Chapter Review For Final Exam:

Ch. 1 → (Inverse Functions)

Ch. 2 → (Radical Functions)

Ch. 7 → (Exponential Functions) $y = 2^x$

Ch. 8 → (Logarithmic Functions) $y = \log_2 x$

Ch. 4 → (Trig + Unit Circle)

Ch. 5 → (Trig Functions)

Ch. 6 → (Trig Identities)

Ch. 2

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

Ch. 7

$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

Ch. 8

$y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

Ch. 5

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

$y = \cos x$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$y = af[b(x-h)]+k$ Ch.1

$y = a\sqrt{b(x-h)}+k$ Ch.2

$y = ac^{b(x-h)}+k$ Ch.7

$y = a\log_c[b(x-h)]+k$ Ch.8

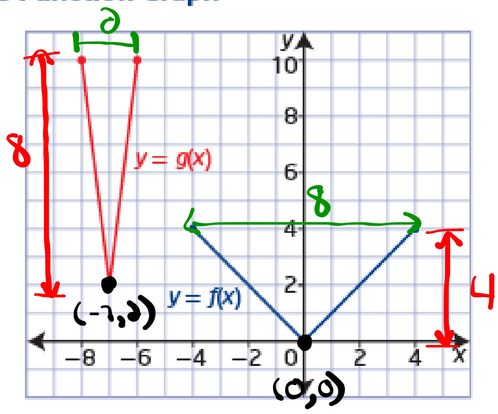
$y = a\sin[b(x-h)]+k$ Ch.5

Chapter 1:

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

① Reflection: None

② VSF $= \frac{8}{4} = 2$ $(a=2)$

③ HSF $= \frac{2}{8} = \frac{1}{4}$ $(b=4)$

④ HT: $(\underline{0},0) \rightarrow (\underline{-7},2)$ 7 units left $(h=-7)$

⑤ VT: $(0,\underline{0}) \rightarrow (-7,\underline{2})$ 2 units up $(k=2)$

⑥ Equation: $\boxed{g(x) = 2f[4(x+7)]+2}$

Chapter 2 Radical Functions

Solve for x: $(\sqrt{x+8})^2 = (x+6)^2$

$x+8 = (x+6)(x+6)$

$x+8 = x^2+12x+36$

$0 = x^2+11x+28$

$\underline{4} + \underline{7} = 11$
$\underline{4} \times \underline{7} = 28$

$0 = (x+4)(x+7)$

$x+4=0$	$x+7=0$
$x=-4$	$x=-7$

is a solution

Test $x=-4$

$\sqrt{x+8} = x+6$

$\sqrt{-4+8}$	$-4+6$
$\sqrt{4}$	2 ✓
2 ✓	

is not a solution (extraneous)

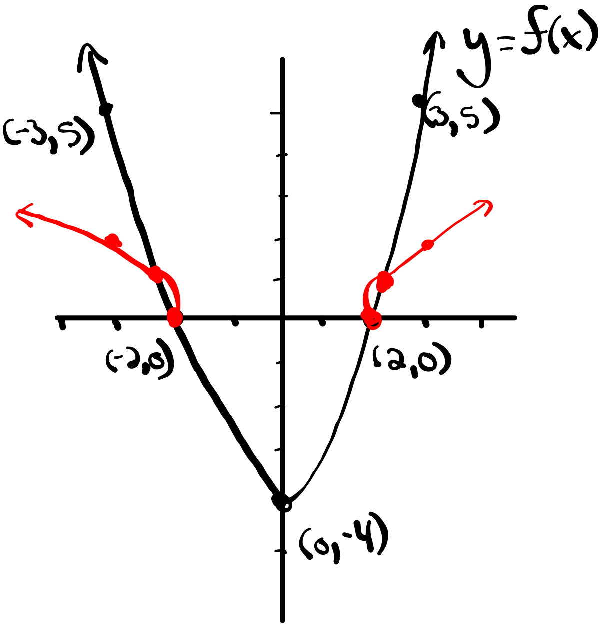
Test $x=-7$

$\sqrt{x+8} = x+6$

$\sqrt{-7+8}$	$-7+6$
$\sqrt{1}$	-1 ✗
1	

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. State the domain and range of each.



$y = f(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -4, y \in R\}$ or $[-4, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \leq -2, x \geq 2, x \in R\}$ or $(-\infty, -2] \cup [2, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

Ch. 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81(27^{x})$

a) $2^{2x+2} + 7 = 71$

$2^{2x+2} = 64$

$\cancel{2}^{2x+2} = (\cancel{2})^{6}$

$2x + 2 = 6$

$\frac{2x}{2} = \frac{4}{2}$

$\boxed{x = 2}$

b) $9^{2x+1} = 81(27^{x})$

$(3^{2})^{2x+1} = (3^{4})(3^{3})^{x}$

$3^{4x+2} = 3^{4} \cdot 3^{3x}$

$\cancel{3}^{4x+2} = \cancel{3}^{3x+4}$

$4x + 2 = 3x + 4$

$\boxed{x = 2}$

* Be sure to test your answers!

Ch. 8 → Logarithmic Functions

4. Rewrite each expression as a single logarithm.

$$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$$

$$\log_5 x^3 + \log_5 (x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5 x^3(x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5\left[\frac{x^3(x-1)^{1/2}}{x^2+1}\right] \quad \text{or} \quad \log_5\left[\frac{x^3\sqrt{x-1}}{x^2+1}\right]$$

Ch. 8

7. Solve the following equation (be sure to test your answers).

$$\log_{10}(x+2)+\log_{10}(x-1)=1$$

$$\log_{10}(x+2)(x-1) = 1$$

$$\log_{10}\underbrace{(x^2+x-2)}=1$$ (log. form)

Base Ans Exp.

$$10^1 = x^2+x-2$$ (exp. form)

$$10 = x^2+x-2$$

$$0 = x^2+x-12$$

$-3 + 4 = 1$
$-3 \times 4 = -12$

$$0=(x-3)(x+4)$$

$x-3=0$ | $x+4=0$

$\boxed{x=3}$ | $x=-4$

is a solution | extraneous

* Be sure to test your answers!

