Review 1

① a) $x^2+2xy+y^2=7$

$$2x+2y+2x\frac{dy}{dx}+2y\frac{dy}{dx}=0$$

$$2x\frac{dy}{dx}+2y\frac{dy}{dx}=-2x-2y$$

$$\frac{dy}{dx}(2x+2y)=-2x-2y$$

$$\frac{dy}{dx}=\frac{-2x-2y}{2x+2y}$$

$$\frac{dy}{dx}=\frac{-\cancel{2}\cancel{(x+y)}}{\cancel{2}\cancel{(x+y)}}=\boxed{-1}$$

b) $6x^3+3x^2y=4y^2$

$$18x^2+6xy+3x^2\frac{dy}{dx}=8y\frac{dy}{dx}$$

$$18x^2+6xy=8y\frac{dy}{dx}-3x^2\frac{dy}{dx}$$

$$18x^2+6xy=\frac{dy}{dx}(8y-3x^2)$$

$$\boxed{\frac{18x^2+6xy}{8y-3x^2}=\frac{dy}{dx}}$$

Review 1

② $s = t^3 - 4.5t^2 + 6t$

$s' = v = 3t^2 - 9t + 6$

$s'' = a = 6t - 9$

a) v @ 3 sec.

$v = 3(3)^2 - 9(3) + 6$

$v = 6\,m/s$

b) a @ 4 sec.

$a = 6(4) - 9$

$a = 15\,m/s^2$

c) at rest (v=0)

$0 = 3t^2 - 9t + 6$

$0 = 3(t^2 - 3t + 2)$

$0 = 3(t-1)(t-2)$

$t-1=0 \quad | \quad t-2=0$

$t=1 \quad | \quad t=2$

d) a > 0

$6t - 9 > 0$

$6t > 9$

$t > 1.5\,s$

Review 1

③ Given:

$\frac{dV}{dt} = 2m^3/min$

$r = 2h$

$\frac{dh}{dt} = ?$

$h = 4m$

$V = \frac{1}{3}\pi r^2 h$

Express in terms of h only

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{1}{3}\pi 4h^2 \cdot h$

$V = \frac{4\pi h^3}{3}$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$2 = 4\pi (4)^2 \frac{dh}{dt}$

$2 = 64\pi \frac{dh}{dt}$

$2/64\pi \ m/min = \frac{dh}{dt}$

$0.009947 \ m/min = \frac{dh}{dt}$

Review 1

④ Given:

$l = 5cm$

$\frac{dv}{dt} = 125cm^3/s$

$V = l^3$

$\frac{dv}{dt} = 3l^2 \frac{dl}{dt}$

$125 = 3(5)^2 \frac{dl}{dt}$

$125 = 75 \frac{dl}{dt}$

$\frac{5}{3} cm/s = \frac{dl}{dt}$

$1.\overline{6} cm/s = \frac{dl}{dt}$

Review 1

⑤ Given:

$\frac{dr}{dt} = 10cm/s$

$t = 3s$

(i) Find r:

$r = 10cm/s \times 3s$

$r = 30cm$

(ii) Find $\frac{da}{dt}$:

$a = \pi r^2$

$\frac{da}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{da}{dt} = 2\pi(30)(10)$

$\frac{da}{dt} = 600\pi \ cm^2/s$

$\boxed{\frac{da}{dt} = 1884.96cm^2/s}$

Review 1

⑥ Given:

$\frac{dV}{dt} = 6m^3/min$

$\frac{dr}{dt} = ?$

$r = 2m$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$6 = 4\pi(2)^2 \frac{dr}{dt}$

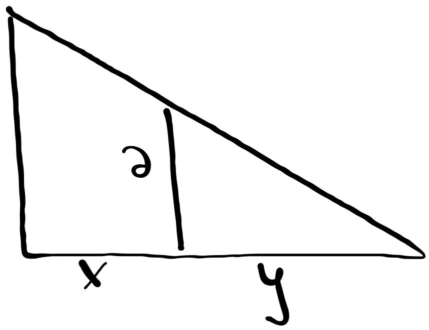
$6 = 16\pi \frac{dr}{dt}$

$\frac{3}{8\pi} m/min = \frac{dr}{dt}$

$0.119 m/min = \frac{dr}{dt}$

Review 1

7



Given:

