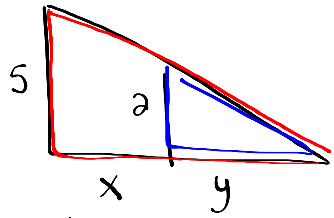
Questions From Homework

①



$\frac{dx}{dt} = 1.5\text{m/s}$

$\frac{x+y}{5} = \frac{y}{2}$

$2x + 2y = 5y$

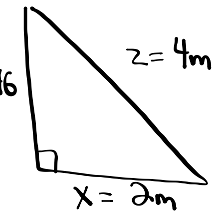
$2x = 3y$

$2\frac{dx}{dt} = 3\frac{dy}{dt}$

$2(1.5) = 3\frac{dy}{dt}$

$\boxed{1\text{m/s} = \frac{dy}{dt}}$

②



$y = 3.46$

$\frac{dy}{dt} =$

$\frac{dx}{dt} = 0.3\text{m/s}$

(1) Find y

$x^2 + y^2 = z^2$

$(2)^2 + y^2 = (4)^2$

$y^2 = 12$

$y = 3.46\text{m}$

(2) Find $\frac{dy}{dt}$

constant

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

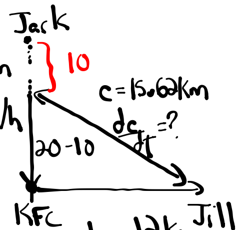
$2(2)(0.3) + 2(3.46)\frac{dy}{dt} = 0$

$1.2 + 6.92\frac{dy}{dt} = 0$

$6.92\frac{dy}{dt} = -1.2$

$\boxed{\frac{dy}{dt} = -0.1734\text{m/s}}$

⑤



$a = 10\text{km}$

$\frac{da}{dt} = -5\text{km/h}$

$b = 12\text{km}$

$\frac{db}{dt} = +6\text{km/h}$

(i) Find c

$a^2 + b^2 = c^2$

$10^2 + 12^2 = c^2$

$100 + 144 = c^2$

$244 = c^2$

$15.62 = c$

(ii) Find $\frac{dc}{dt}$

$a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(10)(-5) + 2(12)(6) = 2(15.62)\frac{dc}{dt}$

$-100 + 144 = 31.24\frac{dc}{dt}$

$44 = 31.24\frac{dc}{dt}$

$\boxed{1.41\text{km/h} = \frac{dc}{dt}}$

Jack is headed south at 60 km/h towards JMH and Jill is headed west towards the school at 50 km/h. At what rate is the distance between them closing when Jack is 2 km and Jill is 3 km from the school?

(Hint: draw a diagram)

A water tank is built in the shape of a circular cone with height 5 m and diameter 6 m at the top. Water is being pumped into the tank at a rate of 1.6 m^3/min. Find the rate at which the water level is rising when the water is 2 m deep?

Let V be the volume of the water and let r and h be the radius of the surface and the height at time t, where t is measured in minutes. We are given the rate of increase of V, that is:

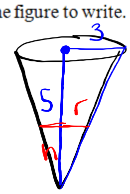
$$\frac{dV}{dt} = 1.6m^3/\min$$

We are asked to find $\frac{dh}{dt}$ when $h = 2m$.

The quantities V and h are related by the equation:

$$V = \frac{1}{3}\pi r^2 h$$

But we have to express V as a function of h alone. To eliminate r we look for a relationship between r and h. We use similar triangles in the figure to write.



$$\frac{r}{h} = \frac{3}{5}$$

$$5r = 3h$$

$$r = \frac{3h}{5}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{3h}{5}\right)^2 h$$

$$V = \frac{\cancel{1}}{\cancel{3}_1}\pi\left(\frac{\cancel{9}^3}{25}h^2\right)h$$

