Chapter Review For Final Exam:

Ch.1 → (Inverse Functions)

Ch.2 → (Radical Functions)

Ch.7 → (Exponential Functions) $y = 2^x$

Ch.8 → (Logarithmic Functions) $y = \log_2 x$

Ch.4 → (Trig + Unit Circle)

Ch.5 → (Trig Functions)

Ch.6 → (Trig Identities)

Ch.2

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

Ch.7

$y = 3^x$

x	y
-2	$1/9$
-1	$1/3$
0	1
1	3
2	9

Ch.8

$y = \log_3 x$

x	y
$1/9$	-2
$1/3$	-1
1	0
3	1
9	2

Ch.5

$y = \sin x$

x	y
$0°$	0
$90°$	1
$180°$	0
$270°$	-1
$360°$	0

$y = \cos x$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$y = af[b(x-h)]+k$ Ch.1

$y = a\sqrt{b(x-h)}+k$ Ch.2

$y = ac^{b(x-h)}+k$ Ch.7

$y = a\log_c[b(x-h)]+k$ Ch.8

$y = a\sin[b(x-h)]+k$ Ch.5

Ch.2 (Radical Functions)

D: $\{x \mid x \geq h, x \in \mathbb{R}\}$ if $b>0$ R: $\{y \mid y \geq k, y \in \mathbb{R}\}$ if $a>0$

$\{x \mid x \leq h, x \in \mathbb{R}\}$ if $b<0$ $\{y \mid y \leq k, y \in \mathbb{R}\}$ if $a<0$

Ch.7 (Exponential Functions)

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y > k, y \in \mathbb{R}\}$ if $a>0$

$\{y \mid y < k, y \in \mathbb{R}\}$ if $a<0$

HA: $y = k$

Ch.8 (Logarithmic Functions)

D: $\{x \mid x > h, x \in \mathbb{R}\}$ if $b>0$ R: $\{y \mid y \in \mathbb{R}\}$

$\{x \mid x < h, x \in \mathbb{R}\}$ if $b<0$

VA: $x = h$

Ch.5 (Sinusoidal Functions)

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid \min \leq y \leq \max, y \in \mathbb{R}\}$

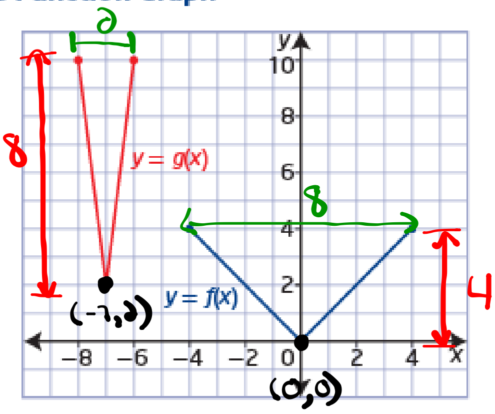
or $\{\theta \mid \theta \in \mathbb{R}\}$

Chapter 1:

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

① Reflection: None

② VSF $= \frac{8}{4} = 2$ $(a=2)$

③ HSF $= \frac{2}{8} = \frac{1}{4}$ $(b=4)$

④ HT: $(0,0) \rightarrow (-7,2)$ 7 units left $(h=-7)$

⑤ VT: $(0,0) \rightarrow (-7,2)$ 2 units up $(k=2)$

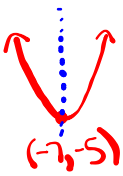
⑥ Equation: $\boxed{g(x) = 2f[4(x+7)]+2}$

Ch. 1

Find the inverse:

1. Replace $f(x)$ with y
2. Switch x's and y's
3. Solve for y
4. Replace y with $f^{-1}(x)$ (if it is a function)

Ex: $f(x) = (x+7)^2 - 5 \rightarrow$ Parabola



Fails HLT

Inverse is not a function

axis of sym: $x = -7$

$$y = (x+7)^2 - 5$$

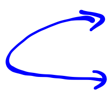
$$x = (y+7)^2 - 5$$

$$x+5 = (y+7)^2$$

$$\pm\sqrt{x+5} = y+7$$

$$-7 \pm \sqrt{x+5} = y$$

$$y = -7 \pm \sqrt{x+5} \rightarrow$$



$$\sqrt{x+5} \cdot \sqrt{x+5} = x+5$$

$$-\sqrt{x+5} \cdot -\sqrt{x+5} = x+5$$

$$f^{-1}(x) = -7 + \sqrt{x+5}, \text{ if } x \geq -7$$

$$f^{-1}(x) = -7 - \sqrt{x+5}, \text{ if } x \leq -7$$

Chapter 2 Radical Functions

Solve for x:

$$\left(\sqrt{x+8}\right)^2 = (x+6)^2$$

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2+12x+36$$

$$0 = x^2+11x+28$$

$\underline{4} + \underline{7} = 11$
$\underline{4} \times \underline{7} = 28$

$$0 = (x+4)(x+7)$$

$x+4=0$	$x+7=0$
$x=-4$	$x=-7$

is a solution

Test $x=-4$

$$\sqrt{x+8} = x+6$$

$\sqrt{-4+8}$	$-4+6$
$\sqrt{4}$	2 ✓
2 ✓	

is not a solution (extraneous)

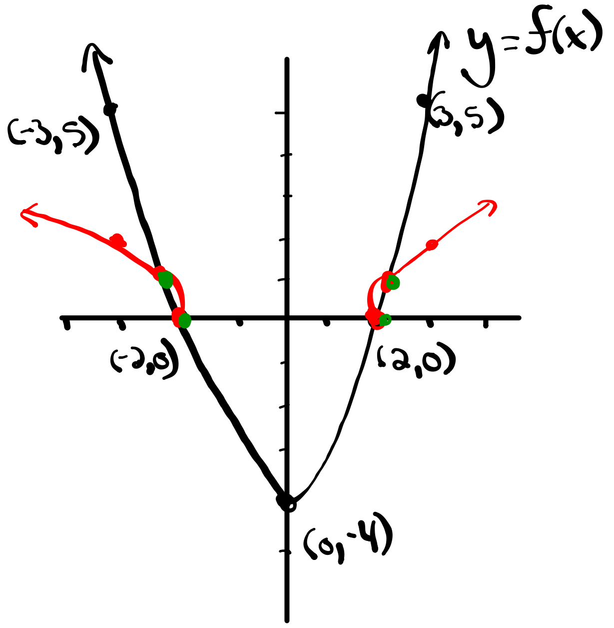
Test $x=-7$

$$\sqrt{x+8} = x+6$$

$\sqrt{-7+8}$	$-7+6$
$\sqrt{1}$	-1 ✗
1	

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. State the domain and range of each.



$y = f(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -4, y \in R\}$ or $[-4, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \leq -2, x \geq 2, x \in R\}$ or $(-\infty, -2] \cup [2, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

Ch. 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81\left(27^{x}\right)$

a) $2^{2x+2} + 7 = 71$

$2^{2x+2} = 64$

$\cancel{2}^{2x+2} = (\cancel{2})^{6}$

$2x + 2 = 6$

$\frac{2x}{2} = \frac{4}{2}$

$\boxed{x = 2}$

b) $9^{2x+1} = 81(27^{x})$

$(3^{2})^{2x+1} = (3^{4})(3^{3})^{x}$

$3^{4x+2} = 3^{4} \cdot 3^{3x}$

$\cancel{3}^{4x+2} = \cancel{3}^{3x+4}$

$4x + 2 = 3x + 4$

$\boxed{x = 2}$

* Be sure to test your answers!

