Questions from Homework

Exercise 2.8

(4) If $f(x) = \sqrt{1+x^3}$ find $f''(2)$

$$f(x) = (1+x^3)^{1/2}$$

$$f'(x) = \frac{1}{2}(1+x^3)^{-1/2}(3x^2)$$

$$f'(x) = \frac{3x^2}{2(1+x^3)^{1/2}}$$

$$f''(x) = \frac{\overset{f'}{6x}\,\overset{g}{\left(2(1+x^3)^{1/2}\right)} - \overset{f}{3x^2}\,\overset{g'}{(1+x^3)^{-1/2}(3x^2)}}{4(1+x^3)}$$

$$f''(x) = \frac{12x(1+x^3)^{1/2} - 9x^4(1+x^3)^{-1/2}}{4(1+x^3)}$$

$4 + 4x^3 - 3x^3$

$$f''(x) = \frac{3x(1+x^3)^{-1/2}\left[4(1+x^3) - 3x^3\right]}{4(1+x^3)}$$

$$f''(x) = \frac{3x(4+x^3)}{4(1+x^3)^{3/2}}$$

$$f''(2) = \frac{3(2)(4+(2)^3)}{4(1+(2)^3)^{3/2}}$$

$$f''(2) = \frac{72}{108}$$

$$\boxed{f''(2) = \frac{2}{3}}$$

Questions from Homework

8. Find a quadratic function f such that

$f(3) = 33$

$f'(3) = 22$

$f''(3) = 8$

$$\boxed{f(x) = 4x^2 - 2x + 3}$$

$f'(x) = 8x - 2$

$f''(x) = 8$

Product Rule:

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Chain Rule:

$$(f \circ g)'(x) = f'(g(x))(g'(x))$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$= (First) x (Second)

" The derivative of the product of two functions is the the first multiplied by the derivative of second, plus the derivative of first multiplied by the second"

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally ...

" The denominator multiplied by the derivative of the numerator, minus the numerator multiplied by the derivative of the denominator, all over the denominator squared"

Combining the Chain Rule With the Product and Quotient Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x)$$

Differentiate the following function and simplify your answer:

$$y = \underbrace{(x^2+1)^3}_{f(x)}\underbrace{(2-3x)^4}_{g(x)}$$

$$\frac{dy}{dx} = \underbrace{3(x^2+1)^2(2x)}_{f'(x)}\underbrace{(2-3x)^4}_{g(x)} + \underbrace{(x^2+1)^3}_{f(x)}\underbrace{(4)(2-3x)^3(-3}_{g'(x)}$$

$$\frac{dy}{dx} = 6x(x^2+1)^2(2-3x)^4 - 12(x^2+1)^3(2-3x)^3$$

$2x - 3x^2$ $\quad -2x^2 - 2$

$$\frac{dy}{dx} = 6(x^2+1)^2(2-3x)^3\left[x(2-3x) - 2(x^2+1)\right]$$

$$\frac{dy}{dx} = 6(x^2+1)^2(2-3x)^3(-5x^2+2x-2)$$

$$\boxed{\frac{dy}{dx} = -6(x^2+1)^2(2-3x)^3(5x^2-2x+2)}$$

Differentiate the following functions and simplify your answers:

$$s=\left(\frac{2t-1}{t+2}\right)^6$$

$$2t+4 \quad -2t \quad +1$$

$$\frac{ds}{dt}=6\left(\frac{2t-1}{t+2}\right)^5\left[\frac{2(t+2)-(2t-1)}{(t+2)^2}\right]$$

$$\frac{ds}{dt}=6\cdot\frac{(2t-1)^5}{(t+2)^5}\cdot\frac{5}{(t+2)^2}$$

$$\boxed{\frac{ds}{dt}=\frac{30(2t-1)^5}{(t+2)^7}}$$

Example 1

Chain Rule

Let $F(x) = f(g(x))$

If $f(2) = 3$, $\underline{f'(2) = 5}$, $\underline{g(1) = 2}$ and $\underline{g'(1) = 4}$ find $\underline{F'(1)}$.