$$V = \frac{9\pi h^3}{75}$$

$$V = \frac{3}{25}\pi h^3$$

$$\frac{dV}{dt} = \frac{9}{25}\pi h^2 \frac{dh}{dt}$$

$$1.6 = \frac{9}{25}\pi(2)^2\frac{dh}{dt}$$

$$1.6 = \frac{36\pi}{25}\frac{dh}{dt}$$

$$1.6 = 4.5239\frac{dh}{dt}$$

$$\boxed{0.3537 \text{ m/min} = \frac{dh}{dt}}$$

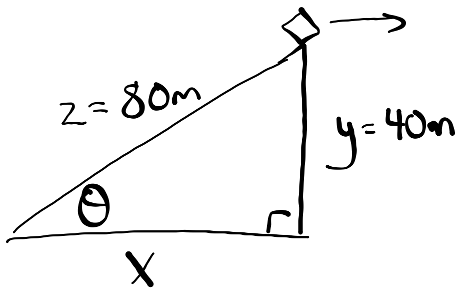
$$\frac{8}{5} = \frac{9}{25}\pi(2)^2\frac{dh}{dt}$$

$$\frac{8}{5} = \frac{36\pi}{25}\frac{dh}{dt}$$

$$\frac{\cancel{8}^{2}}{\cancel{5}_{1}} \cdot \frac{\cancel{25}^{5}}{\cancel{36\pi}_{9\pi}} = \frac{dh}{dt}$$

$$\boxed{\frac{10}{9\pi} \text{ m/min} = \frac{dh}{dt}}$$

A kite 40 m above the ground moves horizontally at a rate of 3 m/s. At what rate is the angle between the string and the horizontal decreasing when 80 m of string is let out?



$\frac{dx}{dt} = 3\text{m/s}$

(i) Find x^2:

$x^2 + y^2 = z^2$

$x^2 = z^2 - y^2$

$x^2 = 80^2 - 40^2$

$x^2 = 6400 - 1600$

$x^2 = 4800$

(ii) Find $\cos^2\theta$:

$\cos^2\theta = \left(\frac{adj}{hyp}\right)^2$

$\cos^2\theta = \frac{x^2}{z^2}$

$\cos^2\theta = \frac{4800}{6400}$

$\cos^2\theta = \frac{3}{4}$

$\tan\theta = \frac{40}{x}$ x is changing

$\tan\theta = 40x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -40x^{-2}\frac{dx}{dt}$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{40}{x^2}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \boxed{\frac{1}{\sec^2\theta}}\left(\frac{-40}{x^2}\right)\left(\frac{dx}{dt}\right)$

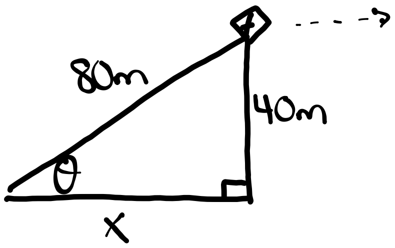
$\frac{d\theta}{dt} = \cos^2\theta\left(\frac{-40}{x^2}\right)\left(\frac{dx}{dt}\right)$

$\frac{d\theta}{dt} = \left(\frac{\cancel{3}^{1}}{\cancel{4}_{1}}\right)\left(\frac{\cancel{-40}^{\cancel{-10}\ -1}}{\cancel{4800}_{\cancel{480}\ 160}}\right)(3)$

$$\boxed{\frac{d\theta}{dt} = \frac{-3}{160} \text{ rads/sec}}$$

$\frac{d\theta}{dt} \approx -0.01875$

A kite 40 m above the ground moves horizontally at a rate of 3 m/s. At what rate is the angle between the string and the horizontal decreasing when 80 m of string is let out?



Find $\frac{d\theta}{dt}$

$\frac{dx}{dt} = 3\text{m/s}$

$\tan\theta = \frac{40}{x}$

$\tan\theta = 40x^{-1}$

$\theta = \tan^{-1}(40x^{-1})$

$x^2 + y^2 = z^2$

$x^2 + (40)^2 = (80)^2$

$x^2 = 6400 - 1600$

$\boxed{x^2 = 4800}$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{40}{x}\right)^2} \cdot -40x^{-2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\frac{1600}{x^2}} \cdot -\frac{40}{x^2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\frac{\cancel{1600}^{1}}{\cancel{4800}_{3}}} \cdot \frac{\cancel{-40}^{-1}}{\cancel{4800}_{120}} \cdot 3$$