Ch.7 or Ch.8   Base = $\frac{1}{2}$ or 0.5 $A_0 = 60mg$

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

$exp = t/5.3$

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

$y = (\text{Initial Amount})(\text{Base})^{exp.}$

$\boxed{y = (60)(0.5)^{t/5.3}}$

b) What amount will be present in 10.6 years? [2] $t = 10.6$

$y = (60)(0.5)^{\frac{10.6}{5.3}}$

$y = (60)(0.5)^2$

$y = (60)(0.25) = \boxed{15mg}$

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

(i) 12.5% of initial Amount:

$= 0.125 \times 60$

$= 7.5 mg$ $(y = 7.5mg)$

(ii) Solve for t:

$y = (60)(0.5)^{t/5.3}$

$\frac{7.5}{60} = \frac{\cancel{60}(0.5)^{t/5.3}}{\cancel{60}}$

$0.125 = (0.5)^{t/5.3}$ $* \ \frac{\log(0.125)}{\log(0.5)} = 3$

$(\cancel{0.5})^3 = (\cancel{0.5})^{t/5.3}$

$(5.3)\,3 = \frac{t}{\cancel{5.3}}\,(\cancel{5.3})$

$\boxed{15.9 \text{ years} = t}$

Ch. 4 → Special Angles

2. Solve for all values of θ in the specified domain.

$\tan^2\theta + \tan\theta = 0$, $0 \le \theta \le 2\pi$ (Radians)

$\tan\theta(\tan\theta + 1) = 0$

$\tan\theta = 0$ (Unit Circle)

$\boxed{\theta = 0, \pi, 2\pi}$

$\tan\theta + 1 = 0$

$\tan\theta = -1$ (Special Triangle)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \tan^{-1}(1)$

$\bar{\theta} = \frac{\pi}{4}$

(ii) where is $\tan\theta < 0$

S	A
T	C

(iii) Find θ:

Q2	Q4
$\theta = \pi - \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi - \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

e. $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta < 360°$ (Degrees)

$\cos\theta\left(\cos\theta + \frac{1}{2}\right) = 0$

$\cos\theta = 0$ (Unit Circle)

$\boxed{\theta = 90°, 270°}$

$\cos\theta + \frac{1}{2} = 0$

$\cos\theta = -\frac{1}{2}$ (Special Triangles)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 60°$

(ii) Where is $\cos\theta < 0$

S	A
T	C

(iii) Find θ:

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 60° = 120°$	$\theta = 180° + 60° = 240°$

Ch. 4 → Special Angles:

$$\frac{5\tan^2\left(\frac{5\pi}{4}\right)}{6\sin\left(\frac{5\pi}{6}\right)+4\sin\left(\frac{4\pi}{3}\right)}$$

$$\frac{5(1)^2}{6\left(\frac{1}{2}\right)+4\left(-\frac{\sqrt{3}}{2}\right)}$$

$$\frac{5}{3-\frac{4\sqrt{3}}{2}}$$

$$\frac{5\,(3+2\sqrt{3})}{(3-2\sqrt{3})(3+2\sqrt{3})}$$

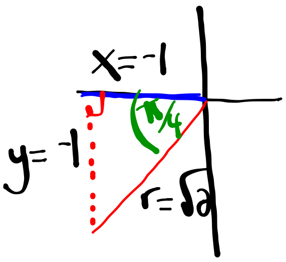
$$\frac{15+10\sqrt{3}}{9+\cancel{6\sqrt{3}}-\cancel{6\sqrt{3}}-4(3)}$$

$$\frac{15+10\sqrt{3}}{-3}$$

$$\boxed{\frac{-15-10\sqrt{3}}{3}}$$

(i) $\frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}$

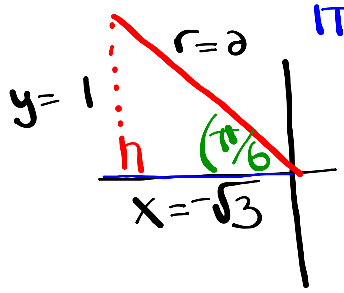
1π



$\tan\left(\frac{5\pi}{4}\right) = \frac{-1}{-1} = 1$

(ii) $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

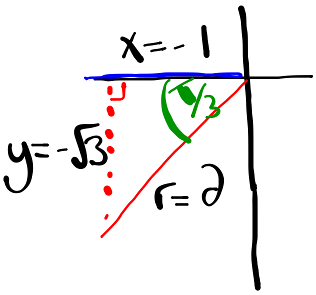
1π



$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

(iii) $\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

1π



$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Ch.4 → Special Angles

$$\sec(15\pi)+\sqrt{2}\sin\left(\frac{39\pi}{4}\right)\sin\left(\frac{21\pi}{2}\right)-\csc^2\left(\frac{100\pi}{3}\right)$$

$$(-1)+\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1)-\left(\frac{2}{-\sqrt{3}}\right)^2$$

$$-1-\frac{\sqrt{2}}{\sqrt{2}}-\frac{4}{3}$$

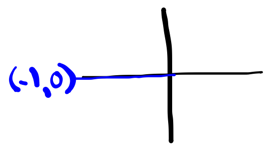
$$-1-1-\frac{4}{3}$$

$$-\frac{2}{1}-\frac{4}{3}$$

$$-\frac{6}{3}-\frac{4}{3}$$

$$-\frac{10}{3}$$

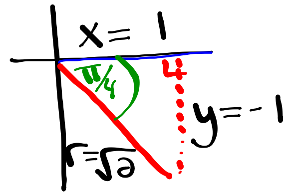
(i) 15π



$\sec(15\pi)=\frac{1}{-1}=-1$

(ii) $\frac{38\pi}{4},\frac{39\pi}{4},\frac{40\pi}{4}$

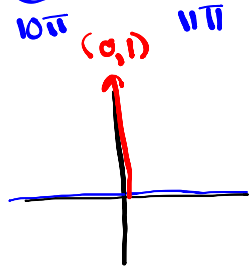
10π



$\sin\left(\frac{39\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

(iii) $\frac{20\pi}{2},\frac{21\pi}{2},\frac{22\pi}{2}$

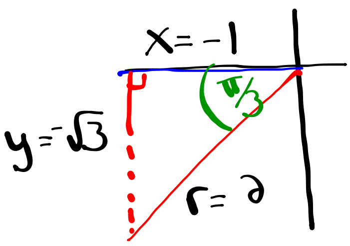
10π 11π



$\sin\left(\frac{21\pi}{2}\right)=1$

(iv) $\frac{99\pi}{3},\frac{100\pi}{3},\frac{101\pi}{3}$

33π

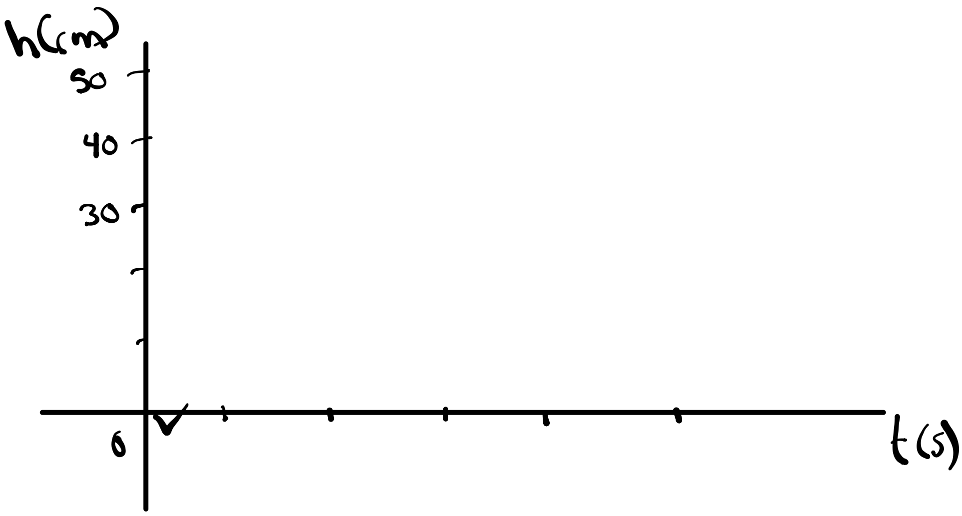


$\csc\left(\frac{100\pi}{3}\right)=\frac{2}{-\sqrt{3}}$

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 sec.

