Warm-Up

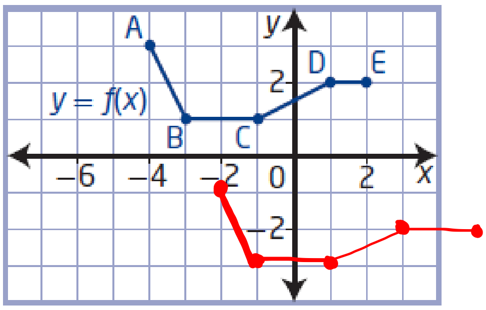
8. Copy and complete the table.

$y = f(x-h)+k$ — horizontal (change sign) for h; vertical for k

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$	K=5 (Up)
horizontal	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$	h=-7 (Left)
horizontal	$y = f(x - 3)$	$(x, y) \rightarrow (x + 3, y)$	h=3 (Right)
vertical	$y = f(x) - 6$	$(x, y) \rightarrow (x, y - 6)$	K=-6 (Down)
horizontal and vertical	$y = f(x + 4) - 9$ $y + 9 = f(x + 4)$	$(x, y) \rightarrow (x - 4, y - 9)$	h=-4 Left, K=-9 Down
horizontal and vertical	$y = f(x - 4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$	h=4 Right, K=-6 Down
h + v	$y = f(x + 2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$	h=-2, K=3
horizontal and vertical	$y = f(x - h) + k$	$(x, y) \rightarrow (x + h, y + k)$	

Questions from Homework

4.



b) $y = f(x-2) - 4$

$h = 2 \quad k = -4$

$(x, y) \rightarrow (x+2, y-4)$

A$(-4, 3)$ →	$(-2, -1)$
B$(-3, 1)$	$(-1, -3)$
C$(-1, 1)$	$(1, -3)$
D$(1, 2)$	$(3, -2)$
E$(2, 2)$	$(4, -2)$

Transformations:

New Functions From Old Functions

✓ Translations h or k

Stretches

✓ Reflections

b<0 (b is negative)
horizontal reflection
in the y-axis

$(x,y) \rightarrow (-x, y)$

a<0 (a is negative)
vertical reflection
in the x-axis

$(x,y) \rightarrow (x, -y)$

Reflections and Stretches

Focus on...

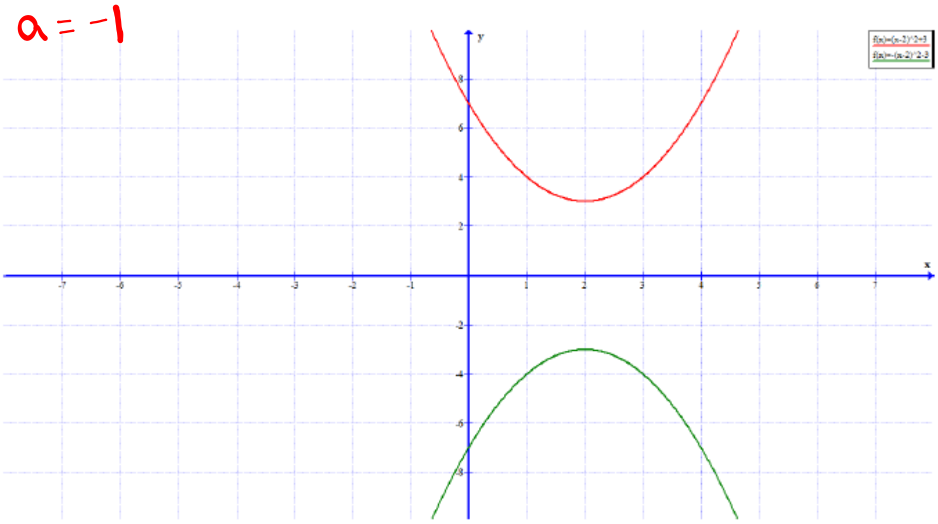
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

Vertical reflection $(x, y) \rightarrow (x, -y)$

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

$a = -1$



Reflect in x-axis:

$f(x) = x^2 - 6x + 3$

$f(1) = (1)^2 - 6(1) + 3$

$f(1) = -2$

$(1, -2)$

New Equation:

$f(x) = -(x^2 - 6x + 3)$

$f(x) = -x^2 + 6x - 3$

Multiply the output by negative 1

$f(1) = -(1)^2 + 6(1) - 3$

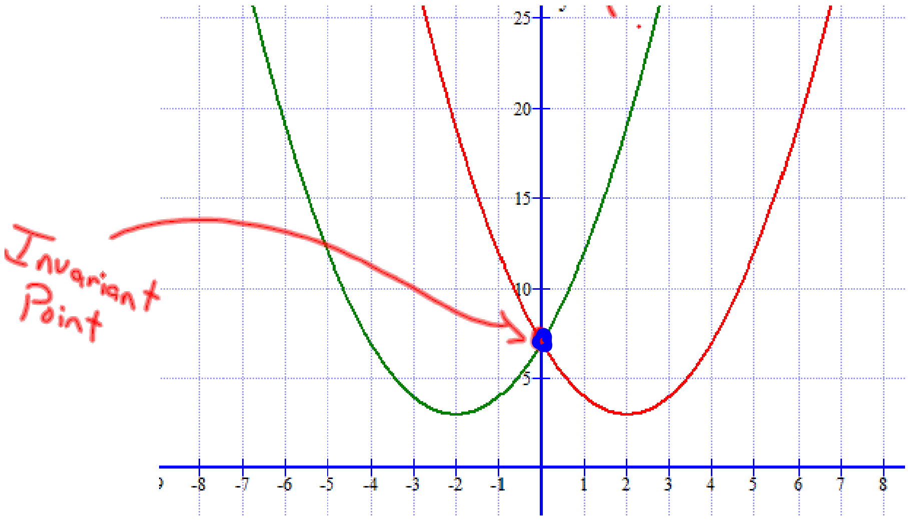
$f(1) = 2$

$(1, 2)$

Horizontal Reflection $(x,y) \to (-x,y)$

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

$b = -1$



invariant point

- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Reflect in y-axis:

$f(x) = x^2 - 6x + 3$

$f(1) = (1)^2 - 6(1) + 3$

$f(1) = -2$

$(1, -2)$

New Equation:

$f(-x) = (-x)^2 - 6(-x) + 3$

$f(-x) = x^2 + 6x + 3$

Multiply the input by negative 1

$f(-1) = (-1)^2 + 6(-1) + 3$

$f(-1) = -2$

$(-1, -2)$

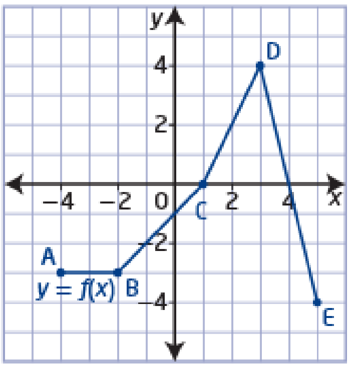
Example 1

vertical horizontal

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?

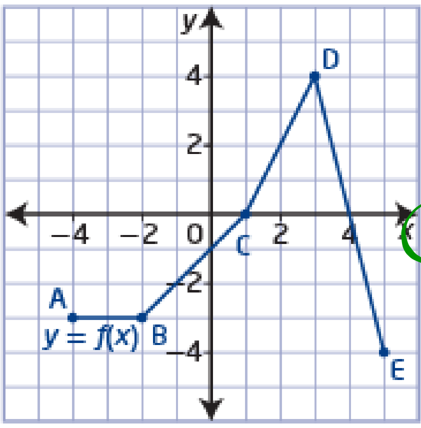


Remember...

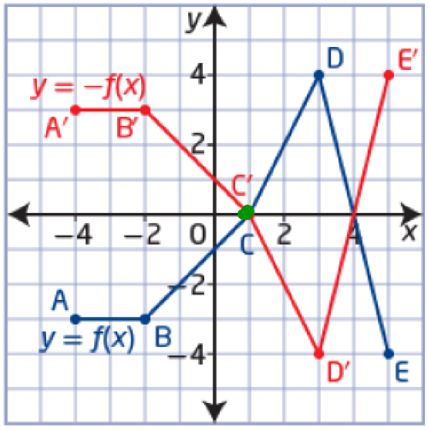
- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below (Vertical Reflection)

$(x, y) \rightarrow (x, -y)$



(-4,-3)	(-4,3)
(-2,-3)	(-2,3)
(1,0)	(1,0)
(3,4)	(3,-4)
(5,-4)	(5,4)



Invariant Point

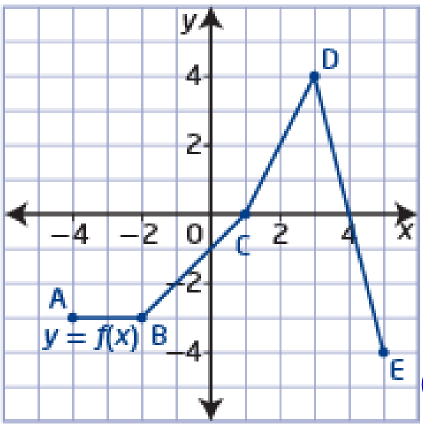
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below