Ch. 8 → Logarithmic Functions

4. Rewrite each expression as a single logarithm.

$$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$$

$$\log_5 x^3 + \log_5 (x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5 x^3(x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5\left[\frac{x^3(x-1)^{1/2}}{x^2+1}\right] \quad \text{or} \quad \log_5\left[\frac{x^3\sqrt{x-1}}{x^2+1}\right]$$

Ch. 8

7. Solve the following equation (be sure to test your answers).

$$\log_{10}(x+2)+\log_{10}(x-1)=1$$

$$\log_{10}(x+2)(x-1) = 1$$

$$\log_{10}(\underbrace{x^2+x-2}) = 1$$ (log. form)

Base Ans Exp.

$$10^1 = x^2+x-2$$ (exp. form)

$$10 = x^2+x-2$$

$$0 = x^2+x-12$$

$-3 + 4 = 1$
$-3 \times 4 = -12$

$$0 = (x-3)(x+4)$$

$x-3=0$	$x+4=0$
$\boxed{x=3}$	$x=-4$
is a solution	extraneous

* Be sure to test your answers!

Ch.7 or Ch.8 Base = $\frac{1}{2}$ or 0.5 $A_0 = 60mg$

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

exp = $t/5.3$

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

$y = (\text{Initial Amount})(\text{Base})^{exp.}$

$\boxed{y = (60)(0.5)^{t/5.3}}$

b) What amount will be present in 10.6 years? [2]

$t = 10.6$

$y = (60)(0.5)^{\frac{10.6}{5.3}}$

$y = (60)(0.5)^2$

$y = (60)(0.25) = \boxed{15mg}$

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

(i) 12.5% of initial Amount:

$= 0.125 \times 60$

$= 7.5\ mg$ $(y = 7.5mg)$

(ii) Solve for t:

$y = (60)(0.5)^{t/5.3}$

$\frac{7.5}{60} = \frac{\cancel{60}(0.5)^{t/5.3}}{\cancel{60}}$

$0.125 = (0.5)^{t/5.3}$

$(\cancel{0.5})^3 = (\cancel{0.5})^{t/5.3}$

$* \ \frac{\log(0.125)}{\log(0.5)} = 3$

$(5.3)\ 3 = \frac{t}{\cancel{5.3}}\ (\cancel{5.3})$

$\boxed{15.9 \text{ years} = t}$

Ch. 4 → Special Angles

2. Solve for all values of θ in the specified domain.

$\tan^2\theta + \tan\theta = 0$, $0 \le \theta \le 2\pi$ (Radians)

$\tan\theta(\tan\theta + 1) = 0$

$\tan\theta = 0$ (Unit Circle)

$\boxed{\theta = 0, \pi, 2\pi}$

$\tan\theta + 1 = 0$

$\tan\theta = -1$ (Special Triangle)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \tan^{-1}(1)$

$\bar{\theta} = \frac{\pi}{4}$

(ii) where is $\tan\theta < 0$

S	A
T	C

(iii) Find θ:

Q2	Q4
$\theta = \pi - \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi - \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

e. $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta < 360°$ (Degrees)

$\cos\theta\left(\cos\theta + \frac{1}{2}\right) = 0$

$\cos\theta = 0$ (Unit Circle)

$\boxed{\theta = 90°, 270°}$

$\cos\theta + \frac{1}{2} = 0$

$\cos\theta = -\frac{1}{2}$ (Special Triangles)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 60°$

(ii) Where is $\cos\theta < 0$

S	A
T	C

(iii) Find θ:

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 60° = 120°$	$\theta = 180° + 60° = 240°$

Ch. 4 → Special Angles:

$$\frac{5\tan^2\left(\frac{5\pi}{4}\right)}{6\sin\left(\frac{5\pi}{6}\right)+4\sin\left(\frac{4\pi}{3}\right)}$$

$$\frac{5(1)^2}{6\left(\frac{1}{2}\right)+4\left(\frac{-\sqrt{3}}{2}\right)}$$

$$\frac{5}{3-\frac{4\sqrt{3}}{2}}$$

$$\frac{5}{(3-2\sqrt{3})}\frac{(3+2\sqrt{3})}{(3+2\sqrt{3})}$$

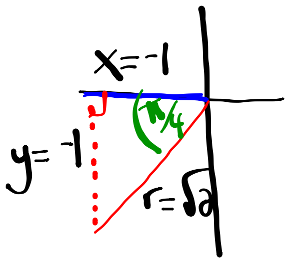
$$\frac{15+10\sqrt{3}}{9+\cancel{6\sqrt{3}}-\cancel{6\sqrt{3}}-4(3)}$$

$$\frac{15+10\sqrt{3}}{-3}$$

$$\boxed{\frac{-15-10\sqrt{3}}{3}}$$

(i) $\frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}$

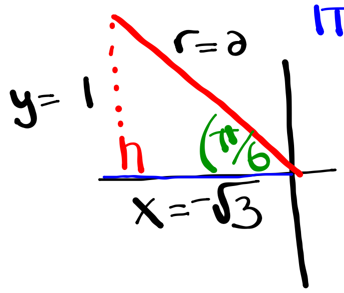
1π



$\tan\left(\frac{5\pi}{4}\right)=\frac{-1}{-1}=\underline{\underline{1}}$

(ii) $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

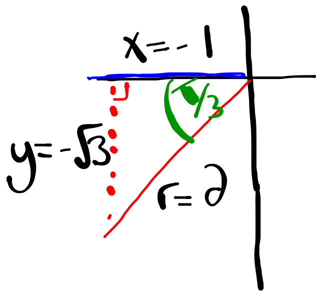
1π



$\sin\left(\frac{5\pi}{6}\right)=\underline{\underline{\frac{1}{2}}}$

(iii) $\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

1π



$\sin\left(\frac{4\pi}{3}\right)=\underline{\underline{-\frac{\sqrt{3}}{2}}}$

Series + Sequence

5 a) $\sum_{n=1}^{5} n^2+1$

$= [(1)^2+1] + [(2)^2+1] + [(3)^2+1] + [(4)^2+1] + [(5)^2+1]$

$= 2+5+10+17+26$

$= 60$

b) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$

$a = 3$

$r = \frac{1}{2}$

$S_n = \frac{a}{1-r}$

$S_n = \frac{3}{1-\frac{1}{2}}$

$S_n = \frac{3}{\frac{2}{2}-\frac{1}{2}}$

$S_n = \frac{3}{\frac{1}{2}} = 3 \times 2 = 6$

Ch.4 → Special Angles

$$\sec(15\pi)+\sqrt{2}\sin\left(\frac{39\pi}{4}\right)\sin\left(\frac{21\pi}{2}\right)-\csc^2\left(\frac{100\pi}{3}\right)$$

$$(-1)+\sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1)-\left(\frac{2}{-\sqrt{3}}\right)^2$$

$$-1-\frac{\sqrt{2}}{\sqrt{2}}-\frac{4}{3}$$

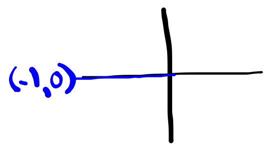
$$-1-1-\frac{4}{3}$$

$$-\frac{2}{1}-\frac{4}{3}$$

$$-\frac{6}{3}-\frac{4}{3}$$

$$-\frac{10}{3}$$

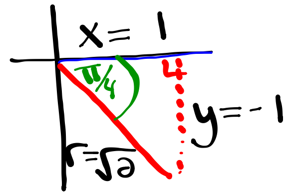
(i) 15π



$\sec(15\pi)=\frac{1}{-1}=-1$

(ii) $\frac{38\pi}{4}, \frac{39\pi}{4}, \frac{40\pi}{4}$

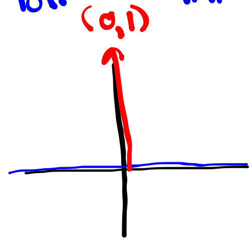
10π



$\sin\left(\frac{39\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

(iii) $\frac{20\pi}{2}, \frac{21\pi}{2}, \frac{22\pi}{2}$

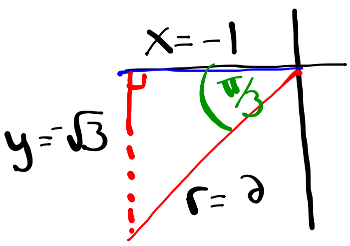
10π $\quad$ 11π



$\sin\left(\frac{21\pi}{2}\right)=1$

(iv) $\frac{99\pi}{3}, \frac{100\pi}{3}, \frac{101\pi}{3}$

33π



$\csc\left(\frac{100\pi}{3}\right)=\frac{2}{-\sqrt{3}}$

Ch. 5 → Trig Functions

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 sec.