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

(b) How high was the weight above the floor when the stopwatch was initially started?



$$\frac{1}{\sec^2\theta\cot\theta} = \frac{\sin\theta - \sin^3\theta}{\cos\theta}$$

Pre-Calculus 12A Chapter 1 Exam Review

1. Given the function $y = f(x)$ write the equation of the form $y = af(b(x-h))+k$ that would result from the following transformations:

A horizontal stretch about the y-axis by a factor of $\frac{1}{4}$ and a horizontal reflection in the y-axis. A vertical stretch about the x-axis by a factor of 3, and a translation of 5 units to the right and 2 units up.

$a = 3$
$b = -4$
$h = 5$
$k = 2$

$\boxed{y = 3f[-4(x-5)] + 2}$

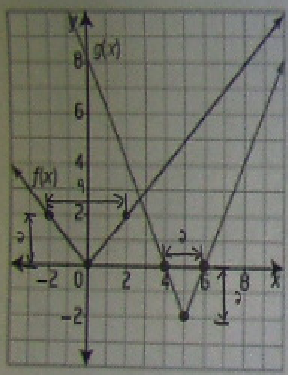
2. Determine the inverse of the function $f(x) = (x-3)^2 - 2$.

$f(x) = (x-3)^2 - 2$
$y = (x-3)^2 - 2$
$x = (y-3)^2 - 2$
$x + 2 = (y-3)^2$
$\pm\sqrt{x+2} = y - 3$

$3 \pm \sqrt{x+2} = y$

$\boxed{y = 3 \pm \sqrt{x+2}}$

3. Write the equation for the graph of $g(x)$ as a transformation of the equation for the graph of $f(x)$.



① Reflections: none

② V.S.F. $= \frac{2}{2} = 1 \rightarrow a = 1$

③ H.S.F. $= \frac{2}{4} = \frac{1}{2} \rightarrow b = 2$

④ HT: $(0,0) \rightarrow (5,-2)$ $h = 5$

⑤ VT: $(0,0) \rightarrow (5,-2)$ $k = -2$

⑥ Equation: $\boxed{g(x) = 1f[2(x-5)] - 2}$

4. The key point (12, -18) is on the graph of $y = f(x)$. Calculate its image point under the following transformation:

4. The key point (12, -18) is on the graph of $y = f(x)$. Calculate its image point under the following transformation:

a) $y+3=-\frac{1}{3}f(2x+12)$

$y = -\frac{1}{3}f[2(x+6)]-3$

$a=-\frac{1}{3}$ $b=2$ $h=-6$ $k=-3$

$(x,y) \rightarrow [\frac{1}{2}x-6, -\frac{1}{3}y-3]$

$(12,-18) \rightarrow \boxed{(0,3)}$

b) $2y-4=6f(6x-12)+4$

$2y = 6f[6(x-2)]+8$

$y = 3f[6(x-2)]+4$

$a=3$ $b=6$ $h=2$ $k=4$

$(x,y) \rightarrow [\frac{1}{6}x+2, 3y+4]$

$(12,-18) \rightarrow \boxed{(4,-50)}$

Radical Functions Exam Review

1. Given that $2y+8=-4\sqrt{-x+3}$, complete the chart shown below. When identifying translations be sure that you indicate both the number of units and direction of the shift.

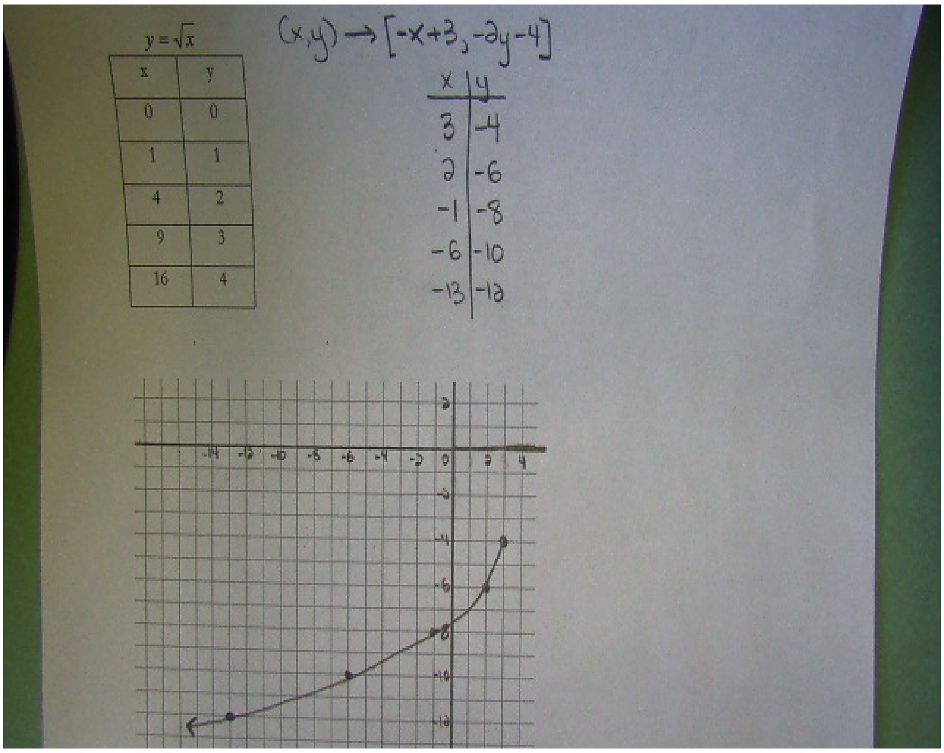
$y=-2\sqrt{-1(x-3)}-4$

$a=-2$
$b=-1$
$h=3$
$k=-4$

Reflected in x-axis	YES or NO (circle correct solution)
Reflected in y-axis	YES or NO (circle correct solution)
Horizontal translation of…	3 units right
Vertical translation of…	4 units down
Horizontally stretched by a factor of…	1 or (no stretch)
Vertically stretched by a factor of …	2
Domain	$\{x \mid x \leq 3, x \in R\}$
Range	$\{y \mid y \leq -4, y \in R\}$

Write a mapping rule and sketch the curve in the space below.

$y=\sqrt{x}$ $(x,y) \rightarrow [-x+3, -2y-4]$



$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4

$(x, y) \rightarrow [-x+3, -2y-4]$

x	y
3	-4
2	-6
-1	-8
-6	-10
-13	-12

2. Solve the following radical equation. $\sqrt{2x-6}+3=x$

$$\sqrt{2x-6} = x-3$$

$$2x-6 = x^2-6x+9$$

$$0 = x^2-8x+15$$

$$0 = (x-3)(x-5)$$

$x-3=0$ | $x-5=0$

$\boxed{x=3}$ | $\boxed{x=5}$

Test $x=3$

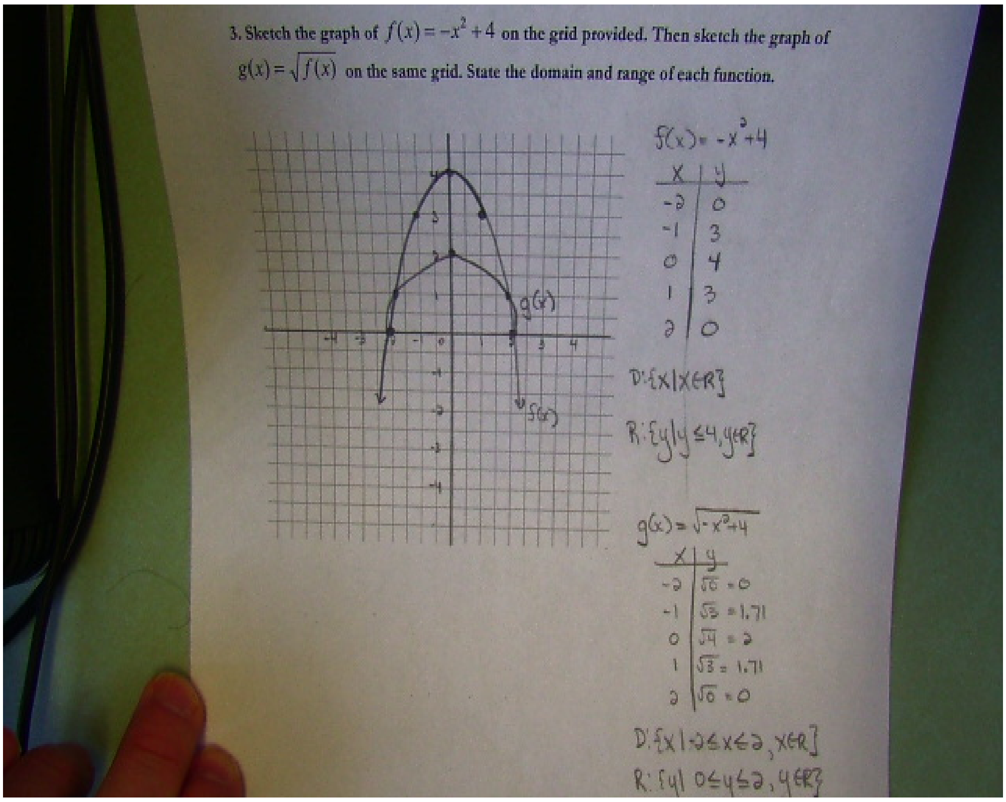
$\sqrt{2(3)-6}+3$	3
$\sqrt{6-6}+3$	
$\sqrt{0}+3$	
$0+3$	
3	

is a solution

Test $x=5$

$\sqrt{2(5)-6}+3$	5
$\sqrt{10-6}+3$	
$\sqrt{4}+3$	
$2+3$	
5	

is a solution



3. Sketch the graph of $f(x) = -x^2 + 4$ on the grid provided. Then sketch the graph of $g(x) = \sqrt{f(x)}$ on the same grid. State the domain and range of each function.

$f(x) = -x^2 + 4$

x	y
-2	0
-1	3
0	4
1	3
2	0

D: $\{x \mid x \in R\}$

R: $\{y \mid y \leq 4, y \in R\}$

$g(x) = \sqrt{-x^2+4}$

x	y
-2	$\sqrt{0} = 0$
-1	$\sqrt{3} = 1.71$
0	$\sqrt{4} = 2$
1	$\sqrt{3} = 1.71$
2	$\sqrt{0} = 0$

D: $\{x \mid -2 \leq x \leq 2, x \in R\}$

R: $\{y \mid 0 \leq y \leq 2, y \in R\}$

Exponential Functions Exam Review

1. Given the exponential function: $\frac{3}{4}(y-1)=6(3)^{4x+8}+9$

(a) Express this function in standard form.

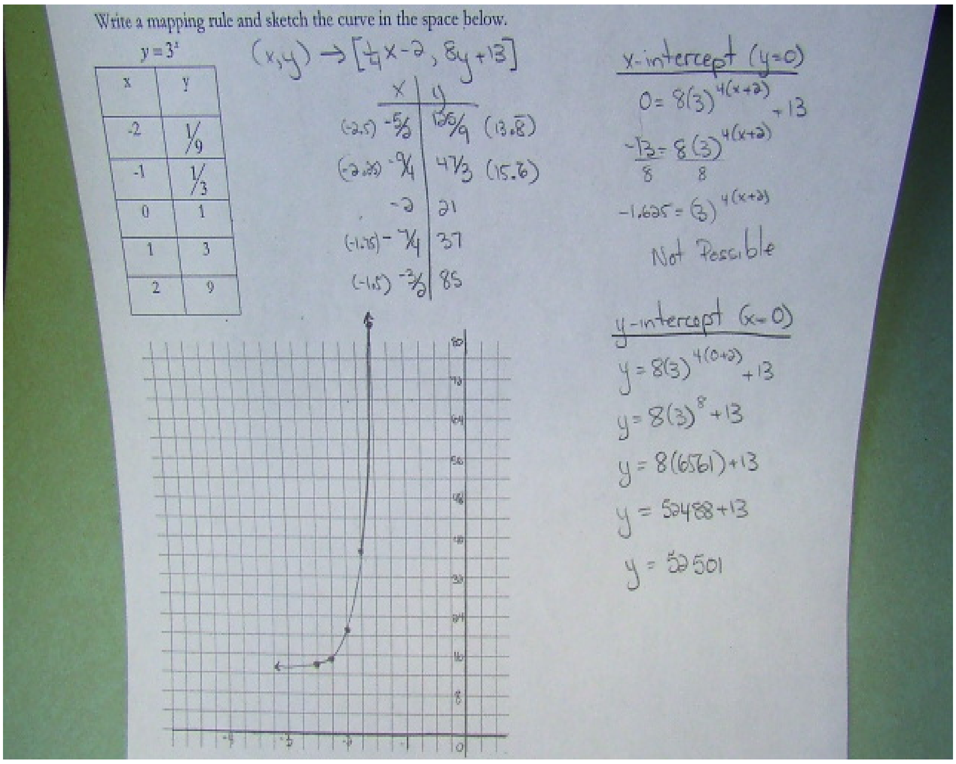
$y-1=8(3)^{4(x+2)}+12$

$y=8(3)^{4(x+2)}+13$

$a=8$ $b=4$ $h=-2$ $k=13$

(b) Complete the chart shown below.

Reflected in x-axis	YES or NO (circle correct solution)
Reflected in y-axis	YES or NO (circle correct solution)
Horizontal translation of…	2 units left
Vertical translation of…	13 units up
Horizontally stretched by a factor of…	$\frac{1}{4}$
Vertically stretched by a factor of …	8
x-intercept (show work)	No x-intercept
y-intercept (show work)	$y=52501$ or $(0, 52501)$
Horizontal Asymptote	$y=13$
Domain	$\{x \mid x \in R\}$
Range	$\{y \mid y>13, y \in R\}$



Write a mapping rule and sketch the curve in the space below.

$y = 3^x$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$(x, y) \rightarrow [\frac{1}{4}x - 2, 8y + 13]$

x	y
(-2.5) $-\frac{5}{2}$	$\frac{125}{9}$ (13.$\bar{8}$)
(-2.25) $-\frac{9}{4}$	$\frac{47}{3}$ (15.$\bar{6}$)
-2	21
(-1.75) $-\frac{7}{4}$	37
(-1.5) $-\frac{3}{2}$	85

x-intercept (y=0)

$0 = 8(3)^{4(x+2)} + 13$

$\frac{-13}{8} = \frac{8(3)^{4(x+2)}}{8}$

$-1.625 = (3)^{4(x+2)}$

Not Possible

y-intercept (x=0)

$y = 8(3)^{4(0+2)} + 13$

$y = 8(3)^8 + 13$

$y = 8(6561) + 13$

$y = 52488 + 13$

$y = 52501$

2. Radioactive carbon-14 has a half-life of 5750 years. When an organism dies, the amount of C-14 present decays exponentially. By measuring the radioactivity of the remains of a fossilized organism and comparing it with the radioactivity of a living organism archaeologists can approximate the age of the artifact. An antique dealer was selling a piece of wood purported to come from a chariot used by Caesar in ancient Rome. Archaeologists found the wood to contain 0.4 mg of C-14, compared with the 0.68 mg found in a new piece of wood. Caesar was assassinated 2048 years ago, could this dealer's claim possibly be true? Provide mathematical proof to back up your claim!

Given:

Base = 0.5

Initial Amount (A_0) = 0.68

Final Amount (A_f) = 0.4

Exponent = $\frac{x}{5750}$

$x = 2048$

Equation:

$y = 0.68(0.5)^{\frac{x}{5750}}$

$y = 0.68(0.5)^{\frac{2048}{5750}}$

$y = 0.68(0.78)$

$\boxed{y = 0.53\text{ mg}}$

or

$0.4 = 0.68(0.5)^{x/5750}$

$0.5882 = (0.5)^{x/5750}$

$(\cancel{0.5})^{0.7655} = (\cancel{0.5})^{x/5750}$

$0.7655 = \frac{x}{5750}$

$\boxed{x = 4402\text{ years}}$

3. Solve each of the following:

3. Solve each of the following:

5750

$\boxed{x = 4402 \text{ years}}$

a) $64^{x-3} = (16)^{x-1}\left(\frac{1}{4}\right)^{2x}$

$(4^3)^{x-3} = (4^2)^{x-1}(4^{-1})^{2x}$

$4^{3x-9} = 4^{2x-2} \cdot 4^{-2x}$

$\cancel{4}^{3x-9} = \cancel{4}^{-2}$

$3x-9 = -2$

$3x = 7$

$\boxed{x = \frac{7}{3}}$

b) $\left(\frac{1}{27}\right)^{x+2} = (3)^{2x-1}(81)^{x}$

$(3^{-3})^{x+2} = (3)^{2x-1}(3^4)^{x}$

$3^{-3x-6} = 3^{2x-1} \cdot 3^{4x}$

$3^{-3x-6} = 3^{6x-1}$

$-3x-6 = 6x-1$

$-5 = 9x$

$\boxed{\frac{-5}{9} = x}$

1. Express the following as a single logarithm in simplest form:

$$8\log_2\sqrt{x}-\frac{2}{3}\left[9\log_2 x^{-2}+6\left(\log_2 x^4-\frac{3}{4}\log_2\sqrt[5]{x}\right)\right]$$

$8\log_2 x^{1/2}-\frac{2}{3}\left[9\log_2 x^{-2}+6\log_2 x^4-\frac{18}{4}\log_2 x^{1/5}\right]$

$8\log_2 x^{1/2}-6\log_2 x^{-2}-4\log_2 x^4+3\log_2 x^{1/5}$

$\log_2 x^4-\log_2 x^{-12}-\log_2 x^{16}+\log_2 x^{3/5}$

$\log_2\left(\frac{x^4\cdot x^{3/5}}{x^{-12}x^{16}}\right)$

$\log_2\left(\frac{\cancel{x^4}\cdot x^{3/5}}{\cancel{x^4}}\right)$

$\log_2 x^{3/5} \longrightarrow \boxed{\frac{3}{5}\log_2 x}$

2. Given that $\log_r x=\underline{-6}$, $\log_r y=\underline{-3}$, and $\log_r z=\underline{8}$, evaluate the expression $\log_r\left(\frac{x^5z^3}{r^{-3}y^5}\right)$.

$\log_r x^5+\log_r z^3-\log_r r^{-3}-\log_r y^5$

$5\log_r x-3\log_r z+3\log_r r-5\log_r y$

$5(-6)-3(8)+3(1)-5(-3)$

$-30-24+3+15$

$\boxed{-36}$

3. Solve for x in the following equations…

$\log_3(2x^2 - x) - \log_3(x+2) = 1$

$\log_3\left(\frac{2x^2-x}{x+2}\right) = 1$

$3^1 = \frac{2x^2-x}{x+2}$

$3x+6 = 2x^2 - x$

$0 = 2x^2 - 4x - 6$

$0 = 2(x^2 - 2x - 3)$

$0 = 2(x-3)(x+1)$

$x-3=0$ | $x+1=0$

$\boxed{x=3}$ | $\boxed{x=-1}$

Test $x=3$

$\log_3(15) - \log_3(5) = 1$

$\log_3\left(\frac{15}{5}\right)$

$\log_3 3$

1

$x=3$ is a solution

Test $x=-1$

$\log_3(3) - \log_3(1) = 1$

$1 - 0$

1

$x=-1$ is a solution

4. Given the logarithmic function: $y - 6 = 2\log_5(3x+15)$

(a) Express this function in standard form. $y = 2\log_5[3(x+5)] + 6$

$a = 2 \quad b = 3 \quad h = -5 \quad K = 6$

(b) Complete the chart shown below.

Reflected in x-axis	YES or NO (circle correct solution)
Reflected in y-axis	YES or NO (circle correct solution)
Horizontal translation of…	5 units left
Vertical translation of…	6 units up
Horizontally stretched by a factor of…	$1/3$
Vertically stretched by a factor of …	2
x-intercept (show work)	$x = \frac{-1874}{375} = -4.997$ or $(-4.997, 0)$
y-intercept (show work)	$y = 9.37$ or $(0, 9.37)$
Horizontal Asymptote	$x = -5$
Domain	$\{x \mid x > -5, x \in R\}$
Range	$\{y \mid y \in R\}$

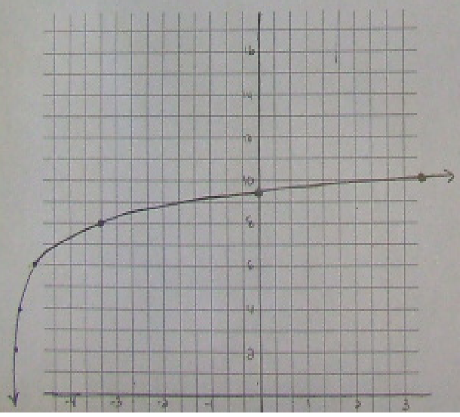
Write a mapping rule and sketch the curve in the space below.

$y = \log_5 x$

x	y
$\frac{1}{25}$	-2
$\frac{1}{5}$	-1
1	0
5	1
25	2

$(x, y) \rightarrow \left[\frac{1}{3}x - 5, 2y + 6\right]$

	x	y
(-4.99)	$-374/75$	2
(-4.93)	$-74/15$	4
$(-4.\bar{6})$	$-14/3$	6
(-3.3)	$-10/3$	8
(3.3)	$10/3$	10



x-intercept (y=0)

$0 = 2\log_5[3(x+5)] + 6$

$-6 = 2\log_5[3(x+5)]$

$-3 = \log_5[3(x+5)]$

$5^{-3} = 3(x+5)$

$\frac{1}{125} \div 3 = \frac{3(x+5)}{3}$

$\frac{1}{375} = x + 5$

$\boxed{-\frac{1874}{375} = x}$

-4.99

y-intercept (x=0)

$y = 2\log_5[3(0+5)] + 6$

$y = 2\log_5(15) + 6$

$y = 2(1.683) + 6$

$y = 3.365 + 6$

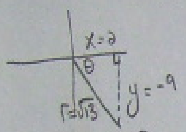
$\boxed{y = 9.37}$

Pre-Calculus 12A
Special Angles Exam Review

$\cos\theta > 0$ and $\sin\theta < 0$ S | A, T | C Q4

1. Given that $\cos\theta = \frac{2}{\sqrt{13}}$ and $\sin\theta < 0$. Sketch the angle and determine the five remaining trig ratios ***as radicals in simplest form.***

Given: $x = 2$, $r = \sqrt{13}$



$x^2 + y^2 = r^2$
$2^2 + y^2 = \sqrt{13}^2$
$4 + y^2 = 13$
$y^2 = 9$
$y = \pm 9$
$\boxed{y = -9}$

$\sin\theta = \frac{-9}{\sqrt{13}} = -\frac{9\sqrt{13}}{13}$

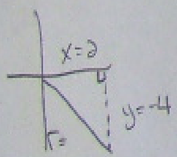
$\tan\theta = -\frac{9}{2}$

$\sec\theta = \frac{\sqrt{13}}{2}$

$\csc\theta = -\frac{\sqrt{13}}{9}$

$\cot\theta = -\frac{2}{9}$

2. The point (2, -4) lies on the terminal arm of an angle. Make a sketch of this angle and determine the 6 trigonometric ratios expressed ***as radicals in simplest form.***



$x^2 + y^2 = r^2$
$2^2 + (-4)^2 = r^2$
$4 + 16 = r^2$
$20 = r^2$
$\sqrt{20} = r$
$2\sqrt{5} = r$

$\sin\theta = \frac{-4}{2\sqrt{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

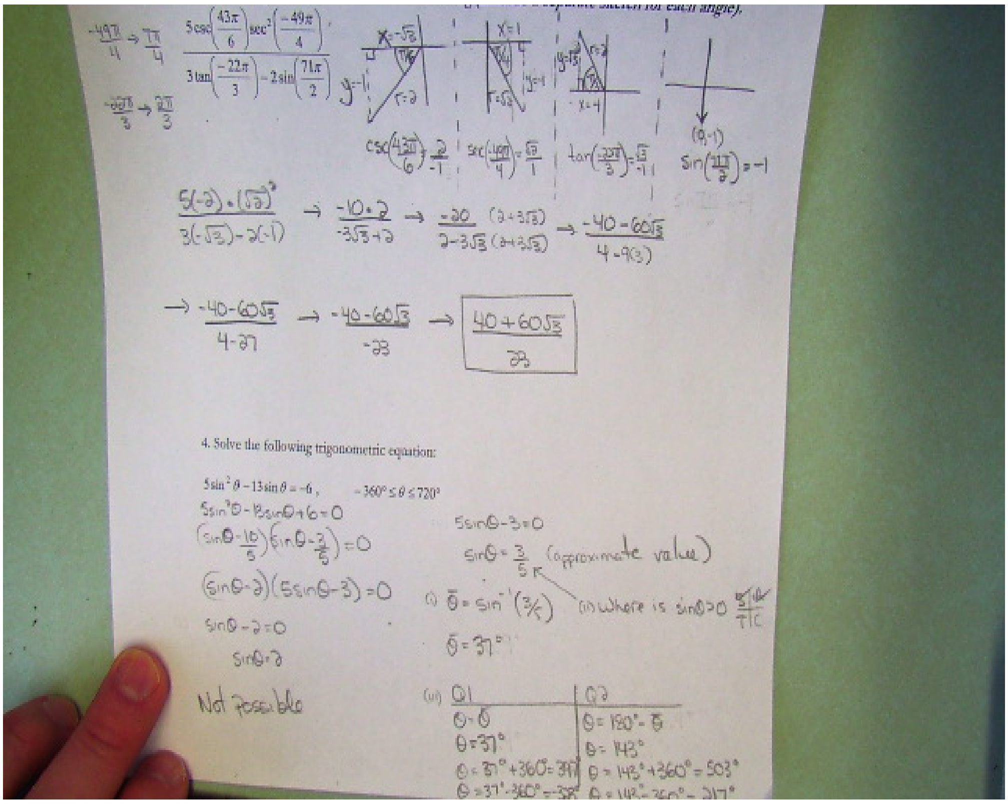
$\cos\theta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{-4}{2} = -2$

$\csc\theta = -\frac{2\sqrt{5}}{4} = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{2\sqrt{5}}{2} = \sqrt{5}$

$\cot\theta = \frac{2}{-4} = -\frac{1}{2}$



$-\frac{49\pi}{4} \rightarrow \frac{7\pi}{4}$

$-\frac{22\pi}{3} \rightarrow \frac{2\pi}{3}$

$$\frac{5\csc\left(\frac{43\pi}{6}\right)\sec^2\left(\frac{-49\pi}{4}\right)}{3\tan\left(\frac{-22\pi}{3}\right)-2\sin\left(\frac{71\pi}{2}\right)}$$

$x=-\sqrt{3}$, $y=-1$, $r=2$ $x=1$, $y=-1$, $r=\sqrt{2}$ $y=\sqrt{3}$, $r=2$, $x=-1$ $(0,-1)$

$\csc\left(\frac{43\pi}{6}\right)=\frac{2}{-1}$ $\sec\left(\frac{-49\pi}{4}\right)=\frac{\sqrt{2}}{1}$ $\tan\left(\frac{-22\pi}{3}\right)=\frac{\sqrt{3}}{-1}$ $\sin\left(\frac{71\pi}{2}\right)=-1$

$$\frac{5(-2)\cdot(\sqrt{2})^2}{3(-\sqrt{3})-2(-1)} \rightarrow \frac{-10\cdot 2}{-3\sqrt{3}+2} \rightarrow \frac{-20}{2-3\sqrt{3}}\frac{(2+3\sqrt{3})}{(2+3\sqrt{3})} \rightarrow \frac{-40-60\sqrt{3}}{4-9(3)}$$

$$\rightarrow \frac{-40-60\sqrt{3}}{4-27} \rightarrow \frac{-40-60\sqrt{3}}{-23} \rightarrow \boxed{\frac{40+60\sqrt{3}}{23}}$$

4. Solve the following trigonometric equation:

$5\sin^2\theta - 13\sin\theta = -6$, $-360° \le \theta \le 720°$

$5\sin^2\theta - 13\sin\theta + 6 = 0$

$\left(\sin\theta - \frac{10}{5}\right)\left(\sin\theta - \frac{3}{5}\right) = 0$

$(\sin\theta - 2)(5\sin\theta - 3) = 0$

$\sin\theta - 2 = 0$

$\sin\theta = 2$

Not Possible

$5\sin\theta - 3 = 0$

$\sin\theta = \frac{3}{5}$ (approximate value)

(i) $\bar{\theta} = \sin^{-1}(3/5)$ (ii) Where is $\sin\theta > 0$ S A / T C

$\bar{\theta} = 37°$

(iii)

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 37°$	$\theta = 143°$
$\theta = 37° + 360° = 397°$	$\theta = 143° + 360° = 503°$
$\theta = 37° - 360° = -323°$	$\theta = 143° - 360° = -217°$

Sinusoidal Functions Exam Review

1 rev. in 24 sec   radius = 15

1. A Ferris wheel completes 2 revolutions in 48 seconds and has a diameter of 30 m. If the bottom of the wheel is 2 m above the ground. When a stopwatch is started you notice that your friend is seated at the middle of the wheel and is going up. $y = \sin x$

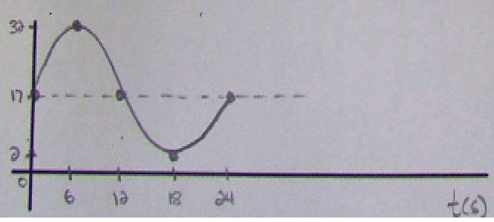
(a) Find the following:

a: ± 15 P: 24   b: $\frac{360}{24} = 15$ min height: 2 m   max height: 32 m (min + diameter)   k: 17 m (min + radius)

(b) What is the equation of the graph?

$y = 15\sin[15(x)] + 17$

(c) Sketch the graph for one period.



2. Graph the following Sinusoidal Function: (One Period) $\frac{\pi}{2} \div 3 = \frac{\pi}{2} \times \frac{1}{3} = \frac{\pi}{6}$

$-3(y+2) = 12\cos\left(3\theta - \frac{\pi}{2}\right) - 3$ $y+2 = -4\cos\left[3\left(\theta - \frac{\pi}{6}\right)\right] + 1$ $a = -4$ $b = 3$ $h = \frac{\pi}{6}$ $k = -1$

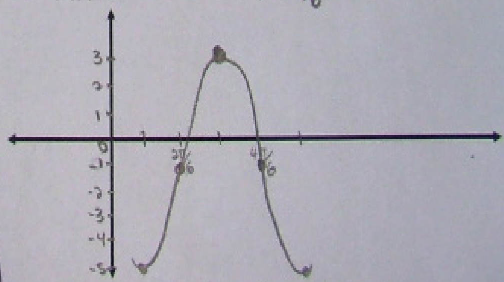
$y = -4\cos\left[3\left(\theta - \frac{\pi}{6}\right)\right] - 1$

DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid -5 \le y \le 3, y \in R\}$
AMPLITUDE	4
PERIOD	$\frac{2\pi}{3}$
PHASE SHIFT	$\pi/6$ right
VERTICAL TRANSLATION	1 down
EQUATION OF SINUSOIDAL AXIS	$y = -1$
MAPPING NOTATION	$(x, y) \rightarrow \left[\frac{1}{3}\theta + \frac{\pi}{6}, -4y - 1\right]$

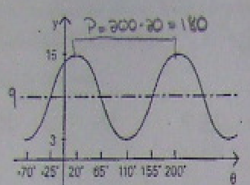
$y = \cos\theta$ $\longrightarrow$

θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

θ	y
$\pi/6$	-5
$2\pi/6$	-1
$3\pi/6$	3
$4\pi/6$	-1
$5\pi/6$	-5



3. Find a ***positive*** sine and a ***positive*** cosine equation from the graph.



$a = \pm 6$

$P = 180°$

$b = \frac{360°}{180°} = 2$

$k = 9$

(i) $y = \sin\theta$ $(h = -25°)$

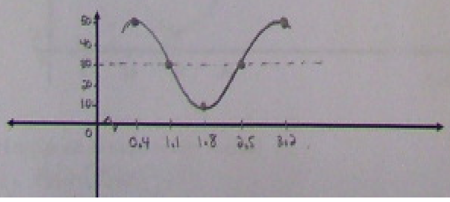
$y = 6\sin[2(\theta + 25°)] + 9$

(ii) $y = \cos\theta$ $(h = 20°)$

$y = 6\cos[2(\theta - 20°)] + 9$

4. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.4 seconds, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 seconds.

(a) Sketch a graph of this sinusoidal function



$h = 0.4$

max = 50 cm

min = 30 cm

$K = \frac{50+30}{2} = \frac{80}{2} = 40$

$a = \pm 10$

$P = 2(1.8 - 0.4) = 2.8$

$b = \frac{360}{2.8} = 128.57$

(b) Write an equation to define the graph.

$y = 10\cos[128.57(x-0.4)]+40$

(c) What was the distance from the floor when you started the stopwatch? $(x=0)$

$y = 10\cos[128.57(0-0.4)]+40$

$\boxed{y = 46.2\text{cm}}$

Extra questions worked out

$0 \le \theta \le 2\pi$

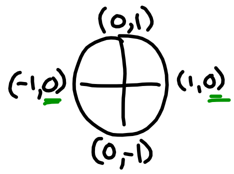
(12) a) $\sin\theta = \sin\theta\tan\theta$

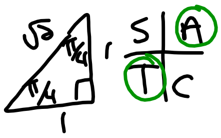
Common Factor

$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

$\sin\theta = 0$	$\tan\theta - 1 = 0$
$\theta = 0, \pi, 2\pi$	$\tan\theta = 1$
	$\theta_R = \frac{\pi}{4}$





Where is $\tan\theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

(13) b) $3\sin^2\theta - 2\sin\theta - 1 = 0$, $0 \le \theta \le 360°$

$(3\sin^2\theta - 3\sin\theta)(+\sin\theta - 1) = 0$

$\frac{-3}{} \times \frac{1}{} = -3$

$\frac{-3}{} + \frac{1}{} = -2$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$3\sin\theta + 1 = 0$	$\sin\theta - 1 = 0$
$\sin\theta = -\frac{1}{3}$	$\sin\theta = 1$
$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$	$\theta = 90°$
$\theta_R = 19$	

Unit Circle

Where is sine negative: S | A / T | C

Q3	Q4
$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 180° + 19$	$\theta = 360° - 19$
$\theta = 199$	$\theta = 340$

① Amp = 11 P = 16 min = -4

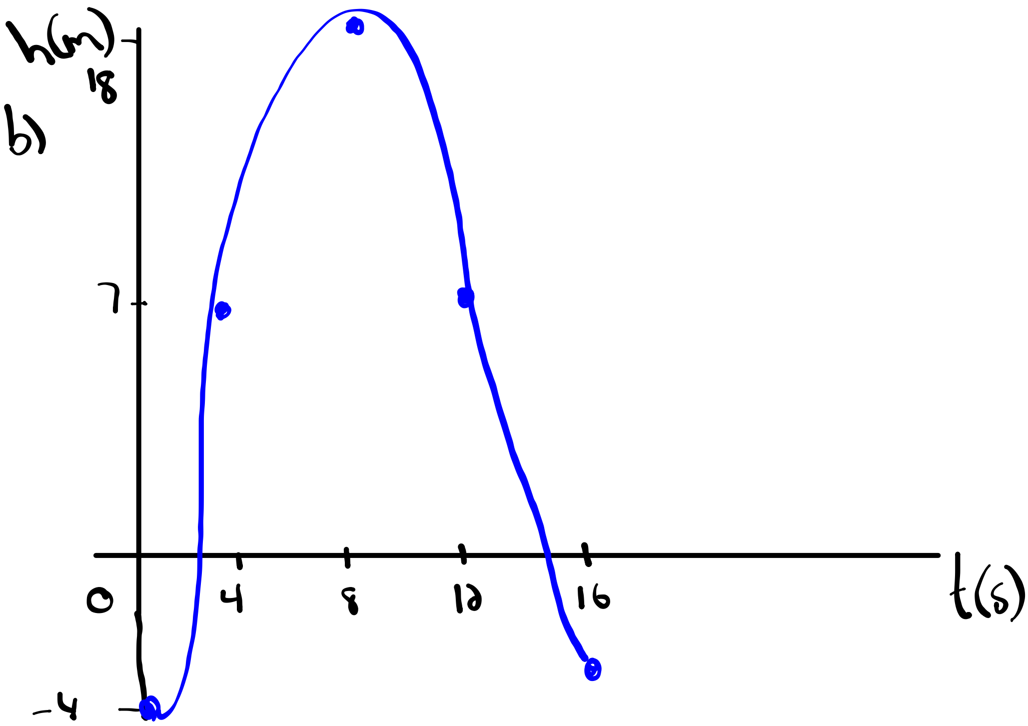
$a = \pm 11$ $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

K = -4 + 11 = 7

h = 0

a) equation: $y = -11\cos[22.5(x)] + 7$

b)



1

4.

max = 68

min = 24

$k = \frac{68+24}{2} = 46$

Amp = 68 − 46 = 22

$a = \pm 22$

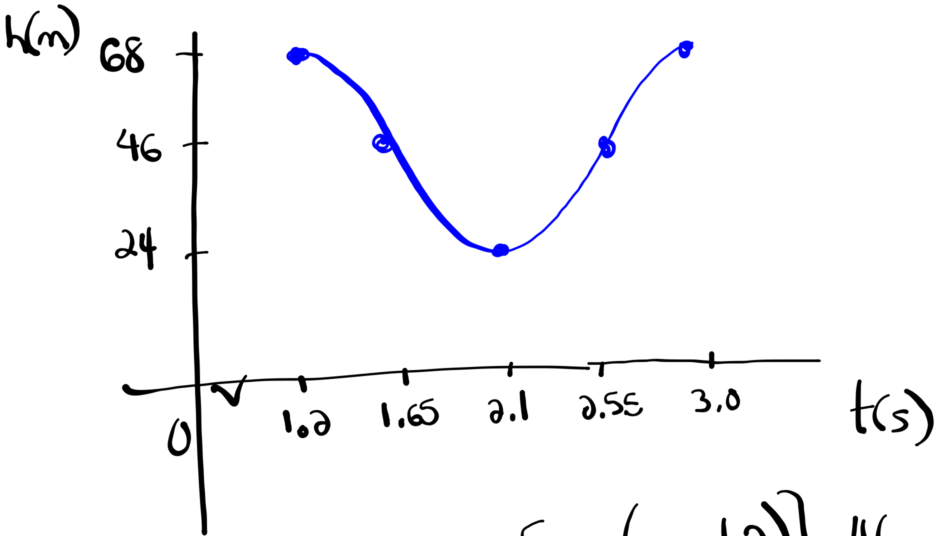
$P = 2(2.1 - 1.2)$

$P = 1.8$

$b = \frac{360}{1.8} = 200$

$h = 1.2$

$\frac{P}{4} = \frac{1.8}{4} = 0.45$



$y = 22\cos[200(x - 1.2)] + 46$

(5) c) $y = \frac{1}{2}\cos(1\theta + \pi) - 4$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

$(x, y) \longrightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$\longrightarrow$

x	y	
$-\pi$	$-7/2$	-3.5
$-\frac{\pi}{2}$	-4	-4
0	$-9/2$	-4.5
$\pi/2$	-4	-4
π	$-7/2$	-3.5