$\frac{dx}{dt} = -1.5\,m/s$ (towards = (−))

using similar triangles:

$$\frac{x+y}{6} = \frac{y}{2}$$

$$2x + 2y = 6y$$

$$2x = 4y$$

$$2\frac{dx}{dt} = 4\frac{dy}{dt}$$

$$2(-1.5) = 4\frac{dy}{dt}$$

$$-3 = 4\frac{dy}{dt}$$

$$-\frac{3}{4}\,m/s = \frac{dy}{dt}$$

$$-0.75\,m/s = \frac{dy}{dt}$$

Review 1

(8) If $x^2y+y^2=22$ find $\frac{dx}{dt}$ when $y=2$ and $\frac{dy}{dt}=4$

(i) Find x:

$x^2(2)+(2)^2=22$

$2x^2+4=22$

$2x^2=18$

$x^2=9$

$x=\pm 3$

(ii) Find $\frac{dx}{dt}$

$x^2y+y^2=22$

$2x\frac{dx}{dt}y+x^2\frac{dy}{dt}+2y\frac{dy}{dt}=0$

(i) $2(3)\frac{dx}{dt}(2)+(3)^2(4)+2(2)(4)=0$

$12\frac{dx}{dt}+36+16=0$

$12\frac{dx}{dt}=-52$

$\boxed{\frac{dx}{dt}=-4.3}$

(ii) $2(-3)\frac{dx}{dt}(2)+(-3)^2(4)+2(2)(4)=0$

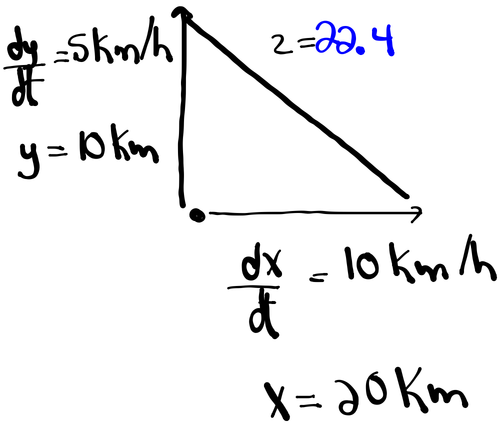
$-12\frac{dx}{dt}+36+16=0$

$-12\frac{dx}{dt}=-52$

$\boxed{\frac{dx}{dt}=4.3}$

Review 1

9



(i) Find z:

$$x^2 + y^2 = z^2$$

$$(20)^2 + (10)^2 = z^2$$

$$400 + 100 = z^2$$

$$500 = z^2$$

$$\pm 22.4 = z$$

$$\boxed{22.4 = z}$$

(ii) Find $\frac{dz}{dt}$:

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(20)(10) + 2(10)(5) = 2(22.4)\frac{dz}{dt}$$

$$400 + 100 = 44.8\frac{dz}{dt}$$

$$500 = 44.8\frac{dz}{dt}$$

$$\boxed{11.2 km/h = \frac{dz}{dt}}$$

Review 2:

① a) $x^3+2x^2y+y^3=12$

$$3x^2+4xy+2x^2\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$$

$$2x^2\frac{dy}{dx}+3y^2\frac{dy}{dx}=-3x^2-4xy$$

$$\frac{dy}{dx}(2x^2+3y^2)=-3x^2-4xy$$

$$\frac{dy}{dx}=\frac{-3x^2-4xy}{2x^2+3y^2}=-\frac{3x^2+4xy}{2x^2+3y^2}$$

b) $2xy^2-y^3=x^2$

$$2y^2+2x\,2y\frac{dy}{dx}-3y^2\frac{dy}{dx}=2x$$

$$2y^2+4xy\frac{dy}{dx}-3y^2\frac{dy}{dx}=2x$$

$$\frac{dy}{dx}(4xy-3y^2)=2x-2y^2$$

$$\boxed{\frac{dy}{dx}=\frac{2x-2y^2}{4xy-3y^2}}$$

Review 2

2) $s = 2t^3 - 21t^2 + 60t$

$v = 6t^2 - 42t + 60$

$a = 12t - 42$

a) $v = 6(3)^2 - 42(3) + 60$

$v = 54 - 126 + 60$

$v = -12\ m/s$

b) $0 = 6t^2 - 42t + 60$

$0 = 6(t^2 - 7t + 10)$

$0 = 6(t-2)(t-5)$

$t - 2 = 0$ | $t - 5 = 0$

$t = 2$ sec | $t = 5$ sec

c) $a = 12(4) - 42$

$a = 48 - 42$

$a = 6\ m/s^2$

d) $12t - 42 > 0$

$12t > 42$

$t > 3.5$ sec

Review 2

③ Given:

$\frac{dA}{dt} = -6m^2/\text{hour}$

$\frac{dr}{dt} = ?$

$r = 3m$

$A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$-6 = 8\pi(3)\frac{dr}{dt}$

$-6 = 24\pi \frac{dr}{dt}$

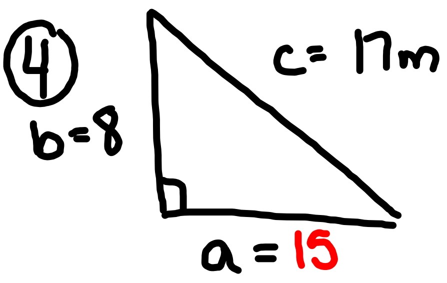
$\frac{-6}{24\pi} = \frac{dr}{dt}$

$-\frac{1}{4\pi}\ m/\text{hour} = \frac{dr}{dt}$

$-0.0796 m/\text{hour} = \frac{dr}{dt}$

Review 2

4



Given

$\frac{db}{dt} = -4 m/s$ $\quad \frac{da}{dt} = ?$

$b = 8$ $\quad a = 15$

$$a^2 + b^2 = c^2$$

$$a^2 + b^2 = 17^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$2(15)\frac{da}{dt} + 2(8)(-4) = 0$$

$$30\frac{da}{dt} = 64$$

$$\frac{da}{dt} = 2.13 \text{ m/s}$$

Review 2

5 Given:

$\frac{r}{\cancel{h}} = \frac{3}{6}$ h

$r = \frac{3}{6}h = \frac{1}{2}h$

$\frac{dV}{dt} = \pi m^3/min$

$h = 3$

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{4} \cdot h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\pi = \frac{1}{4}\pi(3)^2 \frac{dh}{dt}$

$\pi = \frac{9\pi}{4}\frac{dh}{dt}$

$4/9 \text{ m/min} = \frac{dh}{dt}$

$\boxed{0.\overline{4} \text{ m/min} = \frac{dh}{dt}}$

Review 2

⑥ Given:

$\frac{dV}{dt} = 12\,m^3/min$

$r = 4m$

$\frac{dr}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$12 = 4\pi(4)^2 \frac{dr}{dt}$