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(1) = f'(\underline{g(1)}) \cdot \underline{g'(1)}$

$F'(1) = \underline{f'(2)} \cdot (4)$

$F'(1) = 5 \cdot 4$

$\boxed{F'(1) = 20}$

Example 2

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left.\frac{dy}{dx}\right]_{x=1}$

① $\frac{dy}{du} = 10u^9 + 5u^4$

② $\frac{du}{dx} = -6x$

③ Find u when x = 1:

$u = 1 - 3(1)^2$

$u = -2$

④ $\left.\frac{dy}{dx}\right|_{x=1} = \left[\frac{dy}{du}\right] \cdot \left[\frac{du}{dx}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[10u^9 + 5u^4\right]\left[-6x\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[10(-2)^9 + 5(-2)^4\right]\left[-6(1)\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[-5120 + 80\right]\left[-6\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[-5040\right]\left[-6\right]$

$\boxed{\left.\frac{dy}{dx}\right|_{x=1} = 30240}$

Example 3

Differentiate the following: (Implicit Differentiation)

$$5y^2 - (2x^2 y) = 3 + x$$

$$10y\frac{dy}{dx} - \left(4xy + 2x^2\frac{dy}{dx}\right) = 1$$

$$10y\frac{dy}{dx} - 4xy - 2x^2\frac{dy}{dx} = 1$$

$$10y\frac{dy}{dx} - 2x^2\frac{dy}{dx} = 1 + 4xy$$

$$\frac{dy}{dx}\left(10y - 2x^2\right) = 1 + 4xy$$

$$\boxed{\frac{dy}{dx} = \frac{1+4xy}{10y-2x^2}}$$

Homework

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#4-9, 11, 12, and 14

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④ b) $y = 2x^{\pi+1}$

$\frac{dy}{dx} = 2(\pi+1)x^{\pi}$ or $(2\pi+2)x^{\pi}$

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4) e) $y = \sqrt{x}(5-\sqrt{x}) = 5\sqrt{x} - x = 5x^{1/2} - x$

$y' = \frac{5}{2}x^{-1/2} - 1$

$y' = \frac{5}{2\sqrt{x}} - 1$

$y' = \frac{5}{2\sqrt{x}} - \frac{2\sqrt{x}}{2\sqrt{x}} = \frac{5-2\sqrt{x}}{2\sqrt{x}}$

Using Product:

$y = \sqrt{x}(5-\sqrt{x})$ $\quad y' = \frac{1}{2}x^{-1/2}(5-\sqrt{x}) + x^{1/2}\left(-\frac{1}{2}x^{-1/2}\right)$

$y' = \frac{1}{2\sqrt{x}}(5-\sqrt{x}) + \sqrt{x}\left(-\frac{1}{2\sqrt{x}}\right)$

$y' = \frac{5}{2\sqrt{x}} - \underbrace{\frac{\sqrt{x}}{2\sqrt{x}} - \frac{\sqrt{x}}{2\sqrt{x}}}$

$y' = \frac{5}{2\sqrt{x}} - \frac{2\sqrt{x}}{2\sqrt{x}}$

$y' = \frac{5}{2\sqrt{x}} - 1$

f) $y = \frac{x^2 - 2x}{\sqrt{x}}$

$y' = \frac{x^{1/2}(2x-2) - \frac{1}{2}x^{-1/2}(x^2-2x)}{x}$

$y' = \frac{2x^{3/2}\cdot 2x^{1/2} - 2x^{1/2}\cdot 2x^{1/2} - \frac{1}{\cancel{2\sqrt{x}}}(x^2-2x)\cdot \cancel{2x^{1/2}}}{x \cdot 2x^{1/2}}$