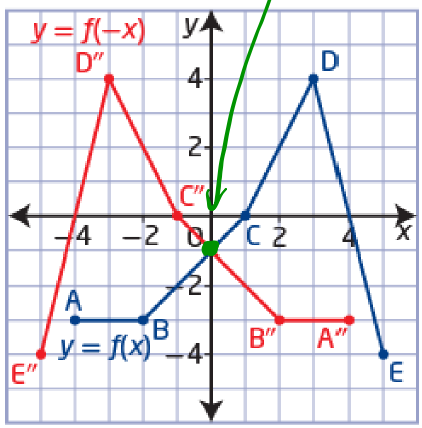
Horizontal reflection

$(x, y) \rightarrow (-x, y)$



(x, y)	$(-x, y)$
(-4, -3)	(4, -3)
(-2, -3)	(2, -3)
(1, 0)	(-1, 0)
(3, 4)	(-3, 4)
(5, -4)	(-5, -4)

Invariant



Homework

Page 28 #1, 3, 4

*$f(-4) = 2(-4)+1$
$= -8+1$
$= -7$

$f(x) = 2x+1$

x	y
-4	-7
-2	-3
0	1
2	5
4	9

Vertical

$g(x) = -f(x)$

x	y
-4	7
-2	3
0	-1
2	-5
4	-9

Horizontal

$h(x) = f(-x)$

x	y
4	-7
2	-3
0	1
-2	5
-4	9

③ a) $f(x) = 3x$

x	y
-2	-6
-1	-3
0	0
1	3
2	6

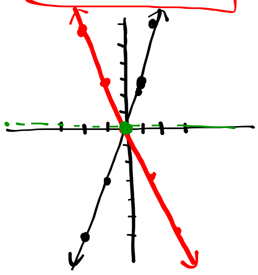
visible (-1 to 1)

$f(x) = -3x$

$(x,y) \rightarrow (x,-y)$

reflection in x-axis is vertical

x	y
-2	6
-1	3
0	0
1	-3
2	-6



D: $\{x \mid x \in R\}$
R: $\{y \mid y \in R\}$

D: $\{x \mid x \in R\}$
R: $\{y \mid y \in R\}$

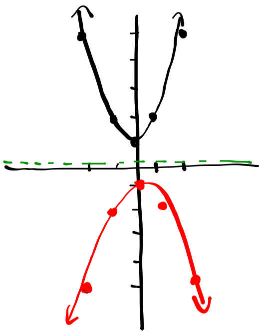
b) $g(x) = x^2+1$

x	y
-2	5
-1	2
0	1
1	2
2	5

$g(x) = -(x^2+1)$

$(x,y) \rightarrow (x,-y)$

x	y
-2	-5
-1	-2
0	-1
1	-2
2	-5



D: $\{x \mid x \in R\}$
R: $\{y \mid y \geq 1, y \in R\}$

D: $\{x \mid x \in R\}$
R: $\{y \mid y \leq 1, y \in R\}$

stretch

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

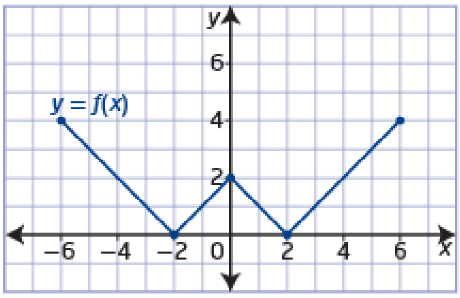
Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

Example 2

Graph $y = af(x)$

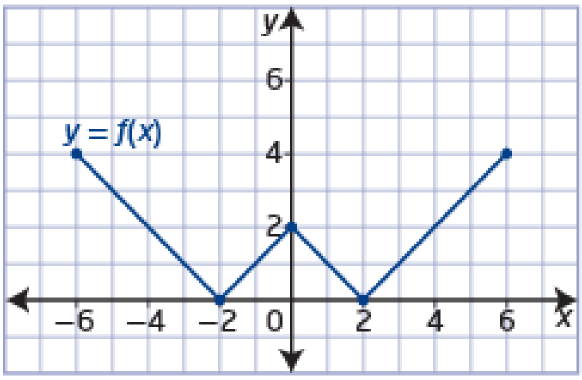
Given the graph of $y = f(x)$,
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$

a) $g(x) = 2f(x)$



[redacted]

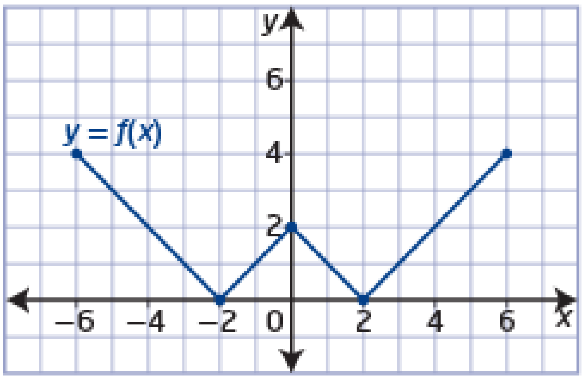
The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

b) $g(x) = \frac{1}{2}f(x)$



[redacted]

The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...