$12 = 64\pi \frac{dr}{dt}$

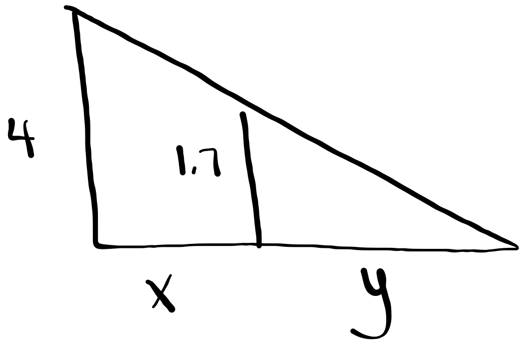
$\frac{12}{64\pi} = \frac{dr}{dt}$

$\frac{3}{16\pi}\,m/min = \frac{dr}{dt}$

$0.0597\,m/min = \frac{dr}{dt}$

Review 2

7



$\frac{dx}{dt} = -0.8\text{m/s}$ $\frac{dy}{dt} = ?$

$$\frac{x+y}{4} = \frac{y}{1.7}$$

$$1.7x + 1.7y = 4y$$

$$1.7x = 2.3y$$

$$1.7\frac{dx}{dt} = 2.3\frac{dy}{dt}$$

$$1.7(-0.8) = 2.3\frac{dy}{dt}$$

$$-1.36 = 2.3\frac{dy}{dt}$$

$$-0.59\text{m/s} = \frac{dy}{dt}$$

Review 2

⑧ $x = y^3 + y$ show that $y'' = \dfrac{-6y}{(3y^2+1)^3}$

$$1 = 3y^2\frac{dy}{dx} + \frac{dy}{dx}$$

$$1 = \frac{dy}{dx}\left[3y^2+1\right]$$

$$\frac{1}{3y^2+1} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = y' = \frac{1}{3y^2+1} = (3y^2+1)^{-1}$$

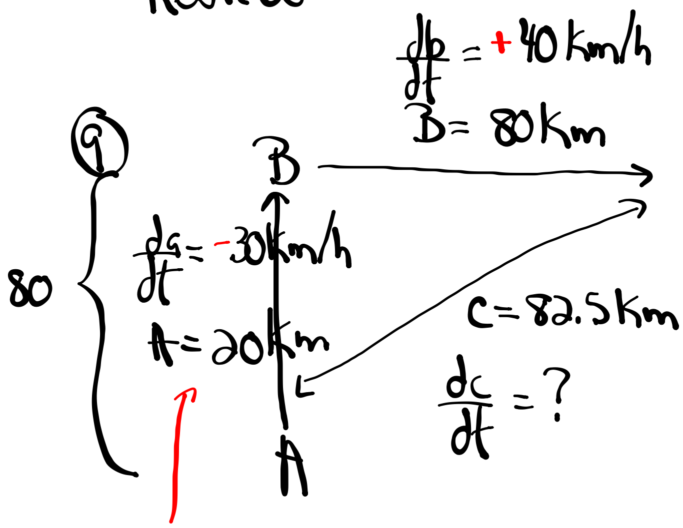
$$y'' = -1(3y^2+1)^{-2}\left(6y\frac{dy}{dx}\right)$$

$$y'' = \frac{-6y\frac{dy}{dx}}{(3y^2+1)^2} = \frac{-6y\left(\frac{1}{3y^2+1}\right)}{(3y^2+1)^2} = \frac{-6y}{3y^2+1} \times \frac{1}{(3y^2+1)^2}$$

$$= \frac{-6y}{(3y^2+1)^3}$$

Review 2

(9)



80 · 60

(i) Find c

$$a^2 + b^2 = c^2$$

$$20^2 + 80^2 = c^2$$

$$400 + 6400 = c^2$$

$$\sqrt{6800} = c$$

$$82.5 = c$$

(ii) Find $\frac{dc}{dt}$

$$a^2 + b^2 = c^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

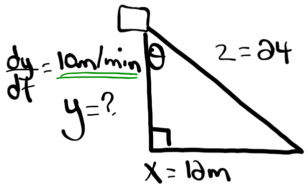
$$2(20)(-30) + 2(80)(40) = 2(82.5)\frac{dc}{dt}$$

$$-1200 + 6400 = 165\frac{dc}{dt}$$

$$5200 = 165\frac{dc}{dt}$$

$$31.5 \text{ km/h} = \frac{dc}{dt}$$

A man stands 12 metres away from a flagpole. He holds onto a long rope attached to the flag. As the flag is raised at a rate of 10 metres per minute, the rope runs tautly through the man's hands (so that it is always kept straight). Find the rate of change of the angle between the rope and the flagpole, at the moment when there is 24 metres of rope between the flag and the man.



