)$

$$\boxed{\frac{dy}{dx} = \frac{6(2x-1)\sec^2(x^2-x)^{-2}}{(x^2-x)^3\tan^4(x^2-x)^{-2}}}$$

Ex 7.3

③ b) $y = \sin x \tan\left(\frac{1}{2}x\right)$ when $x = \frac{\pi}{3}$

(i) Find y:

$y = \sin\left(\frac{\pi}{3}\right)\tan\left(\frac{1}{2}\left(\frac{\pi}{3}\right)\right)$

$y = \left(\frac{\sqrt{3}}{2}\right)\tan\left(\frac{\pi}{6}\right)$

$y = \left(\frac{\cancel{\sqrt{3}}}{2}\right)\left(\frac{1}{\cancel{\sqrt{3}}}\right)$

$y = \frac{1}{2}$

(ii) Find $\frac{dy}{dx}$:

$y = \sin x \tan\left(\frac{1}{2}x\right)$

$\frac{dy}{dx} = \cos(x)\tan\left(\frac{1x}{2}\right) + \frac{1}{2}\sin x \sec^2\left(\frac{1}{2}x\right)$

$\frac{dy}{dx} = \cos\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{6}\right) + \frac{1}{2}\sin\left(\frac{\pi}{3}\right)\sec^2\left(\frac{\pi}{6}\right)$

$\frac{dy}{dx} = \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{3}}\right) + \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right)^2$

$\frac{dy}{dx} = \frac{1}{2\sqrt{3}} + \frac{\cancel{4}^{1}\sqrt{3}}{\cancel{12}\,3}$

$\frac{dy}{dx} = \frac{3}{6\sqrt{3}} + \frac{6}{6\sqrt{3}}$

$\frac{dy}{dx} = \frac{9}{6\sqrt{3}}$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

m

(iii) Find equation:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$

$2y - 1 = \sqrt{3}\left(x - \frac{\pi}{3}\right)$

$2y - 1 = x\sqrt{3} - \frac{\pi\sqrt{3}}{3}$

$6y - 3 = 3x\sqrt{3} - \pi\sqrt{3}$

$$\boxed{0 = 3x\sqrt{3} - 6y + 3 - \pi\sqrt{3}}$$

⑥ Find the points on the curve $y = \frac{x}{x-1}$ where the tangent line is parallel to the line $x + 4y = 1$

① $x + 4y = 1$

$4y = -x + 1$

$y = -\frac{1}{4}x + \frac{1}{4}$

$m = -\frac{1}{4}$

② $y = \frac{x}{x-1}$

$x-1 \quad - x$

$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$

$y' = \frac{-1}{(x-1)^2}$

③ $\frac{-1}{(x-1)^2} = -\frac{1}{4}$

$-(x-1)^2 = -4$

$(x-1)^2 = 4$

$x^2 - 2x + 1 = 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0 \quad | \quad x + 1 = 0$

$x = 3 \quad | \quad x = -1$

④ $y = \frac{x}{x-1}$

$y = \frac{3}{3-1} \quad | \quad y = \frac{-1}{-1-1}$

$y = \frac{3}{2} \quad | \quad y = \frac{1}{2}$

$\boxed{(3, 3/2)} \quad | \quad \boxed{(-1, 1/2)}$

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

① $6x - y = 4$

$6x - 4 = y$

$y = 6x - 4$

$m = 6$

② $y = x\sqrt{x} = x(x)^{1/2} = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

③ $\frac{3\sqrt{x}}{2} = \frac{6}{1}$

$3\sqrt{x} = 12$

$\sqrt{x} = 4$

$x = 16$

④ $y = x\sqrt{x}$

$y = (16)\sqrt{16}$

$y = 64$

$(16, 64)$