Review Binomial Expansion

① a) $\left(\frac{a}{b}+2\right)^3$ $x=\frac{a}{b}$ $y=2$ $n=3$

$$({}_3C_0)\left(\frac{a}{b}\right)^3(2)^0+({}_3C_1)\left(\frac{a}{b}\right)^2(2)^1+({}_3C_2)\left(\frac{a}{b}\right)^1(2)^2+({}_3C_3)\left(\frac{a}{b}\right)^0(2)^3$$

$$(1)\left(\frac{a^3}{b^3}\right)(1)+(3)\left(\frac{a^2}{b^2}\right)(2)+(3)\left(\frac{a}{b}\right)(4)+(1)(1)(8)$$

$$\boxed{\frac{a^3}{b^3}+\frac{6a^2}{b^2}+\frac{12a}{b}+8}$$

① b) $(3x-2y)^5$ $\quad x=3x \quad y=-2y \quad n=5$

$$({}_5C_0)(3x)^5(-2y)^0 + ({}_5C_1)(3x)^4(-2y)^1 + ({}_5C_2)(3x)^3(-2y)^2 + ({}_5C_3)(3x)^2(-2y)^3 + ({}_5C_4)(3x)^1(-2y)^4 + ({}_5C_5)(3x)^0(-2y)^5$$

$$(1)(243x^5)(1) + (5)(81x^4)(-2y) + (10)(27x^3)(4y^2) + (10)(9x^2)(-8y^3) + (5)(3x)(16y^4) + (1)(1)(-32y^5)$$

$$243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$$

Review Composite Functions

(2) Suppose $f(x) = x^2 - 3x + 5$ and $g(x) = 2x - 3$

composed with

a) find $(f \circ g)(x)$

$f(g(x)) = (g(x))^2 - 3(g(x)) + 5$

$f(2x-3) = (2x-3)^2 - 3(2x-3) + 5$

$f(2x-3) = 4x^2 - 12x + 9 - 6x + 9 + 5$

$f(2x-3) = 4x^2 - 18x + 23$

b) find $g(f(x))$

$g(f(x)) = 2(f(x)) - 3$

$g(x^2-3x+5) = 2(x^2-3x+5) - 3$

$g(x^2-3x+5) = 2x^2 - 6x + 10 - 3$

$g(x^2-3x+5) = 2x^2 - 6x + 7$

c) find $f(g(3))$

(i) $g(3) = 2(3) - 3$

$g(3) = 6 - 3$

$g(3) = 3$

(ii) $f(3) = (3)^2 - 3(3) + 5$

$f(3) = 9 - 9 + 5$

$\boxed{f(3) = 5}$

d) find $g(f(-1))$

(i) $f(-1) = (-1)^2 - 3(-1) + 5$

$f(-1) = 1 + 3 + 5$

$f(-1) = 9$

(ii) $g(9) = 2(9) - 3$

$g(9) = 18 - 3$

$\boxed{g(9) = 15}$

Review Combining Functions

③ Suppose: $f(x) = (x+2)^2 - 3$, $g(x) = 3x+1$, $h(x) = \sqrt{x+5}$, $i(x) = \log(x-3)$

a) find $(f \cdot g)(x)$ and state its domain. multiply

b) find $(h-g)(x)$ and state its domain.

c) find $\left(\frac{f}{h}\right)(x)$ and state its domain.

d) find $(f+i)(x)$ and state its domain.

$f(x) = (x+2)^2 - 3$	$g(x) = 3x+1$	$h(x) = \sqrt{x+5}$	$i(x) = \log(x-3)$
$f(x) = x^2+4x+4-3$	(degree 1)	(radical)	(logarithm)
$f(x) = x^2+4x+1$	D: $\{x \mid x \in R\}$	$x+5 \geq 0$	$x-3 > 0$
(degree 2)	or $(-\infty, \infty)$	$x \geq -5$	$x > 3$
D: $\{x \mid x \in R\}$		D: $\{x \mid x \geq -5, x \in R\}$	D: $\{x \mid x > 3, x \in R\}$
or $(-\infty, \infty)$		or $[-5, \infty)$	or $(3, \infty)$

a) $(f \cdot g)(x) = f(x)g(x)$

$(f \cdot g)(x) = (x^2+4x+1)(3x+1)$

$(f \cdot g)(x) = 3x^3 + x^2 + 12x^2 + 4x + 3x + 1$

$(f \cdot g)(x) = 3x^3 + 13x^2 + 7x + 1$ (degree 3)