Find $\frac{d\theta}{dt}$

Find y:

$x^2 + y^2 = z^2$

$y^2 = z^2 - x^2$

$y^2 = 24^2 - 12^2$

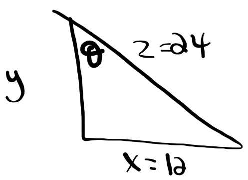
$y^2 = 576 - 144$

$y^2 = 432$

$\cancel{y = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}}$

$\cancel{y = 12\sqrt{3}}$

Find $\cos^2\theta$



$\cos^2\theta = \frac{y^2}{z^2}$

$\cos^2\theta = \frac{432}{576}$

$\cos^2\theta = \frac{3}{4}$

$\tan\theta = \frac{12}{y} = 12y^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -12y^{-2}\frac{dy}{dt}$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{12}{y^2}\frac{dy}{dt}$

$\frac{d\theta}{dt} = \left(-\frac{12}{y^2}\right)\left(\frac{dy}{dt}\right)\left(\frac{1}{\sec^2\theta}\right)$

$\frac{d\theta}{dt} = \left(\frac{-12}{y^2}\right)\left(\frac{dy}{dt}\right)\left(\cos^2\theta\right)$

$\frac{d\theta}{dt} = \left(\frac{-12}{432}\right)(10)\left(\frac{3}{4}\right)$

$\frac{d\theta}{dt} = \frac{-360}{1728}$

$\boxed{\frac{d\theta}{dt} = -\frac{5}{24} \text{ rads/min}}$

$A = l^2$ (Square)

$V = l^3$ (Cube)

$A = \pi r^2$ (Circle)

$A = 4\pi r^2$ (Sphere)

$V = \frac{4}{3}\pi r^3$ (Sphere)

$A = \frac{1}{2}bh$ (Triangle)

$V = \frac{1}{3}\pi r^2 h$ (Cone)

$x^2 + y^2 = z^2 \leftarrow$ z is constant (Ladder)

$x^2 + y^2 = z^2$ (Intersection)

$\frac{h^{\text{of post}}}{x+y} = \frac{h^{\text{of man}}}{y}$ (Lamppost)

Steps in Logarithmic Differentiation

1. Take logarithms of both sides of an equation. (ln)
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y'

Use Logarithmic Differentiation to Differentiate the following:

$$y = \frac{e^x\sqrt{x^2+1}}{\left(x^2+2\right)^3}$$

$$\ln y = \ln\left[\frac{e^x(x^2+1)^{1/2}}{(x^2+2)^3}\right]$$

$$\ln y = \ln e^x + \ln(x^2+1)^{1/2} - \ln(x^2+2)^3$$

$\ln e = 1$

$$\ln y = x\ln e + \frac{1}{2}\ln(x^2+1) - 3\ln(x^2+2)$$

$$\ln y = x + \frac{1}{2}\ln(x^2+1) - 3\ln(x^2+2)$$

$$\frac{y'}{y} = 1 + \frac{1}{2}\left(\frac{2x}{x^2+1}\right) - 3\left(\frac{2x}{x^2+2}\right)$$

$\cancel{y}\cdot$

$$\frac{y'}{\cancel{y}} = \left(1 + \frac{x}{x^2+1} - \frac{6x}{x^2+2}\right)\cdot y$$

$$y' = \left[1 + \frac{x}{x^2+1} - \frac{6x}{x^2+2}\right]\left[\frac{e^x\sqrt{x^2+1}}{(x^2+2)^3}\right]$$

$$y = x^{x^5}$$

$$\ln y = \ln x^{x^5}$$

$$\ln y = x^5 \ln x$$ product rule

$$\frac{y'}{y} = 5x^4 \ln x + x^5\left(\frac{1}{x}\right)$$

$$\cancel{y} \cdot \quad \frac{y'}{\cancel{y}} = 5x^4 \ln x + x^4 \cdot y$$

$$y' = (5x^4 \ln x + x^4)(x^{x^5})$$