$y = \cos x$

$x = 17.2$

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

max = 50

min = 30

sin axis = $k = \frac{30+50}{2} = 40$

Amp = 10

$a = \pm 10$

$h = 0.4$

$P = 2(1.8 - 0.4) = 2.8$

$b = \frac{360}{P} = \frac{360}{2.8} = 128.57$

$y = 10\cos[128.57(x - 0.4)] + 40$

$y = 10\cos[128.57(17.2 - 0.4)] + 40$

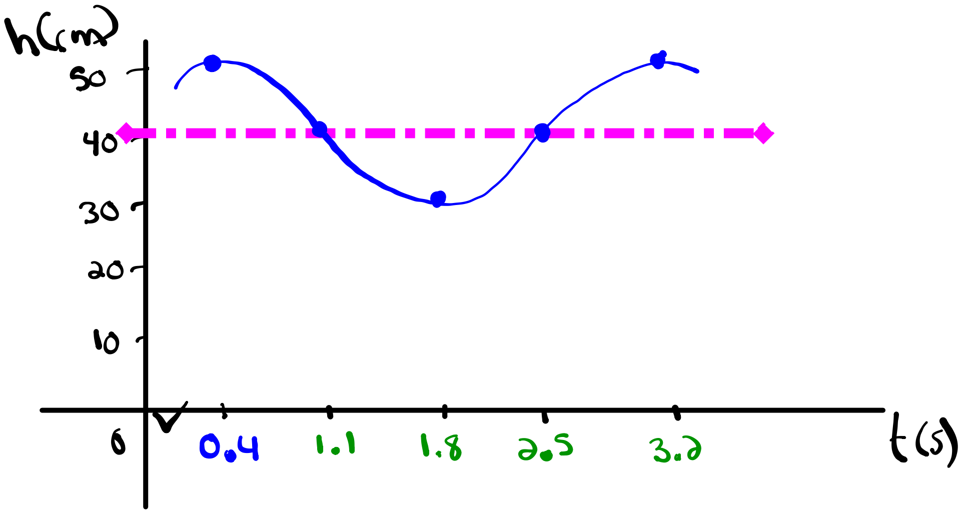
$y = 49.99$ cm

(b) How high was the weight above the floor when the stopwatch was initially started?

$(x = 0)$

$y = 10\cos[128.57(0 - 0.4)] + 40$

$y = 46.23$cm



$\frac{P}{4} = \frac{2.8}{4} = 0.7$

Ch. 6 → Trig Identities

$$\frac{1}{\sec^2\theta\cot\theta} = \frac{\sin\theta - \sin^3\theta}{\cos\theta}$$ ← Factor

$\boxed{\frac{1}{\sec^2\theta}} \cdot \boxed{\frac{1}{\cot\theta}}$

$\cos^2\theta \cdot \tan\theta$

$\cos^{\cancel{2}}\theta \cdot \frac{\sin\theta}{\cancel{\cos\theta}}$

$\boxed{\sin\theta\cos\theta}$

$\frac{\sin\theta\boxed{(1-\sin^2\theta)}}{\cos\theta}$

$\frac{\sin\theta\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}}$

$\boxed{\sin\theta\cos\theta}$

Pre-Calculus 12A

Chapter 1 Exam Review

1. Given the function $y = f(x)$ write the equation of the form $y = af(b(x-h))+k$ that would result from the following transformations:

A horizontal stretch about the y-axis by a factor of $\frac{1}{4}$ and a horizontal reflection in the y-axis. A vertical stretch about the x-axis by a factor of 3, and a translation of 5 units to the right and 2 units up.

$a = 3$
$b = -4$
$h = 5$
$k = 2$

$$\boxed{y = 3f[-4(x-5)]+2}$$

2. Determine the inverse of the function $f(x) = (x-3)^2 - 2$.

$f(x) = (x-3)^2 - 2$

$y = (x-3)^2 - 2$

$x = (y-3)^2 - 2$

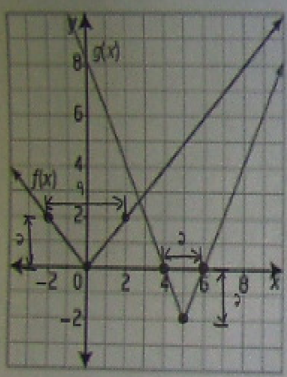
$x+2 = (y-3)^2$

$\pm\sqrt{x+2} = y-3$

$3 \pm \sqrt{x+2} = y$

$$\boxed{y = 3 \pm \sqrt{x+2}}$$

3. Write the equation for the graph of $g(x)$ as a transformation of the equation for the graph of $f(x)$.



① Reflections: none

② V.S.F. $= \frac{2}{2} = 1 \rightarrow a = 1$

③ H.S.F. $= \frac{2}{4} = \frac{1}{2} \rightarrow b = 2$

④ HT: $(0,0) \rightarrow (5,-2)$ $h = 5$

⑤ VT: $(0,0) \rightarrow (5,-2)$ $k = -2$

⑥ Equation: $\boxed{g(x) = 1f[2(x-5)] - 2}$

4. The key point (12, -18) is on the graph of $y = f(x)$. Calculate its image point under the following transformation:

4. The key point (12, -18) is on the graph of $y = f(x)$. Calculate its image point under the following transformation:

a) $y+3=-\frac{1}{3}f(2x+12)$

$y = -\frac{1}{3}f[2(x+6)]-3$

$a=-\frac{1}{3}$ $b=2$ $h=-6$ $k=-3$

$(x,y) \rightarrow [\frac{1}{2}x-6, -\frac{1}{3}y-3]$

$(12,-18) \rightarrow \boxed{(0,3)}$

b) $2y-4=6f(6x-12)+4$

$2y = 6f[6(x-2)]+8$

$y = 3f[6(x-2)]+4$

$a=3$ $b=6$ $h=2$ $k=4$

$(x,y) \rightarrow [\frac{1}{6}x+2, 3y+4]$

$(12,-18) \rightarrow \boxed{(4,-50)}$

Radical Functions Exam Review

1. Given that $2y+8=-4\sqrt{-x+3}$, complete the chart shown below. When identifying translations be sure that you indicate both the number of units and direction of the shift.