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

b) $(h-g)(x) = h(x) - g(x)$

$(h-g)(x) = \sqrt{x+5} - (3x+1)$

$(h-g)(x) = \sqrt{x+5} - 3x - 1$

D: $\{x \mid x \geq -5, x \in R\}$ or $[-5, \infty)$

c) $\left(\frac{f}{h}\right)(x) = \frac{f(x)}{h(x)}$

$\left(\frac{f}{h}\right)(x) = \frac{x^2+4x+1}{\sqrt{x+5}} \rightarrow \sqrt{x+5} \neq 0$

$x+5 \neq 0$

$x \neq -5$

D: $\{x \mid x > -5, x \in R\}$

or $(-5, \infty)$

d) $(f+i)(x) = f(x) + i(x)$

$(f+i)(x) = (x+2)^2 - 3 + \log(x-3)$

$x - 3 > 0$

$x > 3$

D: $\{x \mid x > 3, x \in R\}$

or $(3, \infty)$

Summary of Transformations...

Transformations of the graphs of functions

$f(x)+k$	shift $f(x)$ up k units	vertical trans.	$(x,y)\to(x,y+k)$
$f(x)-k$	shift $f(x)$ down k units		$(x,y)\to(x,y-k)$
$f(x+h)$	shift $f(x)$ left h units	horizontal trans.	$(x,y)\to(x-h,y)$
$f(x-h)$	shift $f(x)$ right h units		$(x,y)\to(x+h,y)$
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal ref.	$(x,y)\to(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical ref	$(x,y)\to(x,-y)$
$af(x)$	When $0<a<1$ – vertical shrinking of $f(x)$ When $a>1$ – vertical stretching of $f(x)$ **Multiply the y values by a**		$(x,y)\to(x,ay)$
$f(bx)$	When $0<b<1$ – horizontal stretching of $f(x)$ When $b>1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**	or multiply by $\frac{1}{b}$	$(x,y)\to(\frac{1}{b}x,y)$

Example 3

Determine the Equation of the Inverse

Algebraically determine the equation of the inverse of each function. Verify graphically that the relations are inverses of each other.

a) $f(x) = 3x + 6$

b) $f(x) = x^2 - 4$

1) Replace $f(x)$ with y.
2) Switch x's and y's.
3) Solve for y.
4) Replace y with $f^{-1}(x)$.
(if the inverse is a function!)

a) $f(x) = 3x+6$

D: $\{x \mid x \in R\}$

R: $\{y \mid y \in R\}$

$y = 3x+6$

$x = 3y+6$ -6 -6

$\frac{x-6}{3} = \frac{3y}{3}$

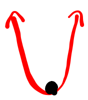
$\frac{x-6}{3} = y$

$y = \frac{x-6}{3}$

$f^{-1}(x) = \frac{x-6}{3}$

$f^{-1}(x) = \frac{1}{3}x - 2$

b) $f(x) = x^2 - 4$



D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

vertex: $(0,-4)$

R: $\{y \mid y \geq -4\}$ or $[-4, \infty)$

$y = x^2 - 4$

$x = y^2 - 4$ $+4$ $+4$

Axis of symmetry $x = 0$

$x+4 = y^2$

$\pm\sqrt{x+4} = y$

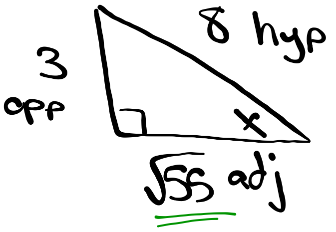
$y = \pm\sqrt{x+4}$

② Find exact value of $\sec\left(\sin^{-1}\left(3/8\right)\right)$

① Let $x = \sin^{-1}\left(\frac{3}{8}\right)$

$\sin x = \frac{3}{8}$

② Draw Diagram:



③ Find "a" or adj side.

$a^2 + b^2 = c^2$

$a^2 + 3^2 = 8^2$

$a^2 + 9 = 64$
$-9 \qquad -9$

$a^2 = 55$

$a = \pm\sqrt{55}$

$a = \sqrt{55}$

④ Find $\sec(x)$

$\sec x = \frac{hyp}{adj}$

$\sec x = \frac{8}{\sqrt{55}}$

$\boxed{\sec x = \frac{8\sqrt{55}}{55}}$