$y' = \frac{4x^2 - 4x - x^2 + 2x}{2x^{3/2}}$

$y' = \frac{3x^2 - 2x}{2x^{3/2}}$

$y' = \frac{x(3x-2)}{2x^{3/2}}$

$y' = \frac{x^{-1/2}(3x-2)}{2}$

$y' = \boxed{\frac{3x-2}{2\sqrt{x}}}$

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4 i) $f(x) = (x^2+x)\sqrt{1-x^2}$

$$f(x) = (x^2+x)(1-x^2)^{1/2}$$

$$f'(x) = (2x+1)(1-x^2)^{1/2} + (x^2+x)\left(\frac{1}{\cancel{2}}\right)(1-x^2)^{-1/2}\overset{-x}{(-\cancel{2}x)}$$

$$f'(x) = (2x+1)(1-x^2)^{1/2} - x(x^2+x)(1-x^2)^{-1/2}$$

$$f'(x) = (1-x^2)^{-1/2}\left[(2x+1)(1-x^2) - x(x^2+x)\right]$$

$$f'(x) = (1-x^2)^{-1/2}\left[2x - 2x^3 + 1 - x^2 - x^3 - x^2\right]$$

$$f'(x) = \frac{(-3x^3 - 2x^2 + 2x + 1)}{(1-x^2)^{1/2}}$$

$$\boxed{f'(x) = -\frac{3x^3 + 2x^2 - 2x - 1}{\sqrt{1-x^2}}}$$

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④ or $R(u) = \sqrt[4]{u+1} - \frac{2}{u^2}$

$$R(u) = (u+1)^{1/4} - 2u^{-2}$$

$$R'(u) = \frac{1}{4}(u+1)^{-3/4}(1) + 4u^{-3}$$

$$R'(u) = \frac{1}{4(u+1)^{3/4}} + \frac{4}{u^3}$$

$$R'(u) = \frac{1}{4\sqrt[4]{(u+1)^3}} + \frac{4}{u^3}$$

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4 r) $F(y) = \dfrac{1}{2+\frac{3}{y}}$

$$F(y) = \left(2+\frac{3}{y}\right)^{-1}$$

$$F'(y) = -\left(2+\frac{3}{y}\right)^{-2}\left(-3y^{-2}\right)$$

$$F'(y) = \frac{3}{y^2\left(2+\frac{3}{y}\right)^2}$$

$$\left(2+\frac{3}{y}\right)\left(2+\frac{3}{y}\right)$$

$$F'(y) = \frac{3}{y^2\left(4+\frac{12}{y}+\frac{9}{y^2}\right)}$$

$$F'(y) = \frac{3}{4y^2+12y+9}$$

$\underline{6} \times \underline{6} = 36$

$\underline{6} + \underline{6} = 12$

$$F'(y) = \frac{3}{\left(y+\frac{6}{4}\right)\left(y+\frac{6}{4}\right)}$$

$$F'(y) = \frac{3}{\left(y+\frac{3}{2}\right)\left(y+\frac{3}{2}\right)}$$

$$F'(y) = \frac{3}{(2y+3)(2y+3)}$$

$$F'(y) = \frac{3}{(2y+3)^2}$$

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5 a) $f(x) = \frac{2x-1}{x^2-5}$

Not permissable values:

$x^2-5 \neq 0$

$x^2 \neq 5$

$x \neq \pm\sqrt{5}$

for $f(x)$:

D: $\{x \mid x \neq \pm\sqrt{5}, x \in \mathbb{R}\}$

or $(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)$

$f'(x) = \frac{2(x^2-5) - 2x(2x-1)}{(x^2-5)^2}$

$f'(x) = \frac{2x^2-10-4x^2+2x}{(x^2-5)^2}$

$f'(x) = \frac{-2x^2+2x-10}{(x^2-5)^2}$

$f'(x) = \frac{-2(x^2-x+5)}{(x^2-5)^2}$

Not Permissable Values

$(x^2-5)^2 \neq 0$

$x^2-5 \neq 0$

$x^2 \neq 5$

$x \neq \pm\sqrt{5}$

for $f'(x)$:

D: $\{x \mid x \neq \pm\sqrt{5}, x \in \mathbb{R}\}$

or $(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)$

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⑥ if $y = u^2 - u^3 + 2u^4$ and $u = \frac{x}{2x-1}$ Find $\left.\frac{dy}{dx}\right|_{x=1}$