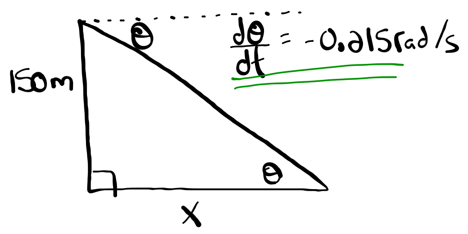
$$\frac{d\theta}{dt} = \frac{1}{4/3} \cdot \frac{-1}{120} \cdot 3$$

$$\frac{d\theta}{dt} = \frac{\cancel{3}^{1}}{4} \cdot -\frac{1}{\cancel{120}_{40}} \cdot 3$$

$$\frac{d\theta}{dt} = \frac{-3}{160} \text{ rads/sec}$$

$$\frac{d\theta}{dt} = -0.01875 \text{ rads/sec}$$

A car passes directly under a police helicopter 150 m above a straight and level highway. After the car has travelled another 20.0 m, the angle of depression of the car from the helicopter is decreasing at the rate of 0.215 rad/s. what is the speed of the car?



$\frac{dx}{dt} = ?$

$\tan\theta = \frac{150}{x}$ x is changing

$\tan\theta = 150x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -150x^{-2}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \boxed{\left(\frac{1}{\sec^2\theta}\right)}\left(\frac{-150}{x^2}\right)\frac{dx}{dt}$

$\frac{d\theta}{dt} = \cos^2\theta\left(\frac{-150}{x^2}\right)\left(\frac{dx}{dt}\right)$

(i) Find z^2 when $x = 20$

$x^2 + y^2 = z^2$

$20^2 + 150^2 = z^2$

$400 + 22500 = z^2$

$22900 = z^2$

(ii) Find $\cos^2\theta$:

$\cos^2\theta = \left(\frac{adj}{hyp}\right)^2$

$\cos^2\theta = \frac{x^2}{z^2}$

$\cos^2\theta = \frac{400}{22900}$

$\cos^2\theta = \frac{4}{229}$

$\cos^2\theta = 0.0175$

$-0.215 = (0.0175)\left(\frac{-150}{400}\right)\left(\frac{dx}{dt}\right)$

$-0.215 = -0.0066\frac{dx}{dt}$

$\boxed{32.6 \text{ m/s} = \frac{dx}{dt}}$

without a calculator

$\frac{-215}{1000} = \left(\frac{4}{229}\right)\left(\frac{-150}{400}\right)\frac{dx}{dt}$

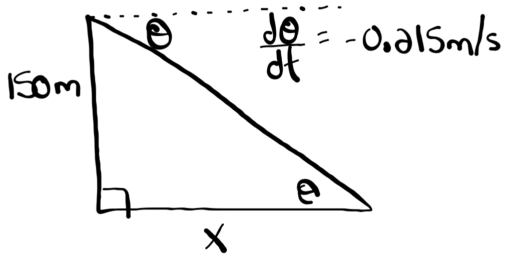
$-\frac{43}{200} = \left(\frac{\cancel{4}^{1}}{229}\right)\left(\frac{-3}{\cancel{8}_{2}}\right)\frac{dx}{dt}$

$-\frac{43}{200} = -\frac{3}{458}\frac{dx}{dt}$

$\frac{-43}{\cancel{200}_{100}} \cdot \frac{-\cancel{458}^{229}}{3} = \frac{dx}{dt}$

$\boxed{\frac{9847}{300} \text{ m/s} = \frac{dx}{dt}}$

A car passes directly under a police helicopter 150 m above a straight and level highway. After the car has travelled another 20.0 m, the angle of depression of the car from the helicopter is decreasing at the rate of 0.215 rad/s. what is the speed of the car?



$$\frac{dx}{dt} = ?$$

$$\tan\theta = \frac{150}{x}$$

$$\tan\theta = 150x^{-1}$$

$$\theta = \tan^{-1}(150x^{-1})$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{150}{x}\right)^2} \cdot -150x^{-2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{22500}{x^2}\right)} \cdot \frac{-150}{x^2} \cdot \frac{dx}{dt}$$

$$-0.215 = \frac{1}{1+\frac{22500}{400}} \cdot \frac{-150}{400} \cdot \frac{dx}{dt}$$

