

Higher Derivatives

Since the derivative of a function f is itself a function f' , we can take its derivative $(f')'$. The result is a function called the **second derivative** of f and is denoted by f''

In Leibniz notation we write:

$$f''(x) = \frac{d^2 y}{dx^2}$$

Find $y'' = f''(x) = \frac{d^2 y}{dx^2}$

Examples

$$y = x^6$$

$$y' = 6x^5$$

$$y'' = 30x^4$$

$$f(x) = 5x^2 + \sqrt{x} = 5x^2 + x^{1/2}$$

$$f'(x) = 10x + \frac{1}{2}x^{-1/2} = 10x + \frac{1}{2}x^{1/2}$$

$$f''(x) = 10 - \frac{1}{4}x^{-3/2} = 10 - \frac{1}{4x^{3/2}}$$

$$f(x) = (2 - x^2)^{10}$$

$$f'(x) = 10(2 - x^2)^9 (-2x) = \underbrace{-20x}_f \underbrace{(2 - x^2)^9}_g$$

$$f''(x) = -20(2 - x^2)^9 - 20x(9)(2 - x^2)^8(-2x)$$

$$= -20(2 - x^2)^9 + 360x^2(2 - x^2)^8$$

$$= -20(2 - x^2)^8 [2 - x^2 - 18x^2]$$

$$= -20(2 - x^2)^8 (2 - 19x^2)$$

Since the first derivative of a function can be interpreted either as the slope of a tangent line or as a rate of change, the second derivative can be interpreted as *the rate of change of the slope of the tangent line*.

This idea will be pursued later where the second derivative gives valuable information about the shape of a graph, or when dealing with acceleration.

Higher derivatives can also be defined. The **third derivative** is the derivative of the second derivative.

$$y''' = f'''(x) = \frac{d^3 y}{dx^3}$$

Find the first five derivatives of

$$y = x^4 + 2x^3 - 5x^2 + 3x - 6$$

$$y' = 4x^3 + 6x^2 - 10x + 3$$

$$y'' = 12x^2 + 12x - 10$$

$$y''' = 24x + 12$$

$$y^{(4)} = 24$$

$$y^{(5)} = 0$$

Homework

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$$\textcircled{1} \text{ c) } f(t) = 2t - \frac{1}{t+1} = 2t - 1(t+1)^{-1}$$

$$f'(t) = 2 + (t+1)^{-2} (1) = 2 + \frac{1}{(t+1)^2}$$

$$f''(t) = -2(t+1)^{-3} (1) = \frac{-2}{(t+1)^3}$$

$$9) \ y = \sqrt{x^2+1} = (x^2+1)^{1/2}$$

$$y' = \frac{1}{2}(x^2+1)^{-1/2} (2x) = x(x^2+1)^{-1/2} = \frac{x}{\sqrt{x^2+1}}$$

$$y'' = 1(x^2+1)^{-1/2} + x(-\frac{1}{2}(x^2+1)^{-3/2} (2x))$$

$$y'' = (x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2} \quad \begin{array}{l} -\frac{1}{2}(-\frac{3}{2}) \\ -\frac{1}{2} + \frac{3}{2} \\ \frac{2}{2} \\ 1 \end{array}$$

$$y'' = (x^2+1)^{-3/2} [(x^2+1)' - x^2]$$

$$y'' = \frac{1}{(x^2+1)^{3/2}} = \frac{1}{\sqrt{(x^2+1)^3}}$$