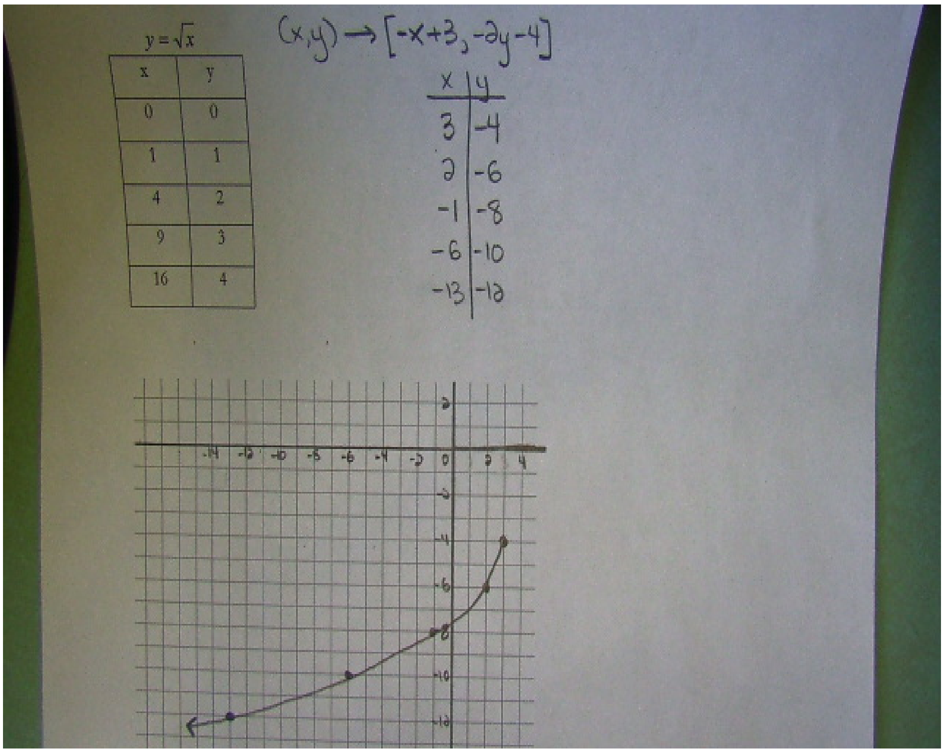
$y=-2\sqrt{-1(x-3)}-4$

$a=-2$
$b=-1$
$h=3$
$k=-4$

Reflected in x-axis	YES or NO (circle correct solution)
Reflected in y-axis	YES or NO (circle correct solution)
Horizontal translation of…	3 units right
Vertical translation of…	4 units down
Horizontally stretched by a factor of…	1 or (no stretch)
Vertically stretched by a factor of …	2
Domain	$\{x \mid x\leq 3, x\in R\}$
Range	$\{y \mid y\leq -4, y\in R\}$

Write a mapping rule and sketch the curve in the space below.

$y=\sqrt{x}$ $(x,y)\rightarrow[-x+3, -2y-4]$



$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4

$(x, y) \rightarrow [-x+3, -2y-4]$

x	y
3	-4
2	-6
-1	-8
-6	-10
-13	-12

2. Solve the following radical equation. $\sqrt{2x-6}+3=x$

$$\sqrt{2x-6} = x-3$$

$$2x-6 = x^2-6x+9$$

$$0 = x^2-8x+15$$

$$0 = (x-3)(x-5)$$

$x-3=0$ | $x-5=0$

$\boxed{x=3}$ | $\boxed{x=5}$

Test $x=3$

$\sqrt{2(3)-6}+3$	3
$\sqrt{6-6}+3$	
$\sqrt{0}+3$	
$0+3$	

3

is a solution

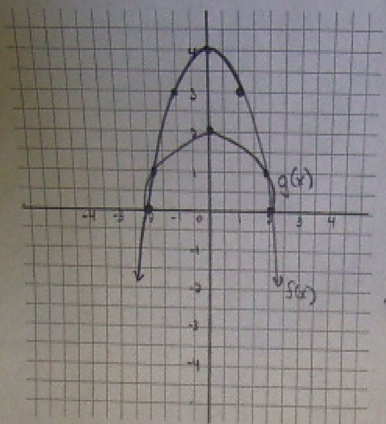
Test $x=5$

$\sqrt{2(5)-6}+3$	5
$\sqrt{10-6}+3$	
$\sqrt{4}+3$	
$2+3$	

5

is a solution

3. Sketch the graph of $f(x) = -x^2 + 4$ on the grid provided. Then sketch the graph of $g(x) = \sqrt{f(x)}$ on the same grid. State the domain and range of each function.



$f(x) = -x^2 + 4$

x	y
-2	0
-1	3
0	4
1	3
2	0

D: $\{x \mid x \in R]$

R: $\{y \mid y \leq 4, y \in R]$

$g(x) = \sqrt{-x^2 + 4}$

x	y
-2	$\sqrt{0} = 0$
-1	$\sqrt{3} = 1.71$
0	$\sqrt{4} = 2$
1	$\sqrt{3} = 1.71$
2	$\sqrt{0} = 0$

D: $\{x \mid -2 \leq x \leq 2, x \in R]$

R: $\{y \mid 0 \leq y \leq 2, y \in R\}$

Exponential Functions Exam Review

1. Given the exponential function: $\frac{3}{4}(y-1)=6(3)^{4x+8}+9$

(a) Express this function in standard form.

$y-1=8(3)^{4(x+2)}+12$

$y=8(3)^{4(x+2)}+13$

$a=8$ $b=4$ $h=-2$ $k=13$

(b) Complete the chart shown below.

Reflected in x-axis	YES or NO (circle correct solution)
Reflected in y-axis	YES or NO (circle correct solution)
Horizontal translation of…	2 units left
Vertical translation of…	13 units up
Horizontally stretched by a factor of…	$\frac{1}{4}$
Vertically stretched by a factor of …	8
x-intercept (show work)	No x-intercept
y-intercept (show work)	$y=52501$ or $(0, 52501)$
Horizontal Asymptote	$y=13$
Domain	$\{x \mid x \in R\}$
Range	$\{y \mid y>13, y \in R\}$

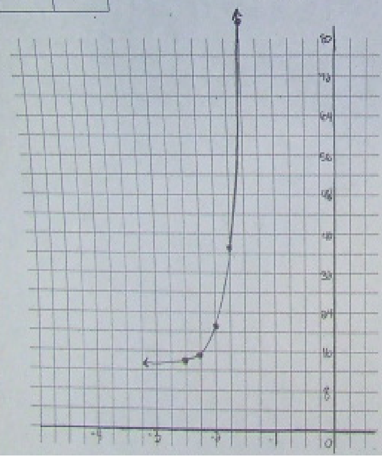
Write a mapping rule and sketch the curve in the space below.

$y = 3^x$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$(x, y) \rightarrow [\frac{1}{4}x - 2,\ 8y + 13]$

	x	y	
(-2.5)	$-\frac{5}{2}$	$\frac{125}{9}$	$(13.\bar{8})$
(-2.25)	$-\frac{9}{4}$	$\frac{47}{3}$	$(15.\bar{6})$
	-2	21	
(-1.75)	$-\frac{7}{4}$	37	
(-1.5)	$-\frac{3}{2}$	85	



<u>x-intercept (y=0)</u>

$0 = 8(3)^{4(x+2)} + 13$

$\frac{-13}{8} = \frac{8(3)^{4(x+2)}}{8}$

$-1.625 = (3)^{4(x+2)}$

Not Possible

<u>y-intercept (x=0)</u>

$y = 8(3)^{4(0+2)} + 13$

$y = 8(3)^8 + 13$

$y = 8(6561) + 13$

$y = 52488 + 13$

$y = 52501$

2. Radioactive carbon-14 has a half-life of 5750 years. When an organism dies, the amount of C-14 present decays exponentially. By measuring the radioactivity of the remains of a fossilized organism and comparing it with the radioactivity of a living organism archaeologists can approximate the age of the artifact. An antique dealer was selling a piece of wood purported to come from a chariot used by Caesar in ancient Rome. Archaeologists found the wood to contain 0.4 mg of C-14, compared with the 0.68 mg found in a new piece of wood. Caesar was assassinated 2048 years ago, could this dealer's claim possibly be true? Provide mathematical proof to back up your claim!