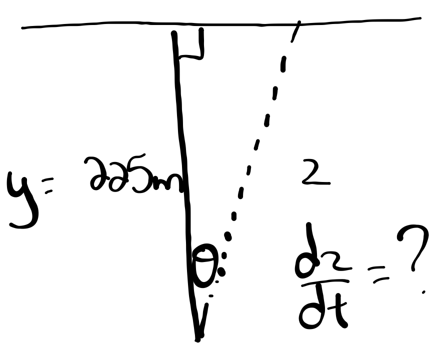
$$-0.215 = \frac{1}{\frac{4}{4}+\frac{225}{4}} \cdot -\frac{3}{8} \cdot \frac{dx}{dt}$$

$$-0.215 = \frac{{}^{1}\cancel{4}}{229} \cdot \frac{-3}{{}_{2}\cancel{8}} \cdot \frac{dx}{dt}$$

$$-0.215 = \frac{-3}{458}\frac{dx}{dt}$$

$$\boxed{32.82\text{ m/s} = \frac{dx}{dt}}$$

A searchlight is 225 m from a straight wall. As the beam moves along the wall, the angle between the beam and the perpendicular to the wall is increasing at the rate of 1.5° /s. How fast is the length of the beam increasing when it is 315 m long?



$\cos\theta = \frac{225}{z}$ ← z is changing

$\cos\theta = 225z^{-1}$

$-\sin\theta \frac{d\theta}{dt} = -225z^{-2}\frac{dz}{dt}$

$-\sin\theta \frac{d\theta}{dt} = \frac{-225}{z^2}\frac{dz}{dt}$

(i) Convert $\frac{d\theta}{dt}$:

$\frac{d\theta}{dt} = 1.5°/s \cdot \frac{\pi}{180}$

$\frac{d\theta}{dt} = 0.02618 \text{ rads/s}$

$\frac{d\theta}{dt} = \left(\frac{-1}{\sin\theta}\right)\left(\frac{-225}{z^2}\right)\left(\frac{dz}{dt}\right)$

$0.02618 = \left(\frac{-1}{\sin 0.77519}\right)\left(\frac{-225}{315^2}\right)\frac{dz}{dt}$

(ii) Find θ when z = 315

$\cos\theta = \frac{adj}{hyp} \quad \frac{y}{z}$

$\cos\theta = \frac{225}{315}$

$\theta = \cos^{-1}\left(\frac{5}{7}\right)$

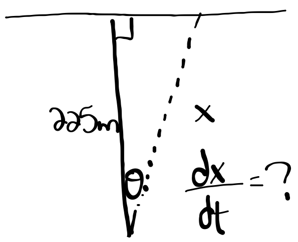
$\theta = 0.77519 \text{ rads}$

$0.02618 = (-1.42887)(-0.00227)\frac{dz}{dt}$

$0.02618 = 0.00324 \frac{dz}{dt}$

$\boxed{8.08 \text{ m/s} = \frac{dz}{dt}}$

A searchlight is 225 m from a straight wall. As the beam moves along the wall, the angle between the beam and the perpendicular to the wall is increasing at the rate of 1.5°/s. How fast the length of the beam increasing when is 315 m long?



$$\cos\theta = \frac{225}{x}$$

$$\cos\theta = 225x^{-1}$$

$$\theta = \cos^{-1}(225x^{-1})$$

$$\frac{d\theta}{dt} = 1.5^\circ/s \cdot \frac{\pi}{180}$$

$$\frac{d\theta}{dt} = 0.02618 \text{ rads/s}$$

$$\frac{d\theta}{dt} = \frac{-1}{\sqrt{1-\left(\frac{225}{x}\right)^2}} \cdot \frac{-225}{x^2} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{1-\left(\frac{225}{315}\right)^2}} \cdot \frac{-225}{315^2} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{1-\frac{50625}{99225}}} \cdot \frac{-\overset{1}{\cancel{225}}}{\underset{441}{\cancel{99225}}} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{\frac{49}{49}-\frac{25}{49}}} \cdot \frac{-1}{441} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{\frac{24}{49}}} \cdot \frac{-1}{441} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\frac{\sqrt{24}}{7}} \cdot \frac{-1}{441} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-\overset{1}{\cancel{7}}}{\underset{2\sqrt{6}}{\sqrt{24}}} \cdot \frac{-1}{\underset{63}{\cancel{441}}} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{1}{126\sqrt{6}}$$

$$\boxed{8.08 \text{ m/s} = \frac{dx}{dt}}$$

Homework