Questions from Homework

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⑥ $s = t^3 - 3t^2 - 5t$, $t \geq 0$

$s' = 3t^2 - 6t - 5$ (velocity)

$4 = 3t^2 - 6t - 5$

$0 = 3t^2 - 6t - 9$ factor

$0 = 3(t^2 - 2t - 3)$

$0 = 3(t+1)(t-3)$

$t+1=0$ | $t-3=0$

$t=-1$ | $t=3$

⑦ $s = t^2 - 4t + 4$, $t \geq 0$

$s' = 2t - 4$ (velocity)

a) $s'(1) = 2(1) - 4$ | $s'(3) = 2(3) - 4$

$s'(1) = -2\ m/s$ | $s'(3) = 2\ m/s$

b) Let v or $s' = 0$

$s' = 2t - 4$

$0 = 2t - 4$

$0 = 2(t - 2)$

$t - 2 = 0$

$t = 2$

c) Let v or $s' > 0$

$2t - 4 > 0$

$2t > 4$

$t > 2$

⑧ $s = t^3 - 15t^2 + 63t$

$v = 3t^2 - 30t + 63$

a) Let $v = 0$

$0 = 3t^2 - 30t + 63$

$0 = 3(t^2 - 10t + 21)$

$0 = 3(t-7)(t-3)$

$t-7=0$ | $t-3=0$

$t=7s$ | $t=3s$

b) $v > 0$

$3t^2 - 30t + 63 > 0$

$3(t-7)(t-3) > 0$

① $t-7>0$ | $t-3>0$

$t>7$ | $t>3$

② $t-7<0$ | $t-3<0$

$t<7$ | $t<3$

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Velocity

④ $h = 24.5t - 4.9t^2$

$v = 24.5 - 9.8t$

b) Let $v = 0$

$0 = 24.5 - 9.8t$

$9.8t = 24.5$

$t = 2.5s$

c) Find $h(2.5)$

$h = 24.5(2.5) - 4.9(2.5)^2$

$h = 30.6m$

d) Let $h = 0$

$0 = 24.5t - 4.9t^2$

$0 = t(24.5 - 4.9t)$

$t = 0$ | $24.5 - 4.9t = 0$

$24.5 = 4.9t$

$5 = t$

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③ d) $s = \sqrt{t^2+t} = (t^2+t)^{1/2}$

$v = \frac{1}{2}(t^2+t)^{-1/2}(2t+1)$

$v = \frac{2t+1}{2(t^2+t)^{1/2}}$

$a = \frac{2(t^2+t)^{1/2}(2) - (2t+1)(1)(t^2+t)^{-1/2}(2t+1)}{[2(t^2+t)^{1/2}]^2}$

$a = \frac{4(t^2+t)^{1/2} - (2t+1)^2(t^2+t)^{-1/2}}{4(t^2+t)}$

FOIL

$a = \frac{(t^2+t)^{-1/2}[4(t^2+t) - (2t+1)^2]}{4(t^2+t)}$

$a = \frac{4t^2+4t-4t^2-4t-1}{4(t^2+t)^{3/2}}$

$a = \frac{-1}{4(t^2+t)^{3/2}}$

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④ d) $s = \frac{5t}{1+t}$

$s' = \frac{5(1+t) - 5t(1)}{(1+t)^2} = \frac{5+5t-5t}{(1+t)^2} = \frac{5}{(1+t)^2}$ (velocity)

$s'' = \frac{(0)(1+t)^2 - 5(2)(1+t)(1)}{(1+t)^4} = \frac{-10(1+t)}{(1+t)^{4\,3}} = \frac{-10}{(1+t)^3}$

$s''(4) = \frac{-10}{(1+4)^3} = \frac{-10}{125} = -\frac{2}{25}\ m/s^2$

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time.*

Differentiate with respect to time

If $xy^2 = 12$ and $\frac{dy}{dt} = 6$, Find $\frac{dx}{dt}$ when $y = 2$

(i) Find x

$x(2)^2 = 12$

$\frac{4x}{4} = \frac{12}{4}$

$x = 3$

(ii) differentiate wrt

$(xy^2) = 12$

$\frac{dx}{dt} y^2 + x 2y \frac{dy}{dt} = 0$

$y^2 \frac{dx}{dt} + 2xy \frac{dy}{dt} = 0$

(iii) Solve for $\frac{dx}{dt}$

$y^2 \frac{dx}{dt} + 2xy \frac{dy}{dt} = 0$

$(2)^2 \frac{dx}{dt} + 2(3)(2)(6) =$

$4\frac{dx}{dt} + 72 = 0$

$4\frac{dx}{dt} = -72$

$\frac{dx}{dt} = -18$

If $x^3 + y^3 = 9$ and $\frac{dx}{dt} = 4$, Find $\frac{dy}{dt}$ when ~~$x = 2$~~

(i) Find y

$x^3 + y^3 = 9$

$(2)^3 + y^3 = 9$

$8 + y^3 = 9$

$y^3 = 1$

$y = 1$

(ii) differentiate wrt. t

$x^3 + y^3 = 9$

$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$

(iii) Solve for $\frac{dy}{dt}$

$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$

$3(2)^2(4) + 3(1)^2\frac{dy}{dt} = 0$

$48 + 3\frac{dy}{dt} = 0$

$3\frac{dy}{dt} = -48$

$\frac{dy}{dt} = -16$