Warm Up

2. Factor each of the following:

$x^{27} - 1$ $\qquad$ $(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$

$(x^9-1)(x^{18}+x^9+1)$

$(x^3-1)(x^6+x^3+1)(x^{18}+x^9+1)$

$(x-1)(x^2+x+1)(x^6+x^3+1)(x^{18}+x^9+1)$

$(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$

$(x^2+1)^{-1/2}(x^2+1+3)$

$(x^2+1)^{-1/2}(x^2+4)$

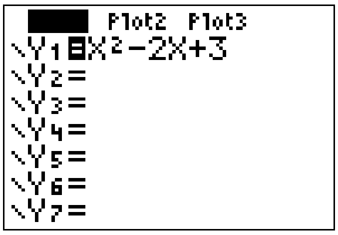
$\frac{(x^2+1)^{1/2}}{(x^2+1)^{-1/2}} = (x^2+1)^1$

$\frac{3(x^2+1)^{-1/2}}{(x^2+1)^{-1/2}} = 3(x^2+1)^0$

$= 3$

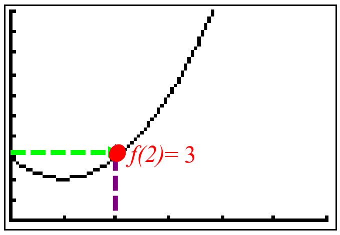
Limit of a Function

Let's examine the function $f(x) = x^2 - 2x + 3$ (parabola)



X	Y1
0	3
1	2
2	3
3	6
4	11
5	18
6	27

X=0



We can see that *f(2)*= 3...let's check the behaviour of *f* as we get closer and closer to $x = 2$.

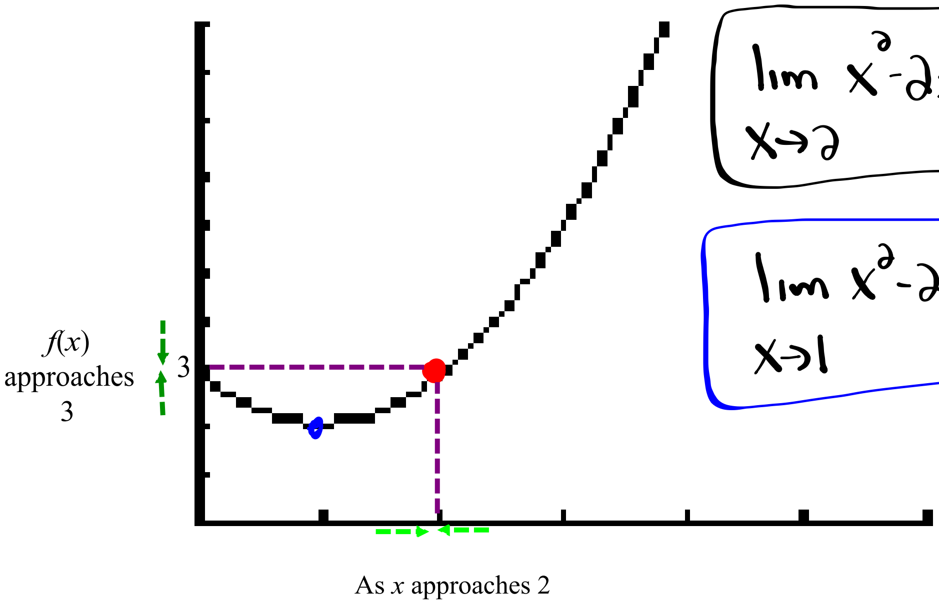
X	Y1
1.85	2.7225
1.9	2.81
1.95	2.9025
2	3
2.05	3.1025
2.1	3.21
2.15	3.3225

X=1.85

As *x* gets closer to 2 from the left *y* is getting closer to 3.

As *x* gets closer to 2 from the right *y* is getting closer to 3.

From the above, the notion of the limit of a function arises...



height

$$\lim_{x \to 2} x^2 - 2x + 3 = \underline{\underline{3}}$$

height

$$\lim_{x \to 1} x^2 - 2x + 3 = \underline{\underline{2}}$$

Notation: $\lim_{x \to 2} f(x) = 3$

"The limit of the function *f*(*x*) as *x* approaches 2 is equal to 3."

The common sense definition of a limit...

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What *is* a limit?

A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ arbitrarily close to L

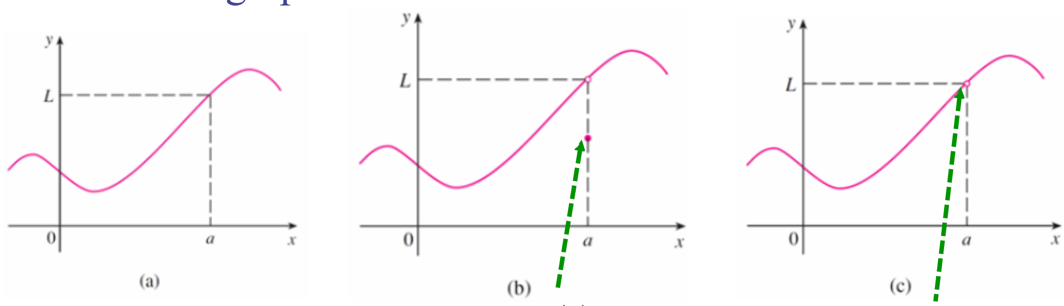
- (as close to L as we like)

by taking x to be sufficiently close to a

- (on either side of a)

but not equal to a.