(i) $\frac{dy}{du} = 2u - 3u^2 + 8u^3$

(ii) $\frac{du}{dx} = \frac{1(2x-1) - 2x}{(2x-1)^2}$

$\cancel{2x} - 1 - \cancel{2x}$

$\frac{du}{dx} = \frac{-1}{(2x-1)^2}$

(iii) Find u

$u = \frac{x}{2x-1}$

$u = \frac{(1)}{2(1)-1}$

$u = 1$

(iv) $\left.\frac{dy}{dx}\right|_{x=1} = \left[\frac{dy}{du}\right] \cdot \left[\frac{du}{dx}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[2u - 3u^2 + 8u^3\right]\left[\frac{-1}{(2x-1)^2}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[2(1) - 3(1)^2 + 8(1)^3\right]\left[\frac{-1}{(2(1)-1)^2}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[2 - 3 + 8\right]\left[-1\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = -7$

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Ex: $y = 7x^2$

$\frac{dy}{dx} = 14x$

Product

(7) b) $x^2 - (x^2 y) + y^2 = 1$

$2x - \left(2xy + x^2\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$

$2x - 2xy - x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$-x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 2xy - 2x$

$\frac{\frac{dy}{dx}\cancel{(2y - x^2)}}{\cancel{2y - x^2}} = \frac{2xy - 2x}{2y - x^2}$

$\frac{dy}{dx} = \frac{2xy - 2x}{2y - x^2}$

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1 d) $y\sqrt{x-1} + x\sqrt{y-1} = xy$

$$y(x-1)^{1/2} + x(y-1)^{1/2} = xy$$

$$\frac{dy}{dx}(x-1)^{1/2} + y\left(\tfrac{1}{2}\right)(x-1)^{-1/2}(1) + 1(y-1)^{1/2} + x\left(\tfrac{1}{2}\right)(y-1)^{-1/2}\left(\frac{dy}{dx}\right) = 1y + x\frac{dy}{dx}$$

$$\frac{dy}{dx}(x-1)^{1/2} + \frac{x}{2}(y-1)^{-1/2}\frac{dy}{dx} - x\frac{dy}{dx} = y - \frac{y}{2}(x-1)^{-1/2} - (y-1)^{1/2}$$

$$\frac{dy}{dx}\left[(x-1)^{1/2} + \frac{x}{2}(y-1)^{-1/2} - x\right] = y - \frac{y}{2}(x-1)^{-1/2} - (y-1)^{1/2}$$

$$\frac{dy}{dx} = \frac{y - \frac{y}{2}(x-1)^{-1/2} - (y-1)^{1/2}}{(x-1)^{1/2} + \frac{x}{2}(y-1)^{-1/2} - x}$$

$$\boxed{\frac{dy}{dx} = \frac{y - \frac{y}{2\sqrt{x-1}} - \sqrt{y-1}}{\sqrt{x-1} + \frac{x}{2\sqrt{y-1}} - x}}$$

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8 b) $y = \sqrt{3x+1}$

$y = (3x+1)^{1/2}$

$y' = \frac{1}{2}(3x+1)^{-1/2}(3)$

$y' = \frac{3}{2}(3x+1)^{-1/2}$

$y'' = -\frac{3}{4}(3x+1)^{-3/2}(3)$

$y'' = \frac{-9}{4(3x+1)^{3/2}}$

$y'' = \frac{-9}{4\sqrt{(3x+1)^3}}$

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8) c) $y = \frac{t-1}{t+1}$

$y' = \frac{1(t+1) - 1(t-1)}{(t+1)^2}$

$y' = \frac{t+1-t+1}{(t+1)^2}$

$y' = \frac{2}{(t+1)^2}$

$y' = 2(t+1)^{-2}$

$y'' = -4(t+1)^{-3}(1)$

$$\boxed{y'' = \frac{-4}{(t+1)^3}}$$

Quotient

$y'' = \frac{\cancel{0(t+1)^2} - 2(2)(t+1)(1)}{(t+1)^4}$

$y'' = \frac{-4\cancel{(t+1)}}{(t+1)^{\cancel{4}\,3}}$

$y'' = \frac{-4}{(t+1)^3}$

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8) d) $x^2+y^2=\underline{16}$