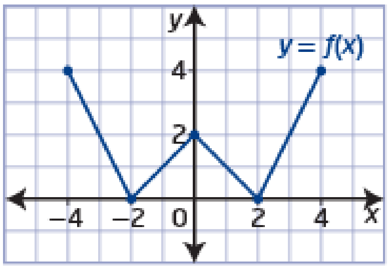
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 3

Graph $y = f(bx)$

Given the graph of $y = f(x)$,

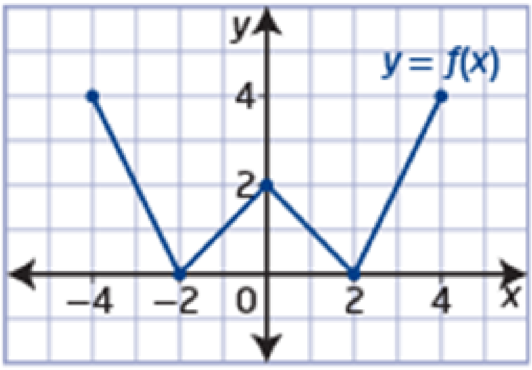
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = f(2x)$

b) $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$



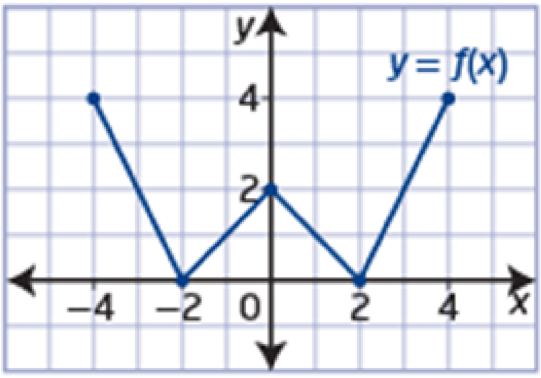
[redacted]

The invariant point is

For $f(x)$, the domain is
or and the range is
or

For $g(x)$, the domain is
or ınd the range is
or

b) $g(x) = f\left(\frac{1}{2}x\right)$



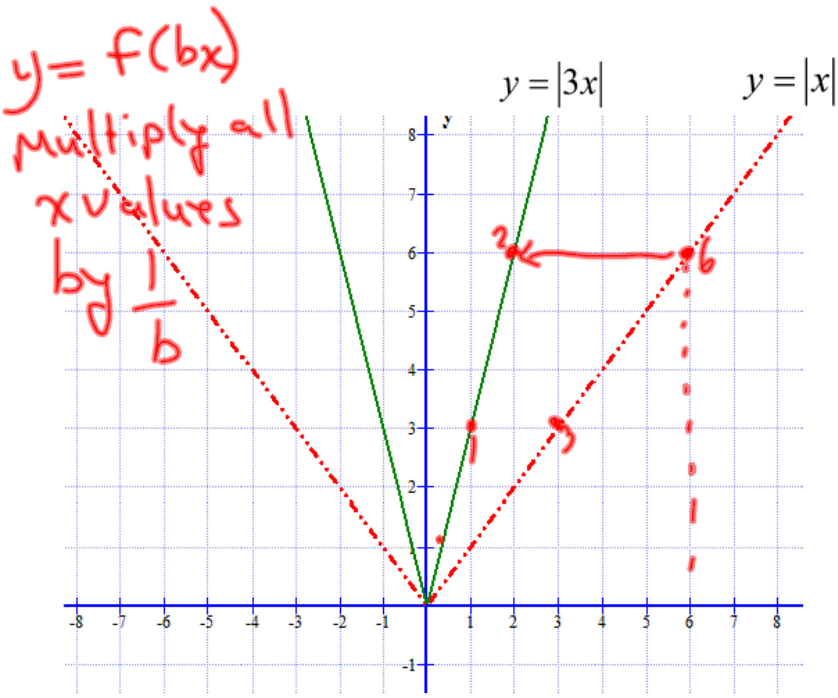
[redacted]

The invariant point is

For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...



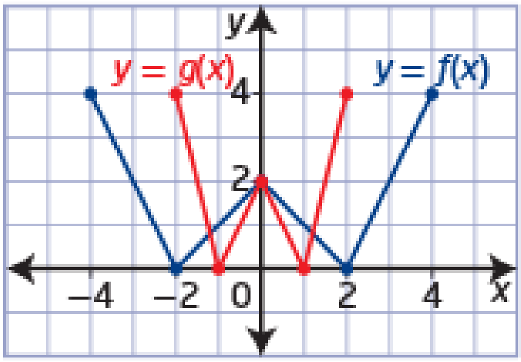
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$y = -3f(-2x) + 7$

Homework

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x