Look at the graphs of these three functions...



Notice $f(a) \neq L$

Notice $f(a)$ is undefined

But in each case, regardless of what happens at a, it is true that

$$\lim_{x \to a} f(x) = L$$

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2} \frac{x^2-2x+1}{x+3}$$

$$\lim_{x\to -2} \frac{(-2)^2-2(-2)+1}{(-2)+3} = \frac{9}{1}$$

(y-value) height

$$\lim_{x\to 3}(16-x^2)$$

$$\lim_{x\to 3}(16-(3)^2) = 7$$

(y-value) height

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

$\Rightarrow$ Factor
$\Rightarrow$ Rationalize
$\Rightarrow$ Expand
$\Rightarrow$ Find Common Denominators

Examples:

$$\lim_{x\to 4} \frac{x^2-16}{x-4}$$

$$\lim_{x\to 4} \frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}$$

(y-value) height

$$\lim_{x\to 4}(4+4) = 8$$

$$\lim_{h\to 0} \frac{(\sqrt{4+h}-2)}{h}\frac{(\sqrt{4+h}+2)}{(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{1}{(\sqrt{4+0}+2)} = \frac{1}{4}$$

Try these...remember to use your algebra skills to try and eliminate the **indeterminate form**.

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$ ← common factor ← diff squares

$$\lim_{x\to 0}\frac{x(x+3)}{((x+2)+(x-2))((x+2)-(x-2))}$$

$$\lim_{x\to 0}\frac{x(x+3)}{(x+2+x-2)(x+2-x+2)}$$

$$\lim_{x\to 0}\frac{x(x+3)}{(2x)(4)}$$

$$\lim_{x\to 0}\frac{\cancel{x}(x+3)}{8\cancel{x}}=\frac{3}{8}$$

$$\lim_{x\to -2}\frac{x^4-16}{x^3+8}$$ ← diff squares ← sum cubes

$$\lim_{x\to -2}\frac{(x^2+4)(x^2-4)}{(x+2)(x^2-2x+4)}$$ ← diff squares

$$\lim_{x\to -2}\frac{(x^2+4)\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x^2-2x+4)}$$

$$=\frac{(8)(-4)}{12}=\frac{-32}{12}=-\frac{8}{3}$$

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$ ← C.F. ← Expand

$$\lim_{x\to 0}\frac{x(x+3)}{\cancel{x^2}+4x+\cancel{4}-(\cancel{x^2}-4x+\cancel{4})}$$

$$\lim_{x\to 0}\frac{\cancel{x}(x+3)}{8\cancel{x}}=\frac{3}{8}$$

$(x+2)(x+2)$
$x^2+2x+2x+4$
x^2+4x+4

$(x-2)(x-2)$
$x^2-2x-2x+4$
x^2-4x+4

$$\lim_{x\to 2}\frac{(x+2)^2-16}{x^2-4}$$

$$\lim_{x\to 2}\frac{(x+2+4)(x+2-4)}{(x+2)(x-2)}$$

$$\lim_{x\to 2}\frac{(x+6)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}=\frac{8}{4}=2$$

$$\lim_{x\to 2}\frac{2x\,\frac{1}{x}-\frac{1}{2}\,2x}{(x-2)2x}$$

$$\lim_{x\to 2}\frac{\overset{-1}{\cancel{(2-x)}}}{2x\cancel{(x-2)}}=-\frac{1}{4}$$

Homework

⑤ $\lim_{h\to 0} \frac{(4+h)^3 - 64}{h}$ ← diff of cubes

$\lim_{h\to 0} \frac{\overset{a}{[(4+h)} \overset{-}{-} \overset{b}{4}][\overset{a^2}{(4+h)^2} \overset{+ab}{+ 4(4+h)} \overset{+b^2}{+ 16}]}{h}$

$\lim_{h\to 0} \frac{\cancel{h}((4+h)^2 + 4(4+h) + 16)}{\cancel{h}} = 16 + 16 + 16 = 48$

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$ ← expanding

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)(x+2)-(x-2)(x-2)}$$

$$\lim_{x\to 0}\frac{x^2+3x}{x^2+4x+4-(x^2-4x+4)}$$

$$\lim_{x\to 0}\frac{x^2+3x}{\cancel{x^2}+\underline{4x}\cancel{+4}-\cancel{x^2}+\underline{4x}\cancel{-4}}$$

$$\lim_{x\to 0}\frac{x^2+3x}{8x}$$ ← common factor

$$\lim_{x\to \underline{\underline{0}}}\frac{\cancel{x}(\underline{x}+3)}{8\cancel{x}} = \frac{3}{8}$$