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{2y\frac{dy}{dx}}{2y}=-\frac{\cancel{2}x}{\cancel{2}y}$$

$$\frac{dy}{dx}=-\frac{x}{y}$$

$$\frac{dy}{dx}=-\frac{x}{y}$$

$$\frac{d^2y}{dx^2}=\frac{-1(y)+x\boxed{\left(\frac{dy}{dx}\right)}}{y^2}$$

$$\frac{d^2y}{dx^2}=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}$$

$$\frac{d^2y}{dx^2}=\frac{-y^{y}-\frac{x^2}{\cancel{y}}y}{y^2\,y}$$

$$\frac{d^2y}{dx^2}=\frac{-y^2-x^2}{y^3}$$

$$\frac{d^2y}{dx^2}=-\frac{\boxed{y^2+x^2}}{y^3}$$

$$\boxed{\frac{d^2y}{dx^2}=-\frac{16}{y^3}}$$

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⑨ c) $y = \frac{1}{\sqrt{x^5}}$, $\left(2, \frac{1}{4\sqrt{2}}\right)$

$y = x^{-5/2}$

① Find $\frac{dy}{dx}$:

$\frac{dy}{dx} = -\frac{5}{2}x^{-7/2}$

$\frac{dy}{dx} = \frac{-5}{2x^{7/2}}$

$2^{7/2}$

$\sqrt{2^7}$

$\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

$8\sqrt{2}$

② Find m:

$\frac{dy}{dx} = \frac{-5}{2(2)^{7/2}}$

$\frac{dy}{dx} = \frac{-5}{2(8\sqrt{2})}$

$\frac{dy}{dx} = \frac{-5}{16\sqrt{2}}$

$m = \frac{-5}{16\sqrt{2}}$

③ Find equation:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{4\sqrt{2}} = \frac{-5}{16\sqrt{2}}(x - 2)$

$16\sqrt{2}$ $16\sqrt{2}$ $16\sqrt{2}$ $16\sqrt{2}$

$y - \frac{1}{4\sqrt{2}} = \frac{-5x}{16\sqrt{2}} + \frac{10}{16\sqrt{2}}$

$16\sqrt{2}y - 4 = -5x + 10$

$\boxed{5x + 16y\sqrt{2} - 14 = 0}$

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9 d, $y = x\sqrt{x^2+5}$, $(-2,-6)$ $x_1 = -2$ $y_1 = -6$

1 Find y'

$$y' = 1\sqrt{x^2+5} + x\left[\frac{1}{\cancel{2}}(x^2+5)^{-1/2} \cdot \cancel{2}x\right]$$

$$y' = \sqrt{x^2+5} + \frac{x^2}{\sqrt{x^2+5}}$$

2 Find m or $y'(-2)$

$$y'(-2) = \sqrt{(-2)^2+5} + \frac{(-2)^2}{\sqrt{(-2)^2+5}}$$

$$y' = 3 + \frac{4}{3} = \boxed{\frac{13}{3}}$$

3 Find equation:

$$y+6 = \frac{13}{3}(x+2)$$

$$y+6 = \frac{13x}{3} + \frac{26}{3} - 6$$

$$\boxed{y = \frac{13x}{3} + \frac{8}{3}}$$

$$3y = 13x + 8$$

$$\boxed{0 = 13x - 3y + 8}$$

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(11) Find the point on $y = 2x^2 - 3x + 6$ where the tangent line is parallel to $7x + y = 1$.

(1) $7x + y = 1$

$y = -7x + 1$

$m = -7$

$m_{\parallel} = -7$

(2) $y = 2x^2 - 3x + 6$

$y' = 4x - 3$

(3) $y' = 4x - 3$

$-7 = 4x - 3$

$-4 = 4x$

$-1 = x$

(4) $y = 2x^2 - 3x + 6$

$y = 2(-1)^2 - 3(-1) + 6$

$y = 2 + 3 + 6$

$y = 11$

(5) Point is $(-1, 11)$

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(12) Find the points on the curve $y = \frac{1}{2x-1}$ where the tangent line is perpendicular to $x - 2y = 1$

(i) Find perpendicular slope:

$x - 2y = 1$

$-2y = -x + 1$

$y = \frac{-x}{-2} + \frac{1}{-2}$

$y = \left(\frac{1}{2}\right)x - \frac{1}{2}$

$m = \frac{1}{2}$

$\boxed{m\perp = -2}$

(ii) Find derivative:

$y = \frac{1}{2x-1} = (2x-1)^{-1}$

$y' = -1(2x-1)^{-2}(2)$

$\boxed{y' = \frac{-2}{(2x-1)^2}}$

(iii) Solve for x

$\frac{-2}{1} = \frac{-2}{(2x-1)^2}$

$-2(2x-1)^2 = -2$

$(2x-1)^2 = 1$

$4x^2 - 4x + 1 = 1$

$4x^2 - 4x = 0$

$4x(x-1) = 0$

$4x = 0 \quad | \quad x - 1 = 0$

$x = 0 \quad | \quad x = 1$

(iv) Solve for y:

if $x = 0$

$y = \frac{1}{2x-1}$

$y = \frac{1}{-1} = -1$

$\boxed{(0,-1)}$

if $x = 1$

$y = \frac{1}{2x-1}$

$y = \frac{1}{1} = 1$

$\boxed{(1,1)}$

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(14) Suppose

$f(3) = 4$

$f'(3) = -1$

$f'(6) = 5$

$g(3) = 6$

$g'(3) = 2$

a) find $(fg)'(3)$ Product

$(fg)'(3) = f'(3)g(3) + f(3)g'(3)$

$= (-1)(6) + (4)(2)$

$= -6 + 8$

$= 2$

c) find $(f \circ g)'(3)$

$(f \circ g)'(3) = f'(g(3)) \cdot g'(3)$

$= f'(6) \cdot (2)$

$= (5) \cdot (2)$

$= 10$

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③ Find $\left.\frac{dy}{dx}\right]_{x=4}$ if $y=u^2-2u^5$ and $u=x-\sqrt{x}$

① Find u

$u=x-\sqrt{x}$

$u=(4)-\sqrt{4}$

$\boxed{u=2}$

② Find $\frac{dy}{du}$

$y=u^2-2u^5$

$\frac{dy}{du}=2u-10u^4$

③ Find $\frac{du}{dx}$

$u=x-x^{1/2}$

$\frac{du}{dx}=1-\frac{1}{2}x^{-1/2}$

$$\left.\frac{dy}{dx}\right]_{x=4}=(2u-10u^4)\left(1-\frac{1}{2\sqrt{x}}\right)$$

$$=(2(2)-10(2)^4)\left(1-\frac{1}{2\sqrt{4}}\right)$$

$$=(4-160)\left(1-\frac{1}{4}\right)$$

$$=(-156)\left(\frac{3}{4}\right)$$

$$=\frac{-468}{4}$$

$$=-117$$

⑨ If $F(x)=f(g(x))$, where $\boxed{g(2)=4}$ $\boxed{g'(2)=3}$ $\boxed{f'(4)=5}$ find $F'(2)$

$$F'(x)=f'(g(x))\cdot g'(x)$$

$$F'(2)=f'(g(2))\cdot g'(2)$$

$$F'(2)=\boxed{f'(4)}\cdot\boxed{g'(2)}$$

$$F'(2)=5\cdot 3=\underline{15}$$

If $f(3)=-2$, $f'(3)=3$, $g(3)=1$, $g'(3)=7$

and $f'(1)=4$

Find:

(i) $(f\cdot g)'(3) = f'(3)g(3) + f(3)\cdot g'(3)$

$= (3)(1) + (-2)(7)$

$= 3-14$

$= -11$

(ii) $\left(\frac{f}{g}\right)'(3) = \frac{f'(3)g(3) - g'(3)f(3)}{[g(3)]^2}$

$= \frac{(3)(1) - (7)(-2)}{(1)^2}$

$= \frac{3+14}{1}$

$= 17$

(iii) $(f\circ g)'(3) = f'(g(3))\cdot g'(3)$

$= f'(1)\cdot g'(3)$

$= 4\cdot 7$

$= 28$