Given:

Base = 0.5

Initial Amount (A_0) = 0.68

Final Amount (A_F) = 0.4

Exponent = $\frac{x}{5750}$

$x = 2048$

Equation: $y = 0.68(0.5)^{\frac{x}{5750}}$

$y = 0.68(0.5)^{\frac{2048}{5750}}$

$y = 0.68(0.78)$

$\boxed{y = 0.53\text{ mg}}$

or

$0.4 = 0.68(0.5)^{x/5750}$

$0.5882 = (0.5)^{x/5750}$

$(0.5)^{0.7655} = (0.5)^{x/5750}$

$0.7655 = \frac{x}{5750}$

$\boxed{x = 4402\text{ years}}$

3. Solve each of the following:

3. Solve each of the following:

5750

$\boxed{x = 4402 \text{ years}}$

a) $64^{x-3} = (16)^{x-1}\left(\frac{1}{4}\right)^{2x}$

$(4^3)^{x-3} = (4^2)^{x-1}(4^{-1})^{2x}$

$4^{3x-9} = 4^{2x-2} \cdot 4^{-2x}$

$\cancel{4}^{3x-9} = \cancel{4}^{-2}$

$3x-9 = -2$

$3x = 7$

$\boxed{x = \frac{7}{3}}$

b) $\left(\frac{1}{27}\right)^{x+2} = (3)^{2x-1}(81)^{x}$

$(3^{-3})^{x+2} = (3)^{2x-1}(3^4)^{x}$

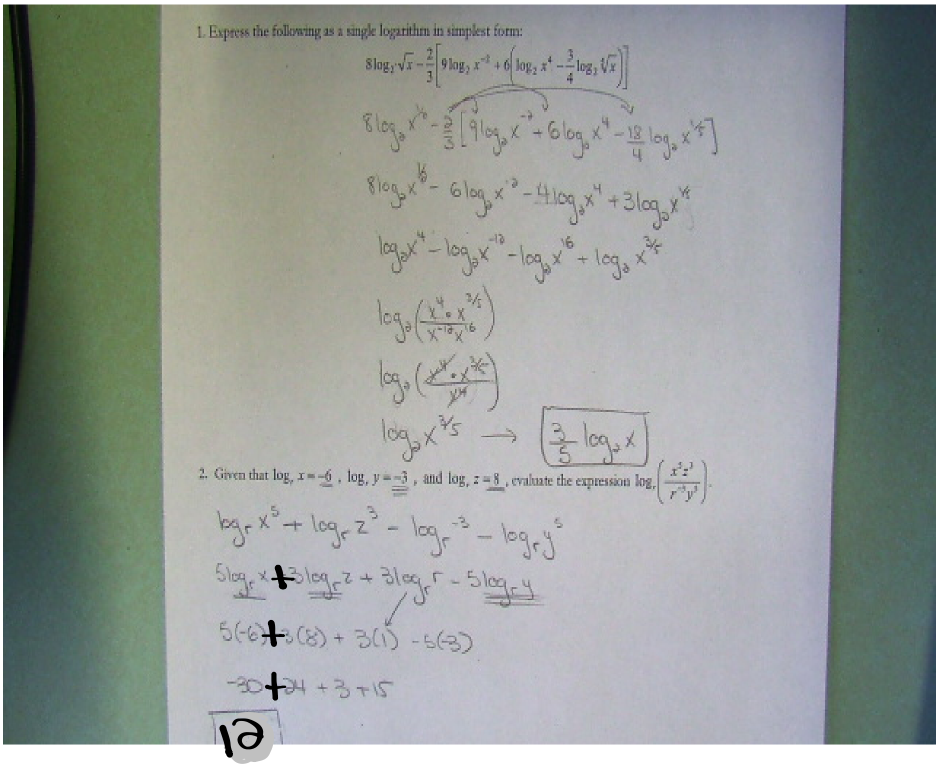
$3^{-3x-6} = 3^{2x-1} \cdot 3^{4x}$

$\cancel{3}^{-3x-6} = \cancel{3}^{6x-1}$

$-3x-6 = 6x-1$

$-5 = 9x$

$\boxed{\frac{-5}{9} = x}$



1. Express the following as a single logarithm in simplest form:

$$8\log_2\sqrt{x}-\frac{2}{3}\left[9\log_2 x^{-3}+6\left(\log_2 x^4-\frac{3}{4}\log_2\sqrt[5]{x}\right)\right]$$

$$8\log_2 x^{1/2}-\frac{2}{3}\left[9\log_2 x^{-3}+6\log_2 x^4-\frac{18}{4}\log_2 x^{1/5}\right]$$

$$8\log_2 x^{1/2}-6\log_2 x^{-2}-4\log_2 x^4+3\log_2 x^{1/5}$$

$$\log_2 x^4-\log_2 x^{-12}-\log_2 x^{16}+\log_2 x^{3/5}$$

$$\log_2\left(\frac{x^4\cdot x^{3/5}}{x^{-12}x^{16}}\right)$$

$$\log_2\left(\frac{\cancel{x^4}\cdot x^{3/5}}{\cancel{x^4}}\right)$$

$$\log_2 x^{3/5}\longrightarrow\boxed{\frac{3}{5}\log_2 x}$$

2. Given that $\log_r x=-6$, $\log_r y=-3$, and $\log_r z=8$, evaluate the expression $\log_r\left(\frac{x^5z^3}{r^{-3}y^5}\right)$.

$$\log_r x^5+\log_r z^3-\log_r r^{-3}-\log_r y^5$$

$$5\log_r x+3\log_r z+3\log_r r-5\log_r y$$

$$5(-6)+3(8)+3(1)-5(-3)$$

$$-30+24+3+15$$

$$\boxed{12}$$

3. Solve for x in the following equations…

$\log_3(2x^2 - x) - \log_3(x+2) = 1$

$\log_3\left(\frac{2x^2-x}{x+2}\right) = 1$

$3^1 = \frac{2x^2-x}{x+2}$

$3x+6 = 2x^2 - x$

$0 = 2x^2 - 4x - 6$

$0 = 2(x^2 - 2x - 3)$

$0 = 2(x-3)(x+1)$

$x-3=0$ | $x+1=0$

$\boxed{x=3}$ | $\boxed{x=-1}$

Test $x=3$

$\log_3(15) - \log_3(5) = 1$

$\log_3\left(\frac{15}{5}\right)$

$\log_3 3$

1

$x=3$ is a solution

Test $x=-1$

$\log_3(3) - \log_3(1) = 1$

$1 - 0$

1

$x=-1$ is a solution

4. Given the logarithmic function: $y-6=2\log_5(3x+15)$

(a) Express this function in standard form. $y=2\log_5[3(x+5)]+6$

$a=2 \quad b=3 \quad h=-5 \quad K=6$

(b) Complete the chart shown below.

Reflected in x-axis	YES or NO (circle correct solution)
Reflected in y-axis	YES or NO (circle correct solution)
Horizontal translation of...	5 units left
Vertical translation of...	6 units up
Horizontally stretched by a factor of...	$\frac{1}{3}$
Vertically stretched by a factor of ...	2
x-intercept (show work)	$x=\frac{-1874}{375}=-4.997$ or $(-4.997, 0)$
y-intercept (show work)	$y=9.37$ or $(0, 9.37)$
Horizontal Asymptote	$x=-5$
Domain	$\{x \mid x>-5, x\in R\}$
Range	$\{y \mid y\in R\}$

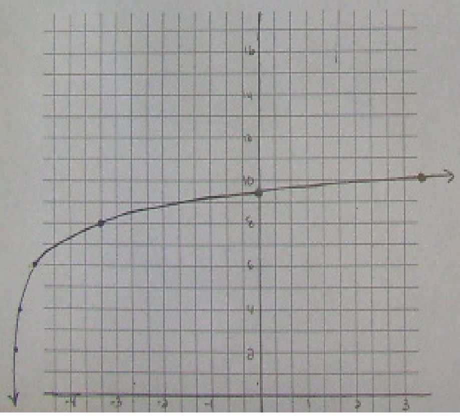
Write a mapping rule and sketch the curve in the space below.

