

# Questions From Homework

# Making a Complete Sketch

**Example:**

Examine the function  $f(x) = 3x^5 - 5x^3$  with respect to...

- Intercepts  $f(x)$
- ~~Symmetry~~
- Asymptotes (No asymptotes for polynomial functions)
- Intervals of Increase or Decrease  $f'(x)$
- Local Maximum and Minimum values  $f'(x)$
- ~~Concavity and Points of Inflection  $f''(x)$~~
- Sketch the Curve

$$f(x) = 3x^5 - 5x^3 \quad f'(x) = 15x^4 - 15x^2 \quad f''(x) = 60x^3 - 30x$$

$$f(x) = x^3(3x^2 - 5) \quad f'(x) = 15x^2(x^2 - 1) \quad f''(x) = 30x(x^2 - 1)$$

$$f'(x) = 15x^2(x-1)(x+1)$$

① x-int ( $y=0$ )

$$f(x) = x^3(3x^2 - 5)$$

$$0 = x^3(3x^2 - 5)$$

$$x^3 = 0 \quad 3x^2 - 5 = 0$$

$$x = 0 \quad 3x^2 = 5$$

$$(0, 0) \quad x^2 = \frac{5}{3}$$

$$x = \pm\sqrt{\frac{5}{3}}$$

$$(1.29, 0) \quad + (-1.29, 0)$$

② y-int ( $x=0$ )

$$f(x) = 3x^5 - 5x^3$$

$$f(0) = 3(0)^5 - 5(0)^3$$

$$f(0) = 0$$

$$(0, 0)$$

③ Intervals of Inc/Dec.

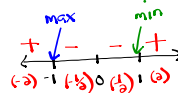
$$f'(x) = 15x^2(x-1)(x+1)$$

$$0 = 15x^2(x-1)(x+1)$$

$$15x^2 = 0 \quad x-1 = 0 \quad x+1 = 0$$

$$x^2 = 0 \quad x = 1 \quad x = -1$$

$$x = 0$$



Increasing on  $(-\infty, -1) + (1, \infty)$   
 Decreasing on  $(-1, 0) + (0, 1)$   
 or  $(-1, 1)$

CV:  $x = -1, 0, 1$

④ max @  $x = -1$

$$f(x) = 3x^5 - 5x^3$$

$$f(-1) = 3(-1)^5 - 5(-1)^3$$

$$f(-1) = -3 + 5$$

$$f(-1) = 2$$

$$(-1, 2)$$

min @  $x = 1$

$$f(x) = 3x^5 - 5x^3$$

$$f(1) = 3(1)^5 - 5(1)^3$$

$$f(1) = 3 - 5$$

$$f(1) = -2$$

$$(1, -2)$$

⑤ Intervals of Concavity:

$$f''(x) = 30x(x^2 - 1)$$

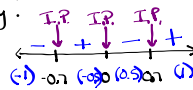
$$0 = 30x(x^2 - 1)$$

$$30x = 0 \quad x^2 - 1 = 0$$

$$x = 0 \quad x^2 = 1$$

$$x = \pm\sqrt{1}$$

$$x = \pm 1$$



CD on  $(-\infty, -1) + (0, 1)$   
 CU on  $(-1, 0) + (1, \infty)$

CV:  $x = -0.7, 0, 0.7$

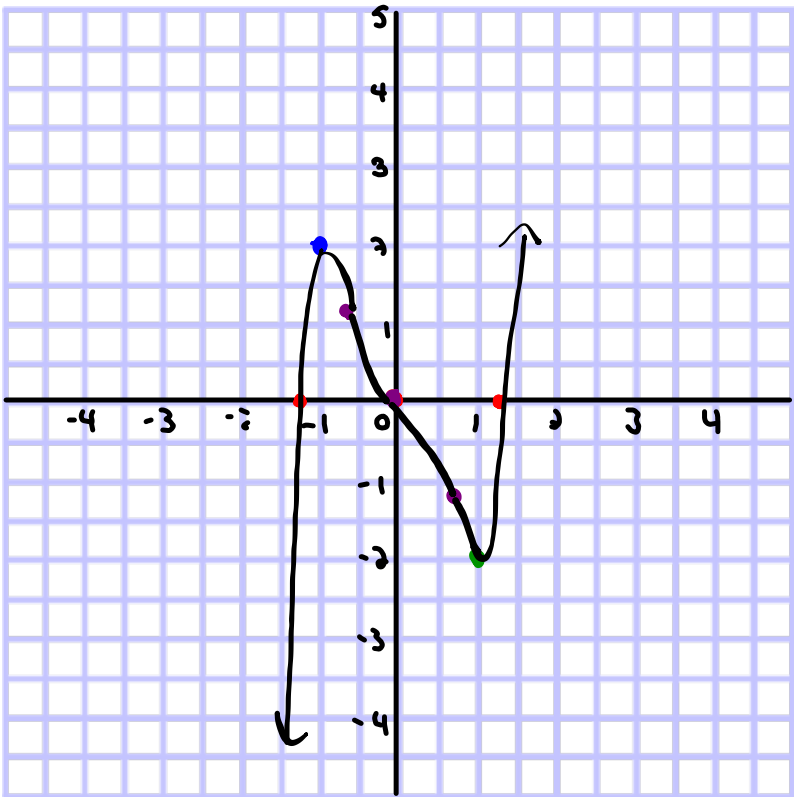
⑥ Inflection Points

$$f(x) = 3x^5 - 5x^3$$

$$f(-0.7) = 3(-0.7)^5 - 5(-0.7)^3 = -0.504 + 1.715 = 1.2 \quad (-0.7, 1.2)$$

$$f(0) = 3(0)^5 - 5(0)^3 = 0 - 0 = 0 \quad (0, 0)$$

$$f(0.7) = 3(0.7)^5 - 5(0.7)^3 = 0.504 - 1.715 = -1.2 \quad (0.7, -1.2)$$



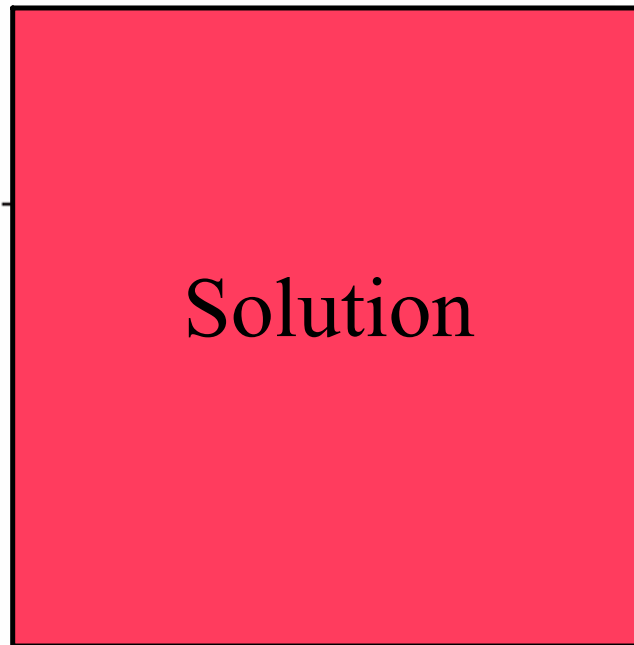
Assignment;

Sketch  $f(x) = x^2 + x^3$

# homework

Examine the function  $f(x) = x^4 - 4x^3$  with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve



# homework

Examine the function  $f(x) = \frac{x^2}{1-x^2}$  with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve



# homework

Examine the function  $f(x) = \frac{x^2}{x-7}$  with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve