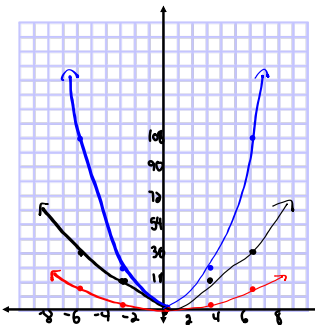


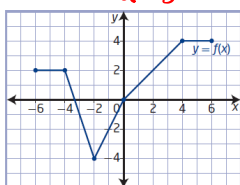
Questions from Homework

2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
-6	36	108	12
-3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

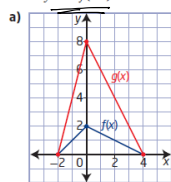


6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a = 2$

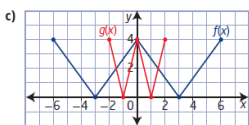


$(x, y) \rightarrow (x, 2y)$
 $f(x)$ $g(x)$
 D: $[-6, 6]$ D: $[-6, 6]$
 R: $[-4, 4]$ R: $[-8, 8]$

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.



$(x, y) \rightarrow (x, 4y)$ A vertical stretch by a factor of 4
 $f(x)$ $g(x)$
 $(-2, 0)$ $(-2, 0)$ $a = 4$
 $(0, 2)$ $(0, 8)$
 $(4, 0)$ $(4, 0)$ $y = 4f(x)$



$(x, y) \rightarrow (\frac{1}{3}x, y)$ A horizontal compression by a factor of 1/3
 $f(x)$ $g(x)$
 $(-6, 4)$ $(-2, 4)$
 $(-3, 0)$ $(-1, 0)$ $b = 3$
 $(0, 4)$ $(0, 4)$
 $(3, 0)$ $(1, 0)$ $y = f(3x)$
 $(6, 4)$ $(2, 4)$

5 a) $y = 4f(x)$

$a = 4 \rightarrow$ A vertical stretch about the x-axis by a factor of 4

$(x, y) \rightarrow (x, 4y)$

b) $y = f(3x)$

$b = 3 \rightarrow$ A horizontal compression about the y-axis by a factor 1/3

$(x, y) \rightarrow (\frac{1}{3}x, y)$

Warm-Up...

$$y = a f[b(x-h)] + k$$

Given that $(-2, 5)$ is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3 \rightarrow$ vertically stretched about the x-axis by a factor of 3

$b = 1 \rightarrow$ no horizontal stretch.

$h = 0 \rightarrow$ no horizontal trans

$k = 0 \rightarrow$ no vertical trans.

$$(x, y) \rightarrow (x, 3y)$$

$$(-2, 5) \rightarrow \boxed{(-2, 15)}$$

(2) $y = f\left(-\frac{1}{3}x\right)$

$a = 1 \rightarrow$ no vertical stretch

$b = -\frac{1}{3} \rightarrow$ horizontally stretched about the y-axis by a factor 3 and a reflection in the y-axis

$h = 0 \rightarrow$ no horizontal trans.

$k = 0 \rightarrow$ no vertical trans.

$$(x, y) \rightarrow (-3x, y)$$

$$(-2, 5) \rightarrow \boxed{(6, 5)}$$

(3) $y = 4f\left(\frac{1}{2}(x+5)\right) - 3$

$a = 4 \rightarrow$ vertically stretched about the x-axis by a factor of 4

$b = \frac{1}{2} \rightarrow$ horizontally stretched about the y-axis by a factor of 2.

$h = -5 \rightarrow$ horizontally translated 5 units left

$k = -3 \rightarrow$ vertically translated 3 units down

$$(x, y) \rightarrow (2x - 5, 4y - 3)$$

$$(-2, 5) \rightarrow \boxed{(-9, 17)}$$

(4) $y = -2f(-2x+6) + 5$

$$y = -2f[-2(x-3)] + 5$$

$a = -2 \rightarrow$ vertically stretched about the x-axis by a factor of 2 and reflected in the x-axis

$b = -2 \rightarrow$ horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and reflected in the y-axis

$h = 3 \rightarrow$ horizontally trans 3 units right

$k = 5 \rightarrow$ vertically trans 5 units up

$$(x, y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$$

$$(-2, 5) \rightarrow \boxed{(4, -5)}$$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

$$g(x) = -3f(4x - 16) - 10$$

factor

$$g(x) = \underline{-3}f[\underline{4}(x - \underline{4})] - \underline{10}$$

$$a = -3 \quad b = 4 \quad h = 4 \quad k = -10$$

a) y-axis

b) $\frac{1}{4}$

c) x-axis

d) 3

e) x-axis

f) 4

g) 10

Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + k$	shift $f(x)$ up k units
$f(x) - k$	shift $f(x)$ down k units
$f(x + h)$	shift $f(x)$ left h units
$f(x - h)$	shift $f(x)$ right h units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$af(x)$	When $0 < a < 1$ - vertical shrinking of $f(x)$
	When $a > 1$ - vertical stretching of $f(x)$
$f(bx)$	When $0 < b < 1$ - horizontal stretching of $f(x)$
	When $b > 1$ - horizontal shrinking of $f(x)$

$(x, y) \rightarrow (x, y+k)$
 $(x, y) \rightarrow (x, y-k)$
 $(x, y) \rightarrow (x-h, y)$
 $(x, y) \rightarrow (x+h, y)$
 $(x, y) \rightarrow (-x, y)$
 $(x, y) \rightarrow (x, -y)$
 $(x, y) \rightarrow (x, ay)$
 $(x, y) \rightarrow (\frac{1}{b}x, y)$

vertical trans.
 horizontal trans.
 horizontal ref.
 vertical ref.
 Multiply the y values by a
 Divide the x values by b or multiply by $\frac{1}{b}$

Transformations:

$$y = f(x) \longrightarrow y = \underline{a}f(\underline{b}(x - \underline{h})) + \underline{k}$$

Mapping Rule:



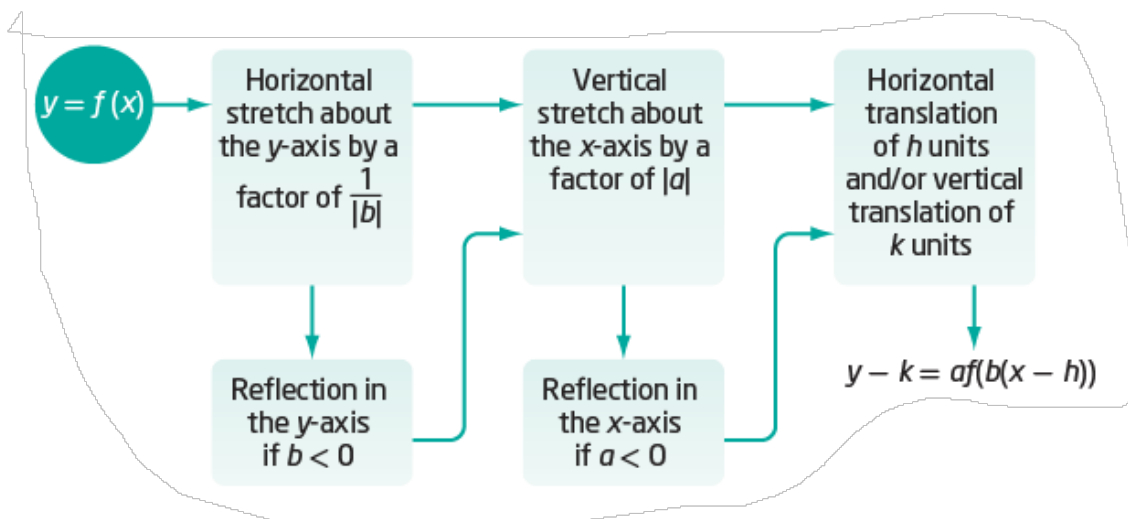
$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k \right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember...RST



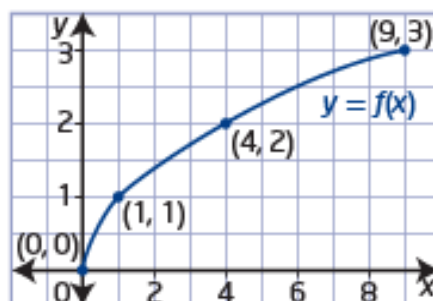
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

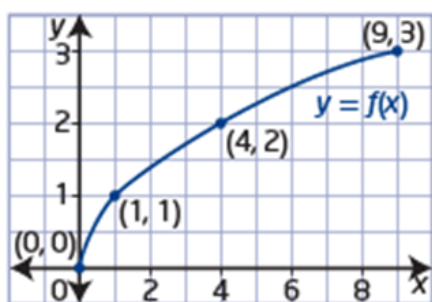
b) $y = f(3x + 6)$



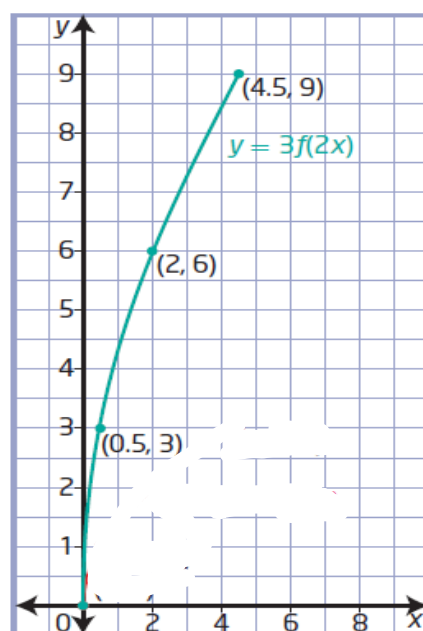
a) $y = 3f(2x)$ $a=3$ $b=2$ $h=0$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$$(x, y) \rightarrow \left[\frac{1}{2}x, 3y \right]$$



$f(x)$	$g(x)$
$(0, 0)$	$(0, 0)$
$(1, 1)$	$(\frac{1}{2}, 3)$
$(4, 2)$	$(2, 6)$
$(9, 3)$	$(\frac{9}{2}, 9)$

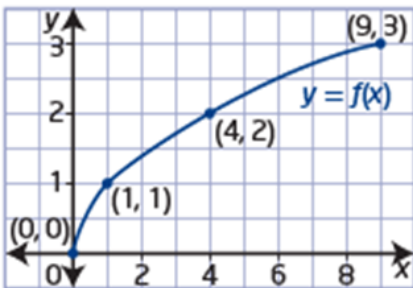


b) $y = f(3x + 6)$ $a=1$ $b=3$ $h=-2$ $k=0$

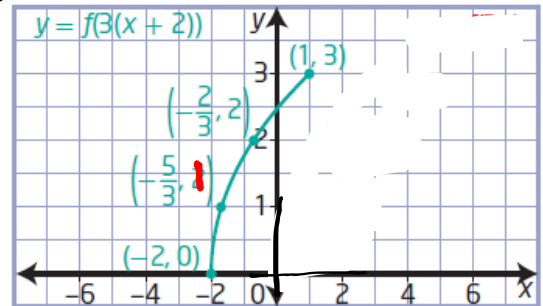
$y = f[3(x+2)] =$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x,y) \rightarrow [\frac{1}{3}x - 2, y]$



$f(x)$	$g(x)$
$(0,0)$	$(-2,0)$
$(1,1)$	$(-\frac{5}{3},1)$
$(4,2)$	$(-\frac{2}{3},2)$
$(9,3)$	$(1,3)$



$\frac{1}{3}x - 2$	$\frac{1}{3}x - 2$
$\frac{1}{3}(1) - 2$	$\frac{1}{3}(4) - 2$
$\frac{1}{3} - \frac{2}{1}$	$\frac{4}{3} - \frac{2}{1}$
$\frac{1}{3} - \frac{6}{3}$	$\frac{4}{3} - \frac{6}{3}$
$-\frac{5}{3}$	$-\frac{2}{3}$

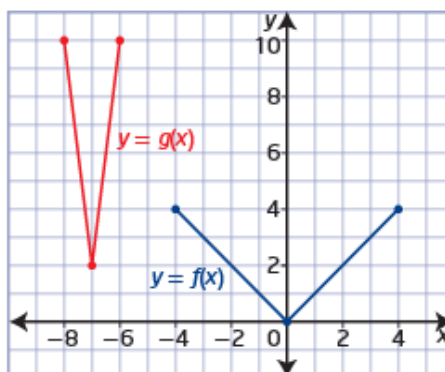
Homework

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Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

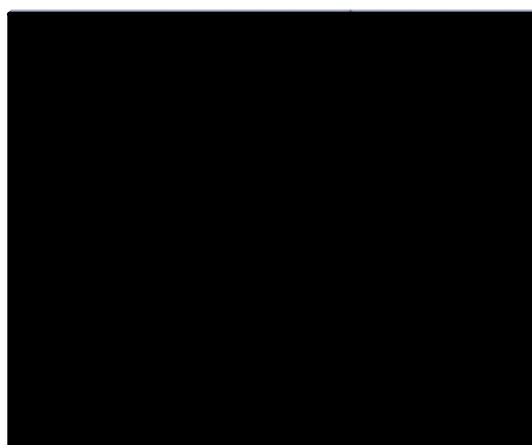
Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$$(-4, 4) \rightarrow (-8, 10)$$

$$(0, 0) \rightarrow (-7, 2)$$

$$(4, 4) \rightarrow (-6, 10)$$

The equation of the transformed function is XXXXXXXXXX



How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x -axis by a factor of 2, stretched horizontally about the y -axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x -axis by a factor of 2. stretched horizontally about the y -axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.