$y = \log_5 x$

x	y
$\frac{1}{25}$	-2
$\frac{1}{5}$	-1
1	0
5	1
25	2

$(x, y) \rightarrow \left[\frac{1}{3}x - 5,\ 2y + 6\right]$

x		y
(-4.99)	$-374/75$	2
(-4.93)	$-74/15$	4
$(-4.\bar{6})$	$-14/3$	6
(-3.3)	$-10/3$	8
(3.3)	$10/3$	10



x-intercept (y=0)

$0 = 2\log_5[3(x+5)] + 6$

$-6 = 2\log_5[3(x+5)]$

$-3 = \log_5[3(x+5)]$

$5^{-3} = 3(x+5)$

$\frac{1}{125} \div 3 = \frac{3(x+5)}{3}$

$\frac{1}{375} = x + 5$

$\boxed{-\frac{1874}{375} = x}$

-4.99

y-intercept (x=0)

$y = 2\log_5[3(0+5)] + 6$

$y = 2\log_5(15) + 6$

$y = 2(1.683) + 6$

$y = 3.365 + 6$

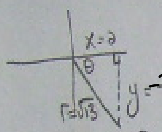
$\boxed{y = 9.37}$

Pre-Calculus 12A
Special Angles Exam Review

$\cos\theta > 0$ and $\sin\theta < 0$ Q4

1. Given that $\cos\theta = \frac{2}{\sqrt{13}}$ and $\sin\theta < 0$. Sketch the angle and determine the five remaining trig ratios ***as radicals in simplest form.***

Given: $x = 2$, $r = \sqrt{13}$



$x^2 + y^2 = r^2$
$2^2 + y^2 = \sqrt{13}^2$
$4 + y^2 = 13$
$y^2 = 9$
$y = \pm 3$
$\boxed{y = -3}$

$\sin\theta = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$

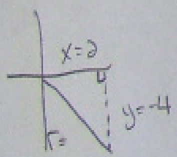
$\tan\theta = \frac{-3}{2}$

$\sec\theta = \frac{\sqrt{13}}{2}$

$\csc\theta = -\frac{\sqrt{13}}{3}$

$\cot\theta = -\frac{2}{3}$

2. The point (2, -4) lies on the terminal arm of an angle. Make a sketch of this angle and determine the 6 trigonometric ratios expressed ***as radicals in simplest form.***



$x^2 + y^2 = r^2$
$2^2 + (-4)^2 = r^2$
$4 + 16 = r^2$
$20 = r^2$
$\sqrt{20} = r$
$2\sqrt{5} = r$

$\sin\theta = \frac{-4}{2\sqrt{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

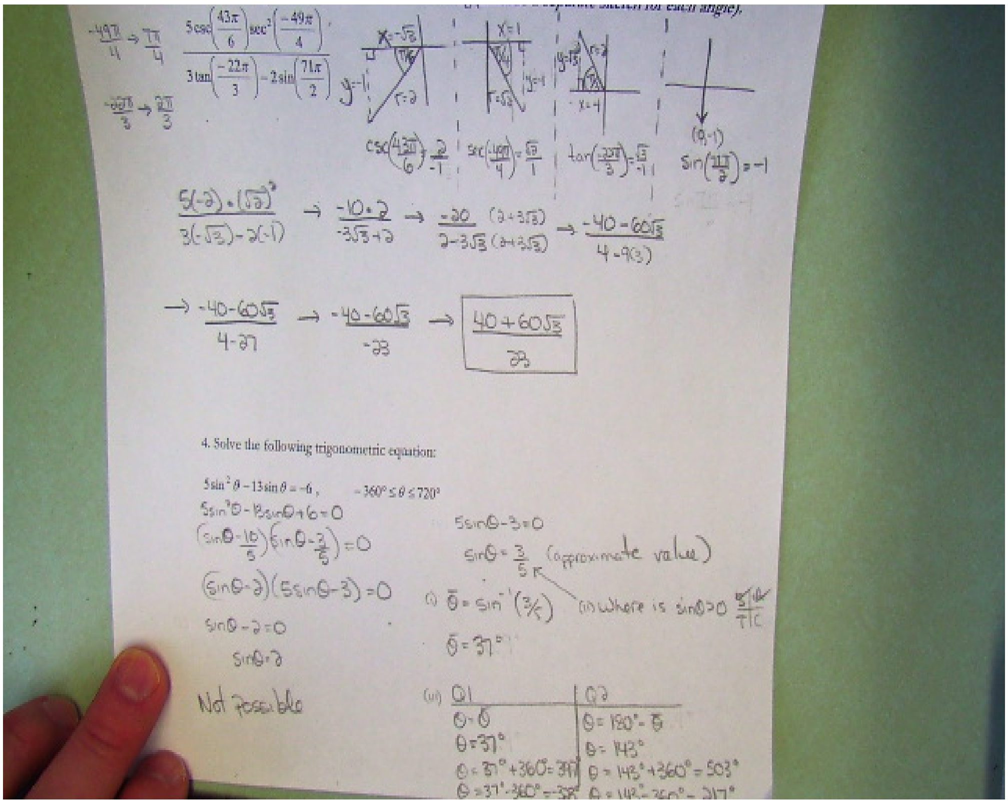
$\cos\theta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\tan\theta = \frac{-4}{2} = -2$

$\csc\theta = -\frac{2\sqrt{5}}{4} = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{2\sqrt{5}}{2} = \sqrt{5}$

$\cot\theta = \frac{2}{-4} = -\frac{1}{2}$



$-\frac{49\pi}{4} \to \frac{7\pi}{4}$

$-\frac{22\pi}{3} \to \frac{2\pi}{3}$

$$\frac{5\csc\left(\frac{43\pi}{6}\right)\sec^2\left(\frac{-49\pi}{4}\right)}{3\tan\left(\frac{-22\pi}{3}\right)-2\sin\left(\frac{71\pi}{2}\right)}$$

$x=-\sqrt{3}$, $y=-1$, $r=2$ | $x=1$, $y=-1$, $r=\sqrt{2}$ | $y=\sqrt{3}$, $x=-1$, $r=2$ | $(0,-1)$

$\csc\left(\frac{43\pi}{6}\right)=\frac{2}{-1}$ | $\sec\left(\frac{-49\pi}{4}\right)=\frac{\sqrt{2}}{1}$ | $\tan\left(\frac{-22\pi}{3}\right)=\frac{\sqrt{3}}{-1}$ | $\sin\left(\frac{71\pi}{2}\right)=-1$

$$\frac{5(-2)\cdot(\sqrt{2})^2}{3(-\sqrt{3})-2(-1)} \to \frac{-10\cdot 2}{-3\sqrt{3}+2} \to \frac{-20}{2-3\sqrt{3}}\,\frac{(2+3\sqrt{3})}{(2+3\sqrt{3})} \to \frac{-40-60\sqrt{3}}{4-9(3)}$$

$$\to \frac{-40-60\sqrt{3}}{4-27} \to \frac{-40-60\sqrt{3}}{-23} \to \boxed{\frac{40+60\sqrt{3}}{23}}$$

4. Solve the following trigonometric equation:

$5\sin^2\theta - 13\sin\theta = -6, \qquad -360° \le \theta \le 720°$

$5\sin^2\theta - 13\sin\theta + 6 = 0$

$\left(\sin\theta - \frac{10}{5}\right)\left(\sin\theta - \frac{3}{5}\right) = 0$

$(\sin\theta - 2)(5\sin\theta - 3) = 0$

$\sin\theta - 2 = 0$

$\sin\theta = 2$

Not Possible

$5\sin\theta - 3 = 0$

$\sin\theta = \frac{3}{5}$ (approximate value)

(i) $\bar{\theta} = \sin^{-1}(3/5)$ (ii) Where is $\sin\theta > 0$ $\frac{S\,|\,A}{T\,|\,C}$

$\bar{\theta} = 37°$

(iii)

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 37°$	$\theta = 143°$
$\theta = 37° + 360° = 397°$	$\theta = 143° + 360° = 503°$
$\theta = 37° - 360° = -323°$	$\theta = 143° - 360° = -217°$

Sinusoidal Functions Exam Review

1 rev. in 24 sec radius = 15

1. A Ferris wheel completes 2 revolutions in 48 seconds and has a diameter of 30 m. If the bottom of the wheel is 2 m above the ground. When a stopwatch is started you notice that your friend is seated at the middle of the wheel and is going up. $y = \sin x$

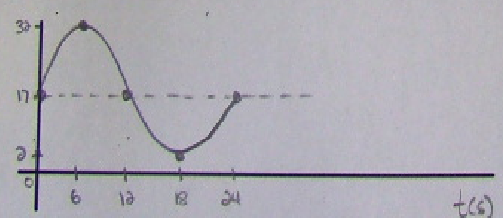
(a) Find the following:

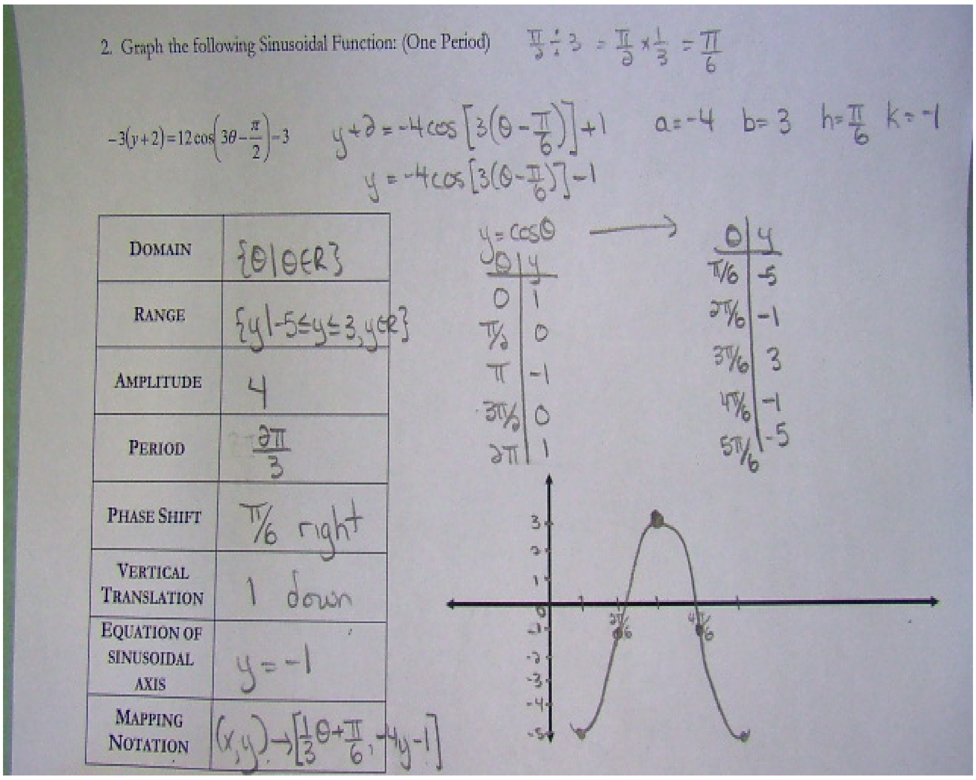
a: ± 15 P: 24 b: $\frac{360}{24} = 15$ min height: 2m max height: 32 m (min + diameter) k: 17m (min + radius)

(b) What is the equation of the graph?

$y = 15\sin[15(x)] + 17$

(c) Sketch the graph for one period.





2. Graph the following Sinusoidal Function: (One Period) $\frac{\pi}{2} \div 3 = \frac{\pi}{2} \times \frac{1}{3} = \frac{\pi}{6}$

$-3(y+2)=12\cos\left(3\theta-\frac{\pi}{2}\right)-3$ $y+2=-4\cos\left[3\left(\theta-\frac{\pi}{6}\right)\right]+1$ $a=-4$ $b=3$ $h=\frac{\pi}{6}$ $k=-1$

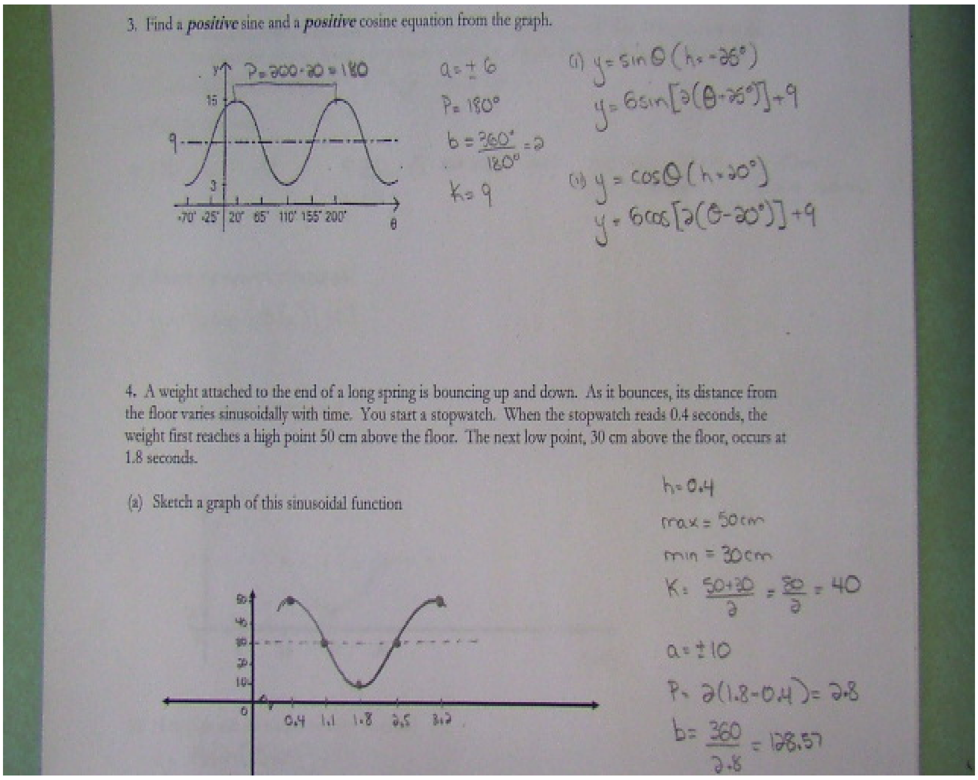
$y=-4\cos\left[3\left(\theta-\frac{\pi}{6}\right)\right]-1$

DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid -5 \le y \le 3, y \in R\}$
AMPLITUDE	4
PERIOD	$\frac{2\pi}{3}$
PHASE SHIFT	$\pi/6$ right
VERTICAL TRANSLATION	1 down
EQUATION OF SINUSOIDAL AXIS	$y=-1$
MAPPING NOTATION	$(x,y) \to \left[\frac{1}{3}\theta+\frac{\pi}{6}, -4y-1\right]$

$y=\cos\theta$ ⟶

θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

θ	y
$\pi/6$	-5
$2\pi/6$	-1
$3\pi/6$	3
$4\pi/6$	-1
$5\pi/6$	-5



3. Find a ***positive*** sine and a ***positive*** cosine equation from the graph.

$a = \pm 6$

$P = 180°$

$b = \frac{360°}{180°} = 2$

$k = 9$

(i) $y = \sin\theta \; (h = -25°)$

$y = 6\sin[2(\theta + 25°)] + 9$

(ii) $y = \cos\theta \; (h = 20°)$

$y = 6\cos[2(\theta - 20°)] + 9$

4. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.4 seconds, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 seconds.

(a) Sketch a graph of this sinusoidal function

$h = 0.4$

$max = 50cm$

$min = 30cm$

$K = \frac{50+30}{2} = \frac{80}{2} = 40$

$a = \pm 10$

$P = 2(1.8 - 0.4) = 2.8$

$b = \frac{360}{2.8} = 128.57$

(b) Write an equation to define the graph.

$y = 10\cos[128.57(x-0.4)]+40$

(c) What was the distance from the floor when you started the stopwatch? $(x=0)$

$y = 10\cos[128.57(0-0.4)]+40$

$\boxed{y = 46.2\text{cm}}$

Extra questions worked out

$0 \le \theta \le 2\pi$

⑫ a) $\sin\theta = \sin\theta\tan\theta$

Common Factor

$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

$\sin\theta = 0$	$\tan\theta - 1 = 0$
$\theta = 0, \pi, 2\pi$	$\tan\theta = 1$
	$\theta_R = \frac{\pi}{4}$

$\sqrt{2}$, $\frac{\pi}{4}$, 1, 1

S | A
T | C

(0,1), (-1,0), (1,0), (0,-1)

Where is $\tan\theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

⑬ b) $3\sin^2\theta - 2\sin\theta - 1 = 0$, $0 \le \theta \le 360°$

$(3\sin^2\theta - 3\sin\theta)(+\sin\theta - 1) = 0$

$\frac{-3}{} \times \frac{1}{} = -3$

$\frac{-3}{} + \frac{1}{} = -2$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$3\sin\theta + 1 = 0$	$\sin\theta - 1 = 0$
$\sin\theta = -\frac{1}{3}$	$\sin\theta = 1$
$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$	$\theta = 90°$
$\theta_R = 19$	

Unit Circle

Where is sine negative:

S | A
T | C

Q3	Q4
$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 180° + 19$	$\theta = 360° - 19$
$\theta = 199$	$\theta = 340$

① Amp = 11   P = 16   min = -4

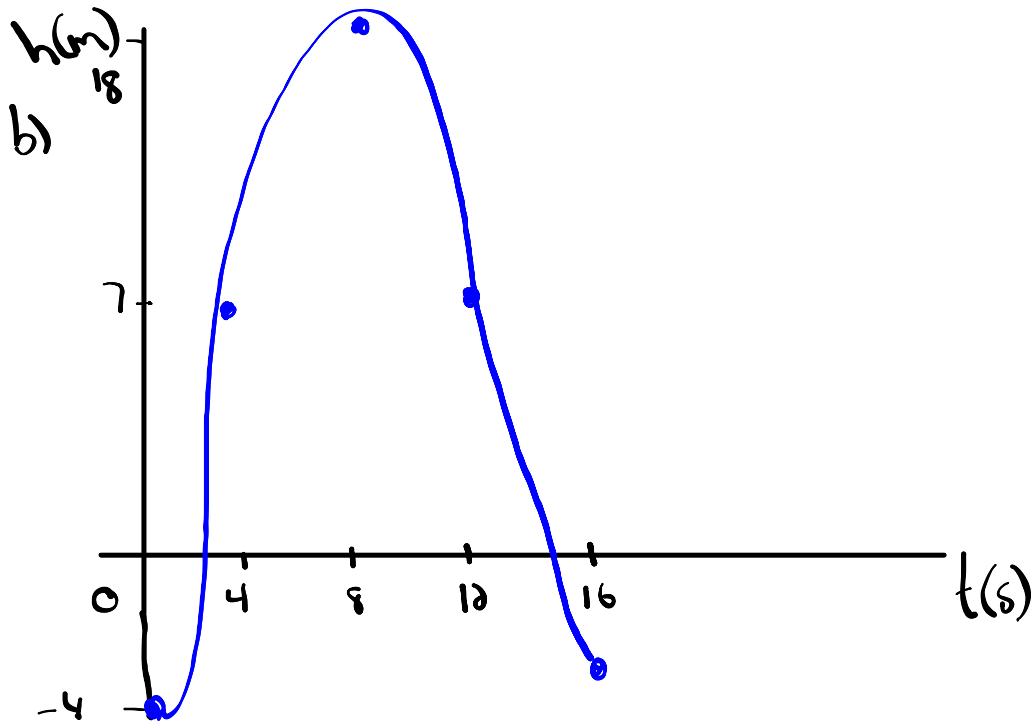
a = ± 11 $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

K = -4 + 11 = 7

h = 0

a) equation: $y = -11\cos[22.5(x)] + 7$

b)



1

4)

max $= 68$

min $= 24$

$K = \dfrac{68+24}{2} = 46$

$Amp = 68 - 46 = 22$

$a = \pm 22$

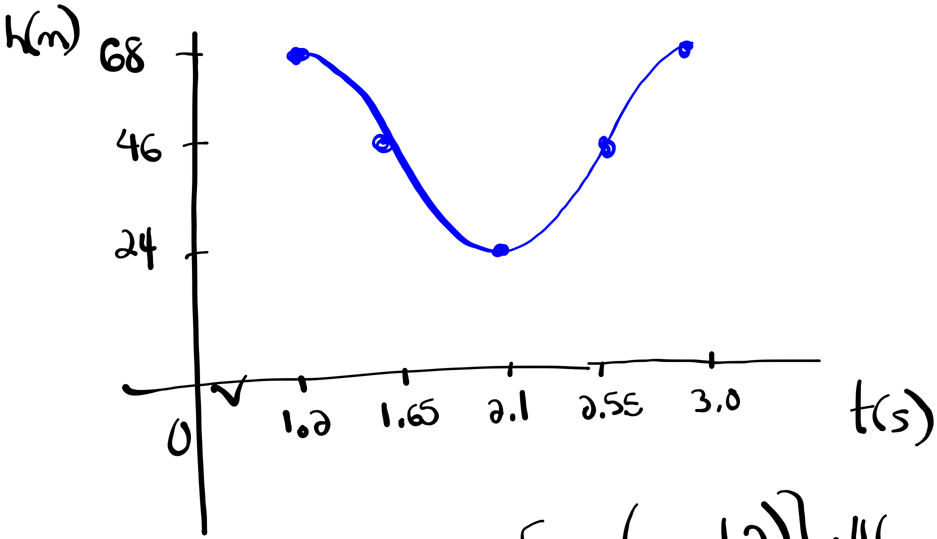
$P = 2(2.1 - 1.2)$

$P = 1.8$

$b = \dfrac{360}{1.8} = 200$

$h = 1.2$

$\dfrac{P}{4} = \dfrac{1.8}{4} = 0.45$



$y = 22\cos[200(x - 1.2)] + 46$

⑤ c) $y = \frac{1}{2}\cos(1\theta + \underline{\underline{\pi}}) - \underline{\underline{4}}$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

$(x, y) \longrightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$\longrightarrow$

x	y	
$-\pi$	$-7/2$	-3.5
$-\frac{\pi}{2}$	-4	-4
0	$-9/2$	-4.5
$\pi/2$	-4	-4
π	$-7